A GRAPH-CUT BASED ALGORITHM FOR APPROXIMATE MRF OPTIMIZATION

Aymen Shabou, Florence Tupin

Institut TELECOM, TELECOM ParisTech
CNRS LTCI, France.

Jérôme Darbon

Department of Mathematics
UCLA, Los Angeles, USA.

ABSTRACT

This paper copes with the approximate minimization of Markovian energy with pairwise interactions. We extend previous approaches that rely on graph-cuts and move making techniques. For this purpose, a new move is introduced that permits us to perform better approximate optimizations. Some experiments show that very good local minima are obtained while keeping the memory usage low.

Index Terms— Optimization methods, Graph-cut, Image restoration.

1. INTRODUCTION

Many vision problems can be formulated as an energy minimization that arises from a Markov Random Field (MRF) formulation [1]. This Markovian approach has been proven to be extremely successful for many vision applications such as stereo, image segmentation, image denoising, and others [2]. Optimizing these energies is generally a difficult problem and we propose an approximate optimization algorithm for a subclass of Markovian energies.

Let us introduce more formally this approach. It is assumed that images are defined on a lattice denoted by $\mathcal{V}$. The value of the image x at the site p is referred to as x_p and takes value in a linearly ordered finite set of labels $\mathcal{L}$, with $\mathcal{L} = \{l_1, l_2, ..., l_k\}$. We endow the lattice with a neighborhood system and pairwise interactions are considered. Two sites p and q that are in interaction with each other are denoted by (p, q). The set of all considered pairwise interactions is referred to as $\mathcal{E}$. A first order Markovian energy is therefore defined as the following:

$$E(x) = \sum_{p \in \mathcal{V}} E_p(x_p) + \sum_{(p,q) \in \mathcal{E}} E_{p,q}(x_p - x_q) \ , \qquad (1)$$

Terms E_p encode the likelihood (i.e., measure the distance of the reconstruction to the observed data) while $E_{p,q}$ correspond to the prior we have on the interactions. In this paper, we only consider priors that are a function of the difference of the labels (see [3] for instance for the general case).

Minimization of an arbitrary Markovian energy of the form of (1) is a difficult problem. Some approaches were proposed to minimize exactly some specific energy functions [4, 5, 3]: more precisely it corresponds to a sub-class of submodular functions. These approaches allow for a global optimization. The idea is to map the original minimization into a s-t minimum-cut (and by duality, a maximum-flow) problem. This approach has been proposed in [6, 7] for minimizing Boolean energies and have been shown to be very efficient in [6]. Extensions of this approach to the case of linearly ordered labels have been proposed for instance in [4, 8, 3]. However, the latter requires to build a huge graph that corresponds to a prohibitive use of memory for practical applications. Thus, several approaches have been proposed to perform approximate minimizations [2], such as iterated conditional modes, belief propagation, move making algorithms [9, 10], etc. that require much less memory. Note that these algorithms are essentially iterative.

Developing graph-based minimization algorithms that provide good approximate solutions while maintaining a low memory requirement is thus a challenge for practical applications. This is the aim of this paper. More precisely, we are interested in optimization algorithms based on the concept of partition move, originally proposed in [9], that allows many pixels to change their values in a single step. Contrary to [9] that essentially limits the number of choices for each pixel to two, we propose a move that allows pixels to take their values in a much larger set. This bigger searching space yields to compute better local minima (in terms of the value of the energy). Experiments show that usually we are able to get the global minimum of the energy with no need of high memory such as exact optimization does.

The remainder of this paper is as follows. In section 2, we briefly review move making algorithms available in the literature and we present our partition move. Section 3 describes an algorithm that compute the new partition move and we discuss its main properties. Finally some experiments for image processing purposes are presented in section 4.

2. APPROXIMATE OPTIMIZATION

Let us consider the family of move making algorithms [9]. A move simply consists of a change of a current labeling. A move is called large if many pixels change their configurations simultaneously and it is said optimal if it yields the

largest possible decrease of the total energy among the possible changes allowed by the move.

