Imitation in a CSMA/CA based cognitive network

Stefano Iellamo[1], Lin Chen[2], Marceau Coupechoux[1]
[1]INFRES, Telecom ParisTech, [2]LRI, Paris XI

June 20th, 2011

Outlines

- Introduction: evolutionary game theory and population games
- Population games applied to CR: model and assumptions
  - Imitation **w/o** channel constraint
  - Imitation **with** channel constraint
  - Channel acces via-CSMA/CA
- Conclusion and future work

Evolutionary games overview

- Evolutionary games formalism is a central mathematical tool developed by biologists for predicting populations dynamics in the context of interactions between populations.
- All players in a population are programmed to use strategies
- Strategies with high payoff will spread within the population. This can be achieved by learning, copying or inheriting strategies.
- The payoff depends on the frequency of the strategies within the population. Since this frequencies change according to the payoffs, this yields a feedback loop.

Population Games

- Population games are a particular class of evolutionary games
- They model strategic interactions in which [1]
  - Population is large
  - The number of strategies is finite
  - Agents interact at random (e.g. pairwise)
  - Payoffs are continuous

Population Games

Definitions

- Strategies: $\mathcal{S} = \{1, ..., n\}$
- Population states: $X = \{x \in R^n_+, \sum_{i=1}^n x_i = 1\}$
- Payoff function: $\pi : X \to R^n$ assigns to each state a payoff vector
- Payoff component for strategy i: $\pi_i : X \to R$
- Average payoff in state x: $\bar{\pi}(x) = \sum_i x_i \pi_i(x)$

Population Games

Evolutionary game dynamics

- Players play mixed strategies
- Players update their strategy according to their environment
- The updating process is called a **revision protocol**
  - Example of revision protocol: the proportional imitation rule (PIR) [2]
- The revision protocol generates a **system dynamic**.
  - PIR, e.g., induces the **replicator dynamic**:

$$\dot{x}_i(t) = \sigma x_i(t)(\pi_i(t) - \bar{\pi}(t))$$

Population Games applied to CRN

The radio cognitive scenario

- There are N SUs and C channels, each with availability $\mu_i \in [0, 1]$
- Strategies: $\mathcal{S} = \{1, ..., C\}$
- Throughput on channel i (normalized): RV T_i
- Payoffs: expectation of the normalized throughput

Step one: simplified model

Model assumptions:

- Imitation can be performed across channels
- Generic MAC protocol. The throughput of the SUs on the same channel i is defined as:

$$\pi_i(\mu_i, x_i) = E[T_i] \approx \mu_i / n_i = \mu_i / (x_i N)$$

Population Games applied to CRN: simplified model

Theorem

In the asymptotic case where N is large, G admits a unique NE. At the NE, there are $x_i^* N$ SUs staying on channel i, where $x_i^* = \frac{\mu_i}{\sum_{l \in \mathcal{C}} \mu_l}$.

- G is a congestion game and also a potential game
- The problem $\max_{\mathbf{x}} P(x)$ $s.t. \sum_{i \in \mathcal{C}} x_i = 1$ has a unique solution, which is the NE of G
- Convergence trend is exponential.

$$\dot{x}_i = \sigma \left(\frac{\mu_i}{N} - x_i \sum_{j \in \mathcal{S}} \frac{\mu_j}{N} \right)$$

$$x_i(t) = K_i e^{-\left(\sum_{j \in \mathcal{S}} \frac{\mu_j}{N}\right)\sigma t} + \frac{\mu_i}{\sum_{j \in \mathcal{S}} \mu_j} \to x_i^* = \frac{\mu_i}{\sum_{j \in \mathcal{S}} \mu_j}$$

Population Games applied to CRN: simplified model

Distributed algorithm based on imitation

Assumptions:

- Each SU estimates its payoff (average norm. throughput)
- Current payoff $\hat{\pi}_i$ is included in the header of each transmitted packet
- Each SU is able to overhear one or two packets of other SUs

---

Algorithm 1 ISAP executed as each SU

1: **Initialization:** Set ϵ_t
2: At each iteration t
3: With probability $1 - \epsilon_t$ perform imitation (PIR or DI)
4: With probability ϵ_t switch to a random channel

