

SIR distribution analysis in cellular networks considering the joint impact of path-loss, shadowing and fast fading

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Abstract

In this paper, we propose an analysis of the joint impact of path-loss, shadowing and fast fading on cellular networks. Two analytical methods are developed to express the outage probability. The first one based on the Fenton–Wilkinson approach, approximates a sum of log-normal

random variables by a log-normal random variable and approximates fast fading coefficients in interference terms by their average value. We denote it FWBM for Fenton-Wilkinson based method. The second one is based on the central limit theorem for causal functions. It allows to approximate a sum of positive random variables by a Gamma distribution. We denote it CLCFM for central limit theorem for causal functions method. Each method allows to establish a simple and easily computable outage probability formula, which jointly takes into account path-loss, shadowing and fast fading. We compute the outage probability, for mobile stations located at any distance from their serving BS, by using a fluid model network that considers the cellular network as a continuum of BS. We validate our approach by comparing all results to extensive Monte Carlo simulations performed in a traditional hexagonal network and we provide the limits of the two methods in terms of system parameters. The proposed framework is a powerful tool to study performances of cellular networks, e.g., OFDMA systems (WiMAX, LTE).

1 Introduction

In this paper, we are interested in characterizing the signal to interference ratio (SIR) distribution on the downlink of a cellular network. The SIR outage probability and the SIR cumulative distribution function (CDF) are important metrics for the performance evaluation of wireless communication systems. In this paper, we define the outage probability as the probability that the SIR at the input of the receiver chain is falling below a given threshold value. This performance parameter is crucial for both coverage and capacity studies. In terms of coverage, mobile stations should be able to decode common control channels (like pilots or broadcast channels) and thus to attain a certain SIR threshold on these channels with high probability. In this case, we are interested in the low SIR region of the SIR distribution in order to evaluate the cell coverage. In terms of capacity and for systems implementing link adaptation on shared downlink channels (such as HSPA or LTE), the whole SIR distribution is

needed for performance evaluation. The ergodic capacity at a certain distance from the base station is indeed evaluated as an expectation of the Shannon classical formula over the channel variations. The cell capacity is obtained by integration over the cell area.

The issue of expressing outage probability in cellular networks has been extensively addressed in the literature. For this study, a difficult task is to take into account the joint impact of path-loss, shadowing and fast fading. There are two classical assumptions: (1) considering only the shadowing effect, (2) considering both shadowing and fast fading effects. In the former case, authors mainly face the problem of expressing the distribution of the sum of log-normally random variables; several classical methods can be applied to solve this issue (see e.g. [1] [2]). In the latter case, formulas usually consist in many infinite integrals, which are uneasy to handle in a practical way (see e.g. [3]). In both cases, outage probability is always an explicit function of all distances from the user to interferers.

As the need for easy-to-use formulas for outage probability is clear, approximations need to be done. Working on the uplink [4], derived the distribution function of a ratio of path-losses with shadowing, which is essential for the evaluation of external interference. For that, authors approximate the hexagonal cell with a disk of same area. Authors of [5] assume perfect power control on the uplink, while neglecting fast fading. On the downlink, Chan and Hanly [6] precisely approximate the distribution of the other-cell interference. They, however, provide formulas that are difficult to handle in practice and do not consider fast fading. Immovilli and Merani [7] take into account both channel effects and make several assumptions in order to obtain simplified formulas. In particular, they approximate interference by its mean value. Outage probability is however an explicit function of all distances from receiver to every interferer. Zorzi [8], proposes a formula essentially valid for packet radio networks rather than for cellular systems. Authors of [9] provide some interesting characterizations and upper bounds of the outage probability but neglects the slowly varying path-gains. In [10], authors consider both shadowing and fast fading but assume a single interferer.