The most two well-known moves are the α-expansion and $\alpha\beta$-swap as described in [6]. These two algorithms are based on binary moves since each pixel is proposed to take values among two labels in each step of the minimization process. In fact, in an α-expansion move, a pixel can keep its current label or change it to the label α. In an $\alpha\beta$-swap move, a pixel that takes value in $\{\alpha, \beta\}$ can exchange its label within this same set. Both algorithms perform binary large optimal moves until convergence to a local optimum.

A binary move exploits a limited number of labels among those in $\mathcal{L}$. Thus, proposing more labels to a pixel in a single move, may yield a better energy minimization result. Hence, the idea is to use a large subset of labels $\mathcal{L}_m$ in $\mathcal{L}$, where m denotes the cardinality of the chosen subset. We call such a move a *multilabel* move.

Using this new move, we propose an approximate optimization algorithm that is iterative and deterministic. It can be seen as an intermediate algorithm between binary move algorithms and exact optimization ones.

It is worth noticing that several different kinds of moves can be considered, such as expansion, swap and jump (see [9] for detailed presentations of such moves in their binary form and also [10]). In this paper, we only consider expansion-based moves. We define an α_m-expansion move as a multilabel move that proposes to each pixel either to keep its current label or to change it by a label in the subset $\mathcal{L}_m$. We refer to this kind of moves as Large and Multilabel Partition Move (LMPM).

3. LMPM OPTIMIZATION

We first present the LMPM algorithm before describing a graph construction whose minimum-cut defines an optimal multilabel move. Some properties of our algorithm are given.

LMPM algorithm. Our algorithm is a generalized form of binary move algorithms (see [6]). This algorithm is iterative. For each iteration (i), a set of labels $\mathcal{L}_m^{(i)}$ of cardinality m is considered. Given $\mathcal{L}_m^{(i)}$ and a current labelling, the optimal $\mathcal{L}_m^{(i)}$-expansion move that produces the largest decrease of the total energy is computed. Once this move is computed, we get another labelling and we iterate until convergence, i.e., no move can produce a decrease of the energy.

Note that we have not specified the label set $\mathcal{L}_m^{(i)}$ for each iteration i . One desirable property is the following: For any current iterate, there is always a consecutive finite series of label sets that spans the original label set $\mathcal{L}$. This assumption guaranties that all labels can be reached through the iterations of the algorithm.

We shall see in the experiments section different choices for defining the label sets $\mathcal{L}_m^{(i)}$. The main characteristic is that they can be generated randomly or chosen deterministically.

Let us now describe how an optimal multilabel move can be computed using a graph-based approach.

Graph construction. We present here a graph construction such that its s-t minimum-cut yields an optimal multilabel move. The graph proposed here is similar to the one described by Ishikawa in [4] for performing global optimization in the case of convex priors.

Recall that our goal consists in finding the optimal expansion move with respect to a subset $\mathcal{L}_m$. In other words, we need to compute a global minimizer where pixels either keep their own values or pick one in a subset of $\mathcal{L}$.

For each iteration, we consider the current labelling x and the proposed expansion set $\mathcal{L}_m$. For each pixel p, we associate its possible sets of labels $\mathcal{L}_m(p)$ defined as follows $\mathcal{L}_m(p) = \mathcal{L}_m \cup \{x_p\}$. We can then apply the graph construction of Ishikawa [4] by replacing the set $\mathcal{L}$ by the set $\mathcal{L}_m(p)$ for each pixel. The graph is layer-based. It is defined as follows: there is a node associated to each p for any label in $\mathcal{L}_m(p)$. Concerning the edges, they are defined in the same way as in [4]. More precisely, three families of edges are created. First, the set of data edges that corresponds to encode the data fidelity terms. Capacities on these edges are defined using the likelihood energy function. Second, a set of constraints edges that guarantees a pixel p to be assigned to a unique element in $\mathcal{L}_m(p)$. Capacities on these edges are set to infinity. Finally, a set of edges that encodes the regularization. An analysis similar to the one conducted in [4, 3] permits to define capacities on these edges and to show that the convexity of the regularization terms $E_{p,q}(\cdot)$ is a sufficient and necessary condition to have non-negative capacities. The latter means that an optimal solution can be computed in polynomial time by computing a maximum-flow on the graph. Figure 1 depicts an example of our graph construction.

