Population Games for Cognitive Radios: Evolution through Imitation

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May 9th, 2011

Outlines

- Introduction: evolutionary game theory and population games
- Population games applied to CR: model and assumptions
  - Simple and double imitation **w/o** channel constraint
  - Simple and double imitation **with** channel constraint
- Conclusion and future work

Evolutionary games overview

- Evolutionary games formalism is a central mathematical tool developed by biologists for predicting populations dynamics in the context of interactions between populations.
- All players in a population are programmed to use strategies
- Strategies with high payoff will spread within the population. This can be achieved by learning, copying or inheriting strategies.
- The payoff depends on the frequency of the strategies within the population. Since this frequencies change according to the payoffs, this yields a feedback loop.

Population Games

- Population games are a particular class of evolutionary games
- They model strategic interactions in which [1]
  - Population is large
  - The number of strategies is finite
  - Agents interact at random (e.g. pairwise)
  - Payoffs are continuous

Population Games

Definitions

- Strategies: $\mathcal{S} = \{1, ..., n\}$
- Population states: $X = \{x \in R^n_+, \sum_{i=1}^n x_i = 1\}$
- Payoff function: $\pi : X \to R^n$ assigns to each state a payoff vector
- Payoff component for strategy i: $\pi_i : X \to R$
- Average payoff in state x: $\bar{\pi}(x) = \sum_i x_i \pi_i(x)$

Population Games

Evolutionary game dynamics

- Players play mixed strategies
- Players update their strategy according to their environment
- The updating process is called a **revision protocol**:
  Let $\rho_{i,j} : R^n \times X \to R_+$ be the switch rate from i to j
  i.e., $\rho_{i,j} dt$ is the probability for a player to switch from i to j in dt.
- The revision protocol generates a **system dynamic** (Kolmogorov):

$$x_i(t + dt) = x_i(t) + \sum_{j \in \mathcal{S}} x_j(t) \rho_{j,i} dt - x_i(t) \sum_{j \in \mathcal{S}} \rho_{i,j} dt$$

$$\dot{x}_i(t) = \sum_{j \in \mathcal{S}} x_j(t) \rho_{j,i} - x_i(t) \sum_{j \in \mathcal{S}} \rho_{i,j}$$

Population Games

Example of revision protocol: the proportional imitation rule (PIR)

- A revising player adopting strategy i picks at random an opponent
- It observes its current strategy j and payoff π_j
- And switches to j iff $\pi_j > \pi_i$ with a probability proportional to the payoff difference
- The switch rate is: $\rho_{i,j} = x_j \sigma [\pi_j - \pi_i]_+$ (σ is a constant)
- The resulting system dynamic is the **replicator dynamic** [2]:

$$\dot{x}_i(t) = \sigma x_i(t)(\pi_i(t) - \bar{\pi}(t))$$

Population Games

Another revision protocol: the double imitation (DI)

- A revising player adopting strategy i picks at random two opponents
- It observes their current strategy j_1 and j_2 and payoffs π_{j_1} and π_{j_2}
- The switch rate is a function of π_i, π_{j_1}, π_{j_2} and two control parameters (α, ω)
- The system dynamic is the **aggregate monotone dynamic** [2]:

$$\dot{x}_i(t) = \frac{x_i(t)}{\omega - \alpha} \left[1 + \frac{\omega - \bar{\pi}(t)}{\omega - \alpha} \right] (\pi_i(t) - \bar{\pi}(t))$$

Population Games applied to CR

Game G and cognitive radio (CR) scenario

- There are N SUs and C channels, each with availability $\mu_i \in [0, 1]$
- Strategies: $\mathcal{S} = \{1, ..., C\}$
- Throughput on channel i (normalized): RV T_i
- Payoffs: expectation of the normalized throughput

Population Games applied to CR: simplified model

Model assumptions:

- Imitation can be performed across channels
- Generic MAC protocol. The throughput of the SUs on the same channel i is defined as:

$$\pi_i(\mu_i, x_i) = E[T_i] \approx \mu_i / n_i = \mu_i / (x_i N)$$

- Payoffs are obtained at the end of each iteration without errors

Population Games applied to CR: simplified model

Theorem

In the asymptotic case where N is large, G admits a unique NE. At the NE, there are $x_i^* N$ SUs staying on channel i, where $x_i^* = \frac{\mu_i}{\sum_{l \in \mathcal{C}} \mu_l}$.