In this paper, we propose two methods to analyze the outage probability of mobile stations located at any distance r from their serving base station (BS). The first method, based on the Fenton–Wilkinson approach [11], approximates a sum of log-normal random variables as a log-normal random variable and approximates fast fading coefficients in interference terms by their average value. We denote it FWBM (for Fenton-Wilkinson Based Method). The second one is based on the central limit theorem for causal functions [12]. It allows to approximate a sum of positive random variables by a gamma distribution. We denote it CLCFM (for central limit theorem for causal functions method). Each method allows to establish a simple and easily computable outage probability formula, which jointly takes into account path-loss, shadowing and fast fading. We compare our proposed formulas with results obtained with extensive Monte Carlo simulations in a classical hexagonal network. At last, we rely on fluid model proposed in [13] [14] in order to express the outage probability as a simple analytical expression depending only on the distance to the serving BS. Such an expression allows further integrations much more easily than with existing formulas. Note that part of the presented results are included in [15].

The paper is organized as follows. FWBM is explained in sect. 3. We derive the outage probability while considering first only path-loss and shadowing and then path-loss, shadowing and fast fading jointly. Sect. 4 develops the CLCF method. Outage probability is calculated while considering first only path-loss and fast fading and then path-loss, shadowing and fast fading jointly. The computation is based on the fluid model (sect. 5). In sect. 6, we validate our approach and compare analytical expressions with results obtained through Monte Carlo simulations.

2 Interference model

We first define the interference model assumed in this paper. We consider a hexagonal cellular network with frequency reuse one and we focus on the downlink. We are interested in evaluating the SIR at a mobile station u , served by base station BS_0 and interfered by N interfering base stations. We assume that mobile stations are attached to their closest BS.

2.0.1 Propagation Model

The power received by u depends on the radio channel state and varies with time due to fading effects (shadowing and fast fading). Let P_j be the transmission power of base station j , the power $p_{j,u}$ received by u can be written as:

$$p_{j,u} = P_j K r_{j,u}^{-\eta} X_{j,u} Y_{j,u}. \quad (1)$$

The term $P_j K r_{j,u}^{-\eta}$, where K is a constant, represents the mean value of the received power at distance $r_{j,u}$ from the transmitter (BS_j). $X_{j,u}$ is a random variable (RV) representing the Rayleigh fading effects, whose pdf is $p_X(x) = e^{-x}$. The term $Y_{j,u} = 10^{\xi_{j,u}/10}$ is a lognormal RV characterizing shadowing. $\xi_{j,u}$ is a *Normal* distributed RV, with zero mean and standard deviation, σ , which is typically between 0 and 8 dB. Parameter η , which is typically between 2.7 and 3.5, is the path-loss exponent.

2.0.2 SIR Calculation

The interference power received by a mobile u can be written as:

$$p_{\text{ext},u} = \sum_{j=1}^N P_j K r_{j,u}^{-\eta} X_{j,u} Y_{j,u}.$$

We consider in this paper that the thermal noise is negligible (in a urban environment) and so we focus on the SIR rather than on the signal to interference plus noise ratio (SINR). Considering moreover

that all BS transmit with the same power P_0 , we can express the SIR expression (dropping index u and setting $r_{0,u} = r$):

$$\gamma = \frac{r^{-\eta} X_0 Y_0}{\sum_{j=1}^N r_j^{-\eta} X_j Y_j}. \quad (2)$$

2.0.3 Outage Probability

In this paper, we define the outage probability as the probability that the SIR at u falls below a given threshold δ . Note that while varying δ , we obtain the definition of the SIR CDF. In this paper, we indifferently speak of outage probability or CDF.

$$\mathbb{P}(\gamma < \delta) = \mathbb{P}\left(\frac{r^{-\eta} X_0 Y_0}{\sum_{j=1}^N r_j^{-\eta} X_j Y_j} < \delta\right). \quad (3)$$

3 Fenton-Wilkinson based method

In this section, we propose a first method based on the Fenton-Wilkinson approximation. We first analyze the path-loss and shadowing impact. We then extend the result to the joint influence of path-loss, shadowing and fast fading based on the previous obtained result.

3.1 Path-loss and shadowing impact

The power p_j received by u can be written in this case:

$$p_j = P_j K r_j^{-\eta} Y_j.$$

The probability density function (PDF) of this slowly varying received power is given by

$$p_Y(s) = \frac{1}{a\sigma s\sqrt{\pi}} \exp\left[-\left(\frac{\ln(s) - am}{\sqrt{2}a\sigma}\right)^2\right]$$

where $a = \frac{\ln 10}{10}$, $m = \frac{1}{a} \ln(KP_j r_j^{-\eta})$ is the (logarithmic) received mean power expressed in decibels (dB), which is related to the path-loss and σ is the (logarithmic) standard deviation of the mean received signal due to the shadowing.