LMPM properties. The first property presents the advantage of making a multilabel move with larger label set. In fact, lets's start with an initial labeling x. It is clear that if we make a same multilabel move (an expansion move for instance) but with different label sets $\mathcal{L}_m$ and $\mathcal{L}_{m'}$ such that $\mathcal{L}_m \subset \mathcal{L}_{m'}$, then the energy of resulted labelings is minimal for the move with the largest label set. This could be seen immediately since the set of optima reached by the move with less labels is included in the set of the other move optima.

Note that if the above inclusions hold at every step, then multilabel large move guaranties a better energy minimum than the one obtained with binary moves. We have also verified experimentally that the global optimum is reached when one is using large label sets. This is a crucial point when dealing with high dimension images, where convergence to global optima, using an exact optimization algorithm, is a high costly memory approach.

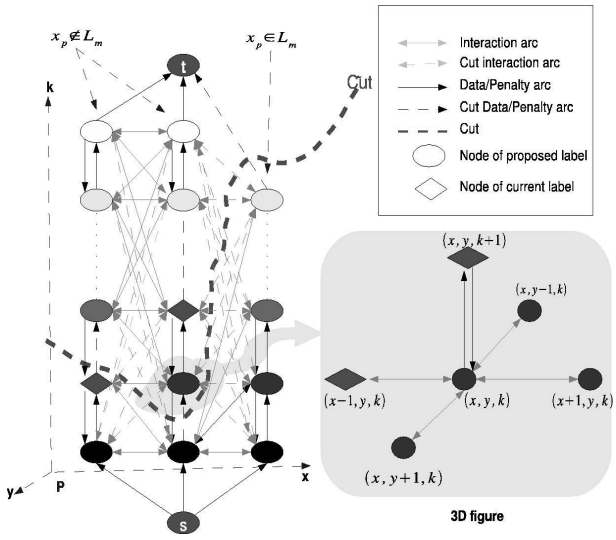

Fig. 1. Graph construction for an optimal multilabel move. On the left, a part of the graph $\mathcal{G}_m$ defined on three pixels. A cut is depicted and arcs are in the cut are dotted whereas continuous ones are not. A part of the graph is highlighted on the right. We distinguish the three arc families: data edges (vertical edges oriented to the top), constraint edges (verticals edges oriented to the buttom) and penalty edges connecting all label nodes of two 4-connexity neighboring pixels.

4. EXPERIMENTS

The efficiency of the proposed approach in term of both optimum quality and memory saving is illustrated for two image processing problems, image restoration and phase unwrapping. For both problems, highly noisy data are considered with specific likelihood energy functions. Results are compared to those obtained using binary partition move (α-expansion) and exact optimization algorithms.

Image restoration. Image used in this experiment is a natural image corrupted by an impulsive noise. This kind of noise is known to be highly destructive. In fact, if a pixel is noisy, we lost information that it carries, as we see in figure 2(b). And, we consider a discrete total variation (DTV) prior energy [8] for the MRF with neighborhood system, the grid of 4-nearest neighbors. The set of all labels is the finite set of integers $\mathcal{L} = \{1..256\}$ (gray levels of pixels). In this experiment, we apply a random perumutation on the label set $\mathcal{L} = \{l_1, ..., l_k\}$. Then, the iterative energy minimization algorithm (α_m-expansion) is performed. As noted in the section describing the LMPM algorithm, the set $\mathcal{L}_m$ consists of firstly selecting the first m values of the set $\mathcal{L}$ obtained after the permutation, and we order labels inside the set. Then, we select the next m values, and so on. Once all the values have been visited, we start the process again until convergence of minimization algorithm.

Results are presented in figure 2. As we see, by using a multilabel expansion move with label sets of size 8 or 32, we obtain better results than the α-expansion based optimization algorithm. It is also important to note the convergence to a local optimum of the energy, which is very close to the global one, only with 32 labels. This is a real gain of memory use to reach results as good as global optimization provides since the graph used with the proposed approach is 4 times smaller than the graph required by an exact optimization algorithm.