- G is a congestion game and also a potential game

$$P(\mathbf{x}) \triangleq \sum_{i \in \mathcal{C}} \int_{\epsilon_0}^{x_i N} \frac{\mu_i}{t} dt \quad \text{and} \quad \frac{\partial P(x)}{\partial x_i} = \mathbb{E}[\pi_j(\mu_i, x_i)]$$

- The problem $\max_{\mathbf{x}} P(x)$ $s.t. \sum_{i \in \mathcal{C}} x_i = 1$ has a unique solution
- Convergence is exponential (DI converges at a higher rate)

Population Games applied to CR: simplified model

Distributed algorithm based on imitation

Assumptions:

- Current payoff π_i is included in the header of each transmitted packet
- Each SU is able to overhear one or two packets of other SUs

Algorithm 1 Imitation Spectrum Access Policy (ISAP) executed at each Secondary User

1: **Initialization:** Set ϵ_t
2: At each iteration t
3: With probability $1 - \epsilon_t$ perform imitation (PIR or DI)
4: With probability ϵ_t switch to a random channel

Population Games applied to CR: simplified model

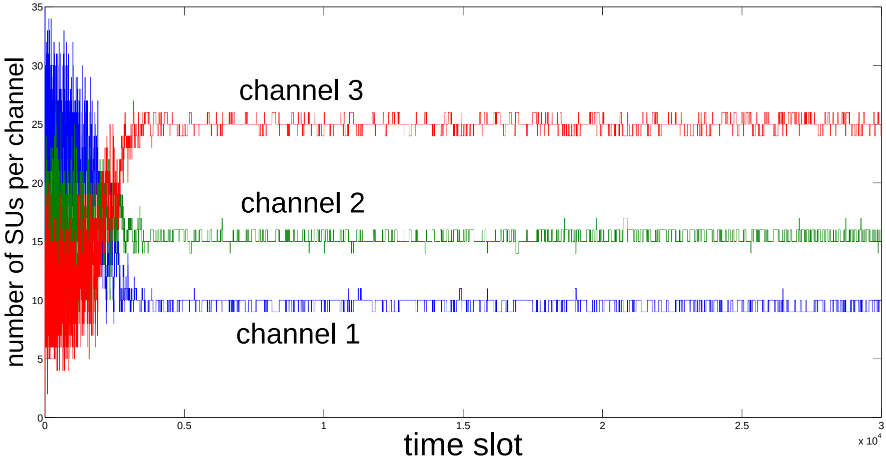

Figure: PIR-ISAP: number of SUs per channel as a function of time

Population Games on CR: imitating neighbors

- SUs can overhear only on the channel on which they stay
- They imitate the payoff obtained at time $t-1$.

Theorem (Dynamics)

In the case of proportional imitation policy it holds that:

$$x_i(t+1) = \sum_{j,l,k \in \mathcal{C}} \frac{x_j^l(t) x_j^k(t)}{x_j(t)} F_{l,k}^i \quad \forall i \in \mathcal{C}$$

Differently, the double imitation policy yields:

$$x_i(t+1) = \sum_{j,l,k,z \in \mathcal{C}} \frac{x_j^l(t) x_j^k(t) x_j^z(t)}{[x_j(t)]^2} F_{l,\{k,z\}}^i \quad \forall i \in \mathcal{C}$$

The system dynamics are well approximated by a double replicator dynamic (PISAP) and by a double aggregate monotone dynamic (DISAP):

- Double replicator, e.g., has the following expression:

$$\begin{cases} x_i(u) = & x_i(u-1) + \sigma x_i(u-1)[\pi_i(u-1) - \bar{\pi}(u-1)] \\ x_i(v) = & x_i(v-1) + \sigma x_i(v-1)[\pi_i(v-1) - \bar{\pi}(v-1)] \end{cases}$$

where $u = 2t$, $v = 2t + 1$.

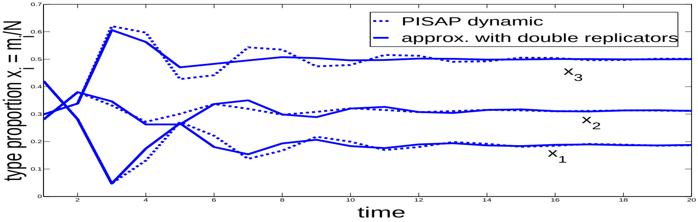

Figure: System dynamic and its approximation by double replicator dynamic.