The SIR at user u is now given by:

$$\gamma = \frac{r^{-\eta} Y_0}{\sum_{j=1}^N r_j^{-\eta} Y_j}.$$

We see that the SIR can be written $\gamma = 1/F$ with

$$F = \frac{\sum_{j=1}^N r_j^{-\eta} Y_j}{r^{-\eta} Y_0}. \quad (4)$$

The factor F is defined for any mobile u and it is thus location dependent. The numerator of this factor is a sum of log-normally distributed RV, which can be approximated by a log-normally distributed RV [2]. The denominator of the factor is a log-normally distributed RV. F can thus be approximated by a log-normal RV. Using the Fenton-Wilkinson [11] method, we can calculate the logarithmic mean and standard deviation, m_f and s_f of F for any mobile at the distance r from its serving BS, BS_0 (see Appendix 1):

$$m_f = \frac{1}{a} \ln(f(r, \eta) H(r, \sigma)), \quad (5)$$

$$s_f^2 = 2(\sigma^2 - \frac{1}{a^2} \ln H(r, \sigma)), \quad (6)$$

where

$$H(r, \sigma) = e^{a^2 \sigma^2 / 2} \left(G(r, \eta) (e^{a^2 \sigma^2} - 1) + 1 \right)^{-\frac{1}{2}}, \quad (7)$$

$$G(r, \eta) = \frac{\sum_j r_j^{-2\eta}}{\left(\sum_j r_j^{-\eta}\right)^2}, \quad (8)$$

$$f(r, \eta) = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}}. \quad (9)$$

From Eqs. (4) and (9), we notice that $f(r, \eta)$ represents the factor F *without shadowing*.

The outage probability is now defined as the probability for the SIR γ to be lower than a threshold value δ and can be expressed as:

$$\begin{aligned} \mathbb{P}(\gamma < \delta) &= 1 - \mathbb{P}\left(\frac{1}{\delta} > F\right) \\ &= 1 - \mathbb{P}\left(10 \log_{10}\left(\frac{1}{\delta}\right) > 10 \log_{10}(F)\right) \\ &= Q\left[\frac{10 \log_{10}\left(\frac{1}{\delta}\right) - m_f}{s_f}\right]. \end{aligned} \quad (10)$$

where Q is the complementary error function: $Q(u) = \frac{1}{2} \text{erfc}\left(\frac{u}{\sqrt{2}}\right)$.

3.2 Path-loss, shadowing and fast fading impact

In this case, the outage probability can be expressed as:

$$\mathbb{P}(\gamma < \delta) = 1 - \mathbb{P}\left(r^{-\eta} X_0 Y_0 > \delta \left(\sum_{j=1}^N r_j^{-\eta} X_j Y_j\right)\right).$$

The interference power received by a mobile u due to fast fading effects varies with time. As a consequence, the fast fading can increase or decrease the power received by u . We consider that the increase of interfering power due to fast fading coming from some base stations are compensated by the decrease of interfering powers coming from other base stations. As a consequence, for the interfering power, only the slow fading effect has a significant impact on the SIR. We thus assume

that $\forall j \neq 0, X_j \approx E[X_j] = 1$ (this assumption will be validated by simulations in the next section), and we can write $\mathbb{P}(r^{-\eta} X_0 Y_0 > \delta (\sum_{j=1}^N r_j^{-\eta} X_j Y_j)) \approx \mathbb{P}(r^{-\eta} X_0 Y_0 > \delta (\sum_{j=1}^N r_j^{-\eta} Y_j))$.