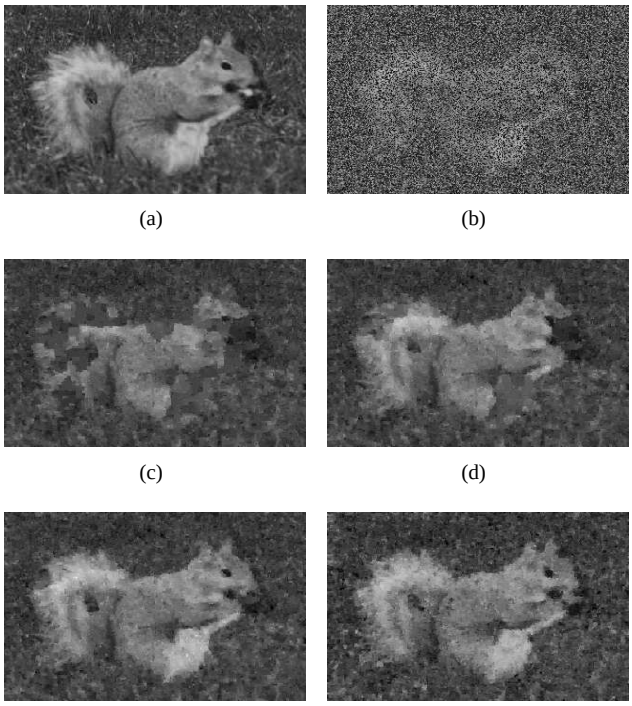

Fig. 2. Restoration results based on α_m-expansion optimization algorithm with different sizes of label set $\mathcal{L}_m$.(a) Original image, (b) Noisy image (70% of pixels are corrupted by the noise) and restorations with (c) α-expansion, (d) α_8-expansion, (e) α_{32}-expansion and (f) exact optimization algorithms.

Considering time running, we note that time needed to compute an optimal move depends on the size of required graph and the number of iterations to converge. For instance, times (in seconds) spent by algorithms in this experiment are 95, 622, 937 and 230 to reach results obtained respectively in 2(c), 2(d), 2(e) and 2(f). For exact minimization, the algorithm takes less time than ours since only one iteration is executed. Compared, for example, to the α_{32}-expansion, the latter took 16 iterations.

Phase unwrapping. In this section, we illustrate the need of such approaches in case of radar image processing, more precisely, for generating digital elevation models (DEM) of earth surface. In this application, data used are images of interferometric phases, were radiometric information is given in the

principal interval [-π, π]. This data is called wrapped phase. In order to restore the relation between interferometric phase and ground height, necessary to generate the DEM, we need to unwrap the phase (i.e. to know the phase in its absolute values). The unwrapping operation is not an easy task. We clearly see, for instance, if the absolute value of phase difference between neighboring pixels is greater than π, the phase unwrapping operation becomes an ill-posed problem. In this case, one possible approach to solve the ill-posed problem is the multichannel phase unwrapping (MCPU) [11] technique by exploiting the availability of different and independent interferograms referred to the same scene. In order to combine these different available channels, a statistical approach with Maximum a Posteriori (MAP) estimation is used. The MAP MCPU problem can thus be seen as an energy minimization problem. We can choose, for the *a priori*, the DTV model.

We simulate height profile data with two frequencies (5GHz and 9GHz) for interferogram generation and we add interferometric noise with a coherence of $\gamma = 0.7$. For each working frequency, we generated 4 azimuth looks leading to a total of 8 independent interferograms. The set of all labels is the finite set of integers $\mathcal{L} = \{1..128\}$. Similarly to the previous experiment, label sets $\mathcal{L}_m$ are selected in the same way, however, no permutation is performed on the original label set $\mathcal{L}$.

In figure 3, phase unwrapping results with expansion move based optimization algorithm made with different sizes of label sets are presented. Results prove the contribution of our algorithm to better minimizing the energy function. We note also in this experiment that the global optimum is reached while using the $1/4^{th}$ of memory needed by an exact minimization algorithm (32 labels instead of 128 labels).