Population Games on CR: imitating neighbors

Finite populations:

- p denotes the population proportions whenever the population is finite.
- It holds that $p \to x$ when N is very large.

Theorem (Finite populations)

For any imitation rule F, if the imitation among SUs of the same type occurs randomly and independently, then $\forall \delta > 0$, $\epsilon > 0$ and any initial state $\{\widetilde{x}_i(0)\}$, $\{\widetilde{x}_i(1)\}$, there exists $N_0 \in \mathbb{N}$ such that if $N > N_0$, $\forall i \in \mathcal{C}$, the event $|p_i(t) - x_i(t)| > \delta$ occurs with probability less than ϵ, where $p_i(0) = x_i(0) = \widetilde{x}_i(0)$, $p_i(1) = x_i(1) = \widetilde{x}_i(1)$.

Population Games on CR: imitating neighbors

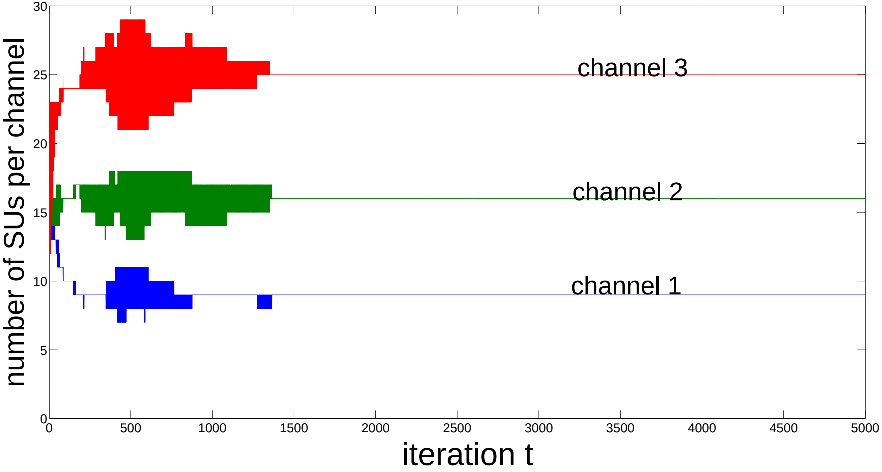

Figure: DISAP: number of SUs per channel as a function of time **with** channel constraint.

Population Games on CR: imitating neighbors

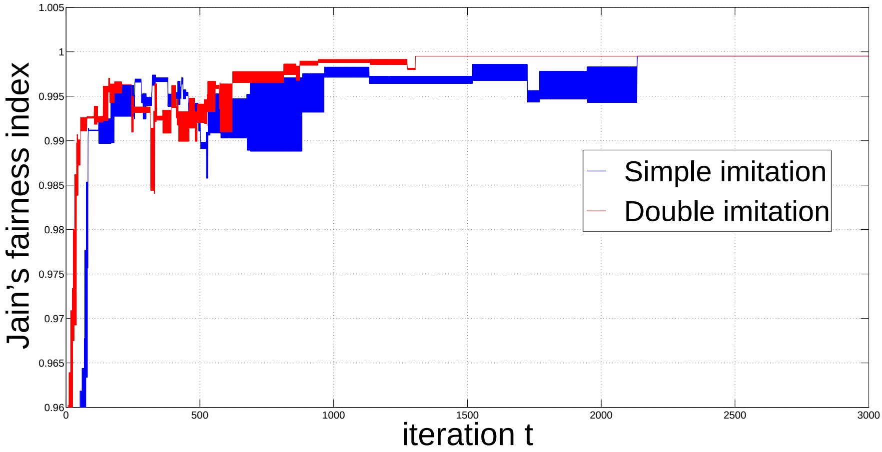

Figure: PISAP and DISAP fairness trends comparison (one realization).

Conclusion and future work

- Imitation-based Spectrum Access Policies allow the SUs to load-balance the system throughput.
- The approach is totally distributed and relies solely on local interactions amongst users
- Our next goals are to make the model more realistic and adapt our algorithms accordingly
  - More realistic MAC protocol (CSMA/CA)
  - Different imitation strategies
  - Non-symmetric topologies
  - Priority schemas

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