So we have:

$$\begin{aligned}
\mathbb{P}(\gamma < \delta) &= 1 - \mathbb{P}\left(X_0 Y_0 > \delta \frac{1}{r^{-\eta}} \sum_{j=1}^N r_j^{-\eta} Y_j\right) \\
&= 1 - \mathbb{P}(X_0 > \delta F) \\
&= 1 - \int_0^{\infty} \mathbb{P}(x > \delta F) p_X(x) dx \\
&= 1 - \int_0^{\infty} \mathbb{P}(10 \log_{10}(\frac{x}{\delta}) > 10 \log_{10}(F)) e^{-x} dx.
\end{aligned}$$

As a consequence, the outage probability for a mobile located at a distance r from its serving BS, taking into account path-loss, shadowing and fast fading can be written as:

$$\mathbb{P}(\gamma < \delta) = \int_0^{\infty} Q\left[\frac{10 \log_{10}(\frac{x}{\delta}) - m_f}{s_f}\right] e^{-x} dx. \quad (11)$$

4 Central limit theorem for causal functions method

In this section, we adopt a different path for deriving the SIR CDF. We first express the outage probability by considering the path-loss and fast fading. Afterwards, we use this result in order to introduce the shadowing impact.

4.1 Path-loss and fast fading impact

Assuming only fast fading channel, the SIR is given by:

$$\gamma = \frac{r^{-\eta} X_0}{\sum_{j=1}^N r_j^{-\eta} X_j} = \frac{S}{I},$$

where:

$$S = r^{-\eta} X_0, \quad I = \sum_{j=1}^N r_j^{-\eta} X_j$$

are two independent RV. To calculate the outage probability, we need to calculate first, the probability distribution function (PDF) $f_S(x)$ of S and the PDF $f_I(x)$ of I . The PDF of the useful power is given by [16]:

$$f_S(x) = \frac{1}{r^{-\eta}} e^{-\frac{x}{r^{-\eta}}}.$$

We now approximate the interference PDF using the central limit theorem for causal functions [12] by a Gamma distribution given by:

$$f_I(y) = \frac{y^{\nu-1}}{\Gamma(\nu)\lambda^\nu} e^{-\frac{y}{\lambda}},$$

where $\nu = \frac{\mathbb{E}[I]^2}{\text{var}(I)}$ and $\lambda = \frac{\text{var}(I)}{\mathbb{E}[I]}$. Since $\mathbb{E}[X_j] = 1$ and $\text{var}(X_j) = 1$ for $j = 1, \dots, N$, the mean of the interference power is given by:

$$\mathbb{E}[I] = \sum_{j=1}^N r_j^{-\eta} \mathbb{E}[X_j] = \sum_{j=1}^N r_j^{-\eta}.$$

The variance of I can be expressed as:

$$\text{var}(I) = \sum_{j=1}^N r_j^{-2\eta} \text{var}(X_j) = \sum_{j=1}^N r_j^{-2\eta}.$$

So we have:

$$\nu = \frac{(\sum_{j=1}^N r_j^{-\eta})^2}{\sum_{j=1}^N r_j^{-2\eta}}, \quad (12)$$

$$\lambda = \frac{\sum_{j=1}^N r_j^{-2\eta}}{\sum_{j=1}^N r_j^{-\eta}}. \quad (13)$$

The outage probability can now be derived as follows:

$$\begin{aligned} \mathbb{P}(\gamma < \delta) &= \int_0^\infty \mathbb{P}(S < I\delta | I = y) f_I(y) dy \\ &= \int_0^\infty F_S(\delta y) f_I(y) dy. \\ &= 1 - \frac{1}{\left(\frac{\lambda}{r^{-\eta}}\delta + 1\right)^\nu}. \end{aligned} \quad (14)$$

4.2 Path-loss, shadowing and fast fading impact

In this section we will consider that the shadowing follows a log-normal distribution. We can again write $\gamma = S/I$ with now

$$S = r^{-\eta} Y_0 X_0, \quad I = \sum_{j=1}^N r_j^{-\eta} Y_j X_j.$$

We again approximate the interference PDF using the Central Limit Theorem for Causal Functions.