Population Games applied to CRN: simplified model

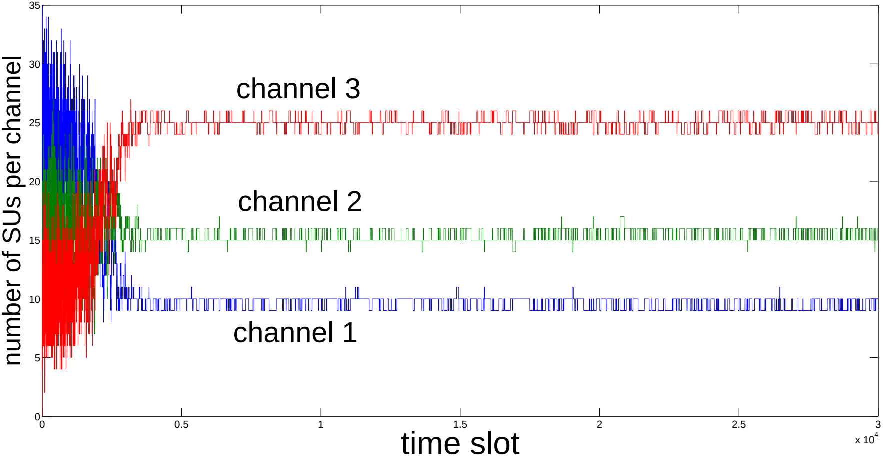

Figure: PIR-ISAP: number of SUs per channel as a function of time

Population Games applied to CRN: simplified model

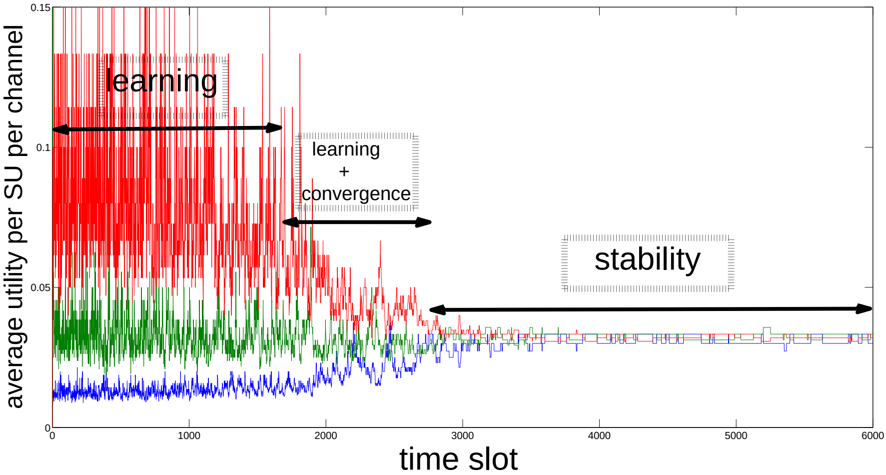

Figure: Three main phases of DI-ISAP

Step two: imitation from neighbors

- SUs can overhear only on the channel on which they stay
- They imitate the payoff obtained at time $t-1$.

Theorem (Dynamics)

In the case of proportional imitation policy it holds that:

$$x_i(t+1) = \sum_{j,l,k \in \mathcal{C}} \frac{x_j^l(t) x_j^k(t)}{x_j(t)} F_{l,k}^i \quad \forall i \in \mathcal{C}$$

- The system dynamics are well approximated by a double replicator dynamic, which has the following expression:

$$\begin{cases} x_i(u) = x_i(u-1) + \sigma x_i(u-1)[\pi_i(u-1) - \bar{\pi}(u-1)] \\ x_i(v) = x_i(v-1) + \sigma x_i(v-1)[\pi_i(v-1) - \bar{\pi}(v-1)] \end{cases}$$

where $u = 2t$, $v = 2t+1$.

Population Games on CRN: imitating neighbors

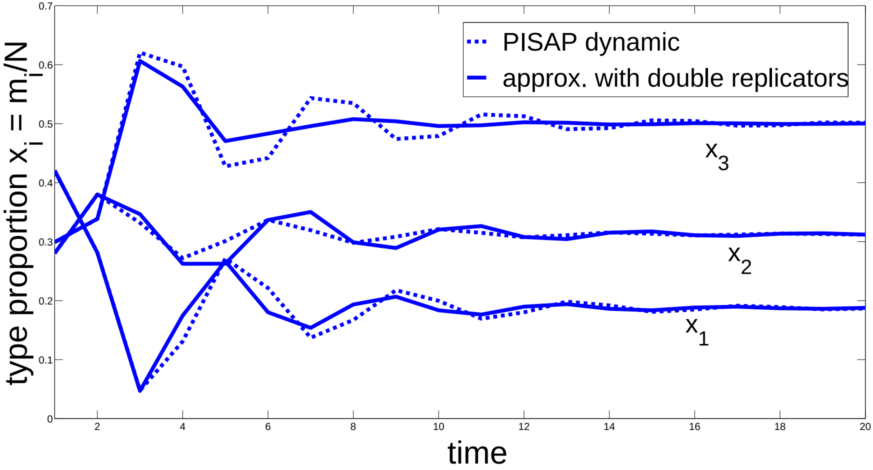

Figure: System dynamic and its approximation by double replicator dynamic.