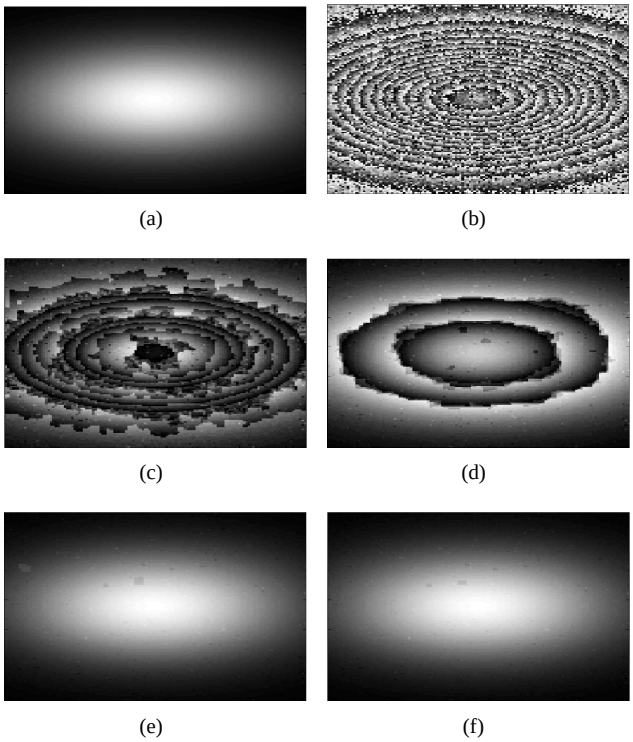

Fig. 3. MCPU results based on α_m-expansion optimization algorithm with different sizes of label set $\mathcal{L}_m$. (a) Original profile, (b) 5GHz noisy interferogram and phase unwrapping results with (c) α-expansion, (d) α_8-expansion, (e) α_{32}-expansion and (f) exact optimization algorithms.

5. CONCLUSION

In this work, a new kind of move is proposed that leads to better minimizing a pairwise Markovian energy than binary large moves do, while consumming much less memory needed by exact minimization. Convincing experimental results are provided showing as the real need of such approaches in specific image processing problems. Further works will be made in this direction to more improving energy optima based on stochastic search combined with these deterministic optimization algorithms.

6. REFERENCES

[1] D. Geman and S. Geman, "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images," *PAMI*, vol. 6, pp. 721–741, 1984.

[2] R. Szeliski, R. Zabih, D. Scharstein, O. Veksler, V. Kolmogorov, A. Agarwala, M. Tappen, and C. Rother, "A comparative study of energy minimization methods for markov random fields with smoothness-based priors," *PAMI.*, vol. 30, no. 6, pp. 1068–1080, 2008.

[3] J. Darbon, "Global optimization for first order Markov random fields with submodular priors," in *IWCIA*, 2008, pp. 229–237.

[4] H. Ishikawa, "Exact optimization for Markov random fields with convex priors," *PAMI*, vol. 25, no. 10, pp. 1333–1336, October 2003.

[5] V. Kolmogorov and R. Zabih, "What energy functions can be minimized via graph cuts?," *PAMI*, vol. 26, no. 2, pp. 147–159, 2004.

[6] Y. Boykov, O. Veksler, and R. Zabih, "Fast approximate energy minimization via graph cuts," *PAMI*, vol. 23, no. 11, pp. 1222–1239, 2001.

[7] J.P. Picard and H.D. Ratlif, "Minimum cuts and related problem," *Networks*, vol. 5, pp. 357–370, 1975.

[8] J. Darbon and M. Sigelle, "Image restoration with discrete constrained Total Variation part II: Levelable functions, convex priors and non-convex cases," *JMIV*, vol. 26, no. 3, pp. 277–291, December 2006.

[9] O. Veksler, *Efficient Graph-Based Energy Minimization Methods in Computer Vision*, Ph.D. thesis, Cornell University, 1999.

[10] O. Veksler, "Graph cut based optimization for mrfs with truncated convex priors," in *IEEE CVPR*, 2007.

[11] G. Ferraiuolo, V. Pascazio, and G. Schirinzi, "Maximum a posteriori estimation of height profiles in InSAR imaging," *IEEE Geoscience and Remote Sensing Letters*, vol. 1, pp. 66–70, april 2004.