We thus need to compute the two quantities $\nu_s = \frac{\mathbb{E}[I]^2}{\text{var}(I)}$ and $\lambda_s = \frac{\text{var}(I)}{\mathbb{E}[I]}$. Since involved RV are independent, the average value of I is given by:

$$\begin{aligned} \mathbb{E}[I] &= \sum_{j=1}^N r_j^{-\eta} \mathbb{E}[Y_j] \mathbb{E}[X_j], \\ &= e^{\frac{a^2 \sigma^2}{2}} \sum_{j=1}^N r_j^{-\eta}. \end{aligned}$$

In the same way, the variance of I is given by:

$$\begin{aligned}\text{var}(I) &= \sum_{j=1}^N r_j^{-2\eta} \left(\mathbb{E}[Y_j^2] \mathbb{E}[X_j^2] - \mathbb{E}[Y_j]^2 \mathbb{E}[X_j]^2 \right), \\ &= \left(2e^{2a^2\sigma^2} - e^{a^2\sigma^2} \right) \sum_{j=1}^N r_j^{-2\eta}.\end{aligned}$$

The two parameters ν_s and λ_s can now be obtained:

$$\nu_s = \frac{1}{2e^{a^2\sigma^2} - 1} \frac{\left(\sum_{j=1}^N r_j^{-\eta} \right)^2}{\sum_{j=1}^N r_j^{-2\eta}}, \quad (15)$$

$$\lambda_s = e^{\frac{a^2\sigma^2}{2}} \left(2e^{a^2\sigma^2} - 1 \right) \frac{\sum_{j=1}^N r_j^{-2\eta}}{\sum_{j=1}^N r_j^{-\eta}}. \quad (16)$$

Recall that Y_0 follows a log-normal distribution with logarithmic mean 0 and standard deviation σ . We can thus average the outage probability over the variations of Y_0 :

$$\begin{aligned}\mathbb{P}(\gamma < \delta) &= \int \mathbb{P}(\gamma < \delta | Y_0) p_Y(y) dy, \\ &= \int \mathbb{P} \left(r^{-\eta} X_0 < \frac{\delta}{y} I \right) p_Y(y) dy, \\ &= 1 - \int_0^\infty \frac{1}{\left(\frac{\lambda_s}{r^{-\eta}} \frac{\delta}{y} + 1 \right)^{\nu_s}} \times \frac{1}{ay\sigma\sqrt{2\pi}} \exp\left(-\frac{\ln(y)^2}{2a^2\sigma^2}\right) dy.\end{aligned} \quad (17)$$

5 Analytical fluid model

With the two proposed methods, we obtain expressions of the SIR CDF at a given distance r from the serving base-station. We see however that expressions depend also on the distances r_i between the considered mobile terminal and all interfering BS. With FWBM, parameters m_f and s_f in (11) depend on the r_i (see Eqs. (5) and (6)). With CLCFM, parameters ν_s and λ_s in (17) depend also on the r_i (see Eqs. (15) and (16)). The presence of all distances r_i make proposed formulas sometimes

uneasy to use for dimensioning purposes. In this section, we thus express the parameters m_f , s_f , ν_s and λ_s as functions dependent only on the distance r using the fluid model.

The fluid model approach has been developed, e.g., in [17]. It consists in replacing on the downlink a given fixed finite number of transmitters (base stations) by an equivalent continuum of transmitters which are distributed according to some distribution function. For a homogeneous and regular cellular network, interferers are now characterized by the interfering BS density ρ_{BS} . Let denote:

$$g(\eta) = \sum_{j=1}^N r_j^{-\eta}. \quad (18)$$

Assuming an infinite network, the fluid model allows us to approximate g by the following function [17] (see Appendix 2 for more details). Introducing the dependence of g on r , we obtain:

$$g(r, \eta) = \frac{2\pi\rho_{BS}}{\eta - 2}(2R_c - r)^{2-\eta}, \quad (19)$$

where R_c is the half inter-BS distance.

