Minimax estimation of the mixing density of a mixture of exponential distributions

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We consider the problem of estimating a mixing density f from i.i.d. observations distributed according to a mixture density $\pi_f(x) = \int_\Theta f(t)\pi_t(x)\mathrm{d}t$, where $\{\pi_t, t \in \Theta\}$ is a parametric collection of densities. In such models, in contrast with finite mixtures models, the distribution of the hidden variable is not bounded to a finite set but is spread out, say, over a given interval. A maximum likelihood method can be applied in this context but it yields a discrete mixing distribution, see [1]. Other approaches rely on smoothness assumptions on the mixing densities f. In [2], some kernel estimators are constructed for mixing densities on a location parameter. For mixtures of discrete distributions, i.e. π_t are densities with respect to the counting measure on a discrete space, orthogonal series estimators have been developed and studied in [3].
In this work, we propose orthogonal series estimators for densities π_t with respect to the Lebesgue measure on the positive half line, where t denotes a scale parameter. That is, we construct an estimator based on an orthonormal sequence $(\psi_k)_k$ by estimating the first m coefficients α_k of the expansion $f = \sum_k \alpha_k\psi_k$. We derive minimax upper and lower bounds of the mean integrated squared error which apply in various contexts. The case of exponential densities $\pi_t(x) = t\mathrm{e}^{-tx}$ is specifically studied. It is shown that in this case the orthogonal series estimator achieves the minimax rate.

References

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