Population Games on CRN: imitating neighbors

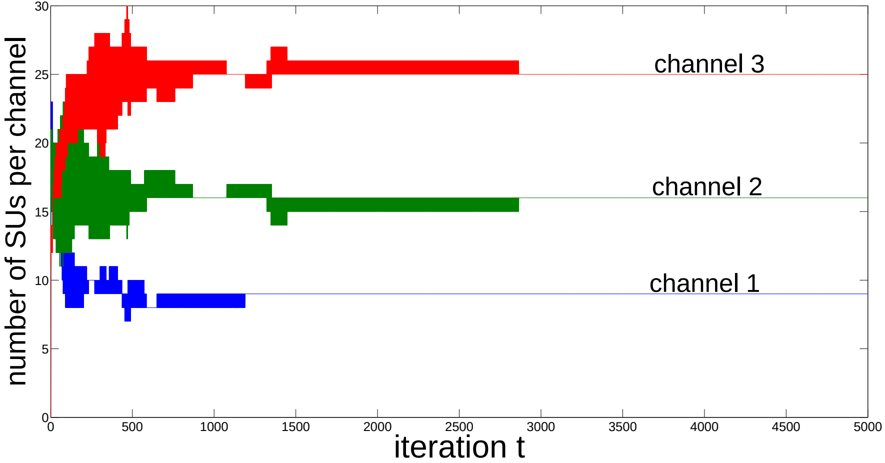

Figure: PISAP: number of SUs per channel as a function of time **with** channel constraint.

Step three: CSMA/CA

- SUs use 802.11 DCF to access the channel
- Payoff is the real throughput:

$$\pi_i = \mu_i p(n_i)$$

- Each iteration encloses a block of PU-slots
- At each iteration each SU estimates the number of neighbors, i.e. the SUs on the same channel
- $\Rightarrow$ convergence is achieved also.

Population Games on CRN: CSMA/CA

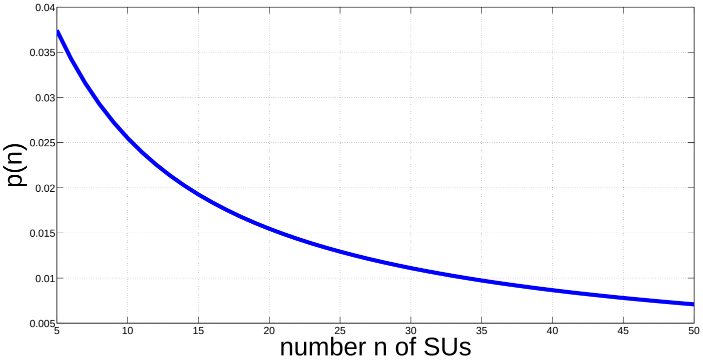

Figure: Probability of successful transmission per SU per coarse slot as a function of the number of users on channel. The contention window has been set according to the DSSS schema parameters

Population Games on CRN: CSMA/CA

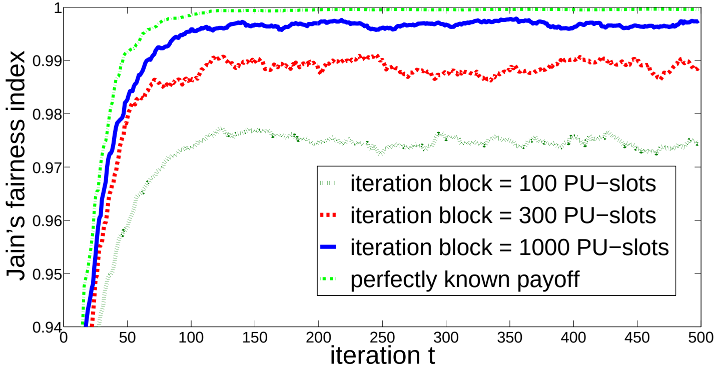

Figure: Fairness convergence of ideal-PISAP and NEPCT-PISAP, the latter plotted for different iteration block sizes. Each curve represents an average over 100 independent realizations

Conclusion and future work

- Imitation-based Spectrum Access Policies allow the SUs to load-balance the system throughput.
- The approach is totally distributed and relies solely on local interactions amongst users
- Our next goals are to make the model more realistic and adapt our algorithms accordingly
  - Different imitation strategies
  - Non-symmetric topologies
  - Priority schemas

References I

[1] W. H. Sandholm.
Local Stability under Evolutionary Game Dynamics.
Theoretical Economics, 5, 2010.

[2] K. H. Schlag.
Why Imitate, and if so, How ?
Discussion paper, University of Bonn, Feb. 1996.