In the FWBM method, parameters $f(r, \eta)$ and $G(r, \eta)$ given by Eqs. (9) and (8), respectively, can be expressed as follows:

$$\begin{aligned} f(r, \eta) &= \frac{g(r, \eta)}{r^{-\eta}}, \\ G(r, \eta) &= \frac{g(r, 2\eta)}{g(r, \eta)^2}. \end{aligned}$$

Parameters m_f and s_f can thus be written as functions only on the distance r to the serving BS. In the same way, for the CLCFM, we have:

$$\nu_s = \frac{1}{2e^{a^2\sigma^2} - 1} \frac{g(r, \eta)^2}{g(r, 2\eta)}, \quad (20)$$

$$\lambda_s = e^{\frac{a^2\sigma^2}{2}} \left(2e^{a^2\sigma^2} - 1 \right) \frac{g(r, 2\eta)}{g(r, \eta)}. \quad (21)$$

6 Performance evaluation

In this section, we compare the figures obtained with analytical expressions (11) and (17) to those obtained by Monte Carlo simulations. It is clear that several approximations have been done in order to obtain easy-to-use closed-form formulas: (1) the Fenton-Wilkinson method is known to be accurate for low standard deviations; (2) the Central Limit Theorem for Causal Functions is an approximation; (3) the fluid model also. It is thus important to know to what extent approximations are acceptable.

6.1 Monte Carlo simulator

The simulator assumes an homogeneous hexagonal network as the one shown in Fig. 1, made of fifteen rings around a central cell. The cell range is denoted R , the half-distance between BS is set to $R_c = 1$ km.

The simulation consists in computing at each snapshot the SIR for a uniformly random location in the central cell. This computation can be done independently of the BS output power because noise is supposed to be negligible both in simulations and analytical study. At each snapshot, shadowing (log-normal distribution with standard deviation σ) and fast fading (exponential distribution of mean 1) RV are independently drawn between the MS and the serving BS and between MS and interfering BS. We do not consider correlation between shadowing coefficient. SIR samples at a given distance from the central BS are recorded in order to compute the outage probability. Five thousand (5,000) snapshots are considered.

6.2 Results

In this section, we compare obtained formulas with results obtained by Monte Carlo simulations in a hexagonal network. We study the robustness of our approaches while varying three important parameters: σ , the standard deviation of the shadowing, η , the path-loss exponent, and r , the distance to the serving BS.

Moreover, we compare the results for the following approaches:

- SIM: results obtained from Monte Carlo simulations;
- FWBM: the Fenton-Wilkinson Based Method in conjunction with the fluid model;
- CLCFM: the Central Limit Theorem for Causal Functions Method in conjunction with the fluid model;

In Fig. 2, we compare SIR CDF obtained with SIM, FWBM and CLCFM at $r = 0.2$ Km, for $\eta = 3.0$ and while varying σ from 3 to 8 dB. Parameter σ is definitely the coefficient that influences the most the difference between analysis and simulations. It is clear that the highest is σ , the highest is the error induced by approximations. For $\sigma = 8$ dB, the CLCFM is not valid anymore if we consider the whole CDF, but remains accurate for low SIR region. For $\sigma = 3$ dB, both methods provide very accurate results.

In Fig. 3, we study the influence of the distance to the serving BS for $\sigma = 4$ dB and $\eta = 3.0$. This distance has a small influence on the accuracy of the proposed methods, all analytical CDF fit well with the CDF obtained by simulations.

In Fig. 4, we study the path-loss exponent η with fixed $\sigma = 4$ dB and distance $r = 0.2$ km. Here again, the parameter has a small influence on the globally good accuracy of the methods. Error seems, however, to increase with η .

For the sake of completeness, we present in Tables 1 and 2 extensive results for the comparison of SIM, FWBM, and CLCFM. We set three probability thresholds (5, 50, and 90%) and we obtain the corresponding SIR thresholds in dB from the different CDF. Reported figures represent the difference in dB between the threshold obtained with SIM on the one side and the threshold obtained with FWBM or CLCFM on the other side. Excessive differences (more than 3 dB) are marked with a star. From these tables, we can draw some conclusions:

- CLCFM provides accurate results for $\sigma \leq 6$ dB and $\eta \leq 3.0$. For all $\sigma \leq 8$ dB, results are still accurate in the low SIR region; this is an interesting result for coverage issues where outage computations are involved.
- FWBM provides accurate results for $\sigma \leq 8$ dB and $\eta \leq 3.0$, η can be greater if σ is strictly less than 8 dB. At $\sigma = 8$ dB and for $\eta \leq 3.5$, results are still accurate in the low SIR region.

Theses results show that proposed methods, especially FWBM, can provide accurate results for typical values of parameters r , σ and η usually considered in cellular networks.

7 Conclusion

In this paper, we establish simple formulas of the outage probability in cellular networks, while considering path-loss, shadowing and fast fading. Using FWBM approach, we take into account path-loss and shadowing to first express the inverse of the SIR of a mobile located at a given distance of its serving BS as a log-normal random variable. We then consider both path-loss, shadowing and fast fading and give an analytical expression of the outage probability at a given distance of the serving BS. Using CLCFM approach, we take into account path-loss and fast fading to express the SIR of a mobile located at a given distance of its serving BS. We then consider path-loss, fast fading and shadowing and give an analytical expression of the outage probability at a given distance of the serving

BS. The fluid model allows us to obtain formulas that only depend on the distance to the serving BS. The analytical model that we propose is validated by comparisons with Monte Carlo simulations. The formulas derived in this paper allow to obtain performances results instantaneously.

The proposed framework is a powerful tool to study performances of cellular networks and to design fine algorithms taking into account the distance to the serving BS, shadowing and fast fading. It can particularly easily be used to study frequency reuse schemes in OFDMA systems.

Appendix 1

In this section, we derive the logarithmic mean and standard deviation of F (see equation (4)) using the Fenton-Wilkinson method. Several classical techniques exist in the literature in order to approximate a sum of log-normal RV, e.g., Schwartz-Yeh [18] and Farley [19]. The former needs complex recursive calculations, the latter assumes identical mean and standard deviation. On the contrary, the Fenton-Wilkinson method provides a closed-form formula for non identical distributed log-normal RV.

Each term of the sum in the numerator of F is a RV following a log-normal distribution. We can write $\ln(r_j^{-\eta} Y_j) \propto \mathcal{N}(am_j, a^2 \sigma^2)$, where $a = \ln(10)/10$ and $m_j = \frac{1}{a} \ln(r_j^{-\eta})$. The sum $S = \sum_{j=1}^N r_j^{-\eta} Y_j$ can itself be approximated by a log-normal RV: $\ln(S) \propto \mathcal{N}(am, a^2 \sigma_s^2)$ with:

$$\begin{aligned} am &= \ln \left(\sum_{j=1}^N e^{am_j} \right) + \frac{a^2 \sigma^2}{2} - \frac{a^2 \sigma_s^2}{2}, \\ a^2 \sigma_s^2 &= \ln \left[\left(e^{a^2 \sigma^2} - 1 \right) \frac{\sum_{j=1}^N e^{2am_j}}{\left(\sum_{j=1}^N e^{am_j} \right)^2} + 1 \right]. \end{aligned}$$

These two expressions simplify to:

$$am = \ln \left(\sum_{j=1}^N r_j^{-\eta} \right) + \frac{a^2 \sigma^2}{2} - \frac{a^2 \sigma_s^2}{2},$$

$$a^2\sigma_s^2 = \ln \left[\left(e^{a^2\sigma^2} - 1 \right) \frac{\sum_{j=1}^N r_j^{-2\eta}}{\left(\sum_{j=1}^N r_j^{-\eta} \right)^2} + 1 \right].$$

Taking now into account the denominator, the logarithmic mean value of F is simply $am_f = am - (-\eta \ln(r))$ and the logarithmic standard deviation of F is $a^2s_f^2 = a^2\sigma_s^2 + a^2\sigma^2$. The ratio of two log-normal RV is indeed a log-normal RV with mean, the difference of the means, and variance, the sum of the variances. With the definitions of f , G and H , we obtain Eqs. (5) and (6). Note that m_f and s_f are expressed in dB.

Appendix 2

In this section, we shortly recall the fluid model approach, that will allow us to simplify Eq. 18. The fluid model approach consists of replacing a given fixed finite number of interfering BS by an equivalent continuum of transmitters which are uniformly distributed with density ρ_{BS} .

We consider a central cell and a round shaped network around this cell with radius R_{nw} . The inter-site distance is $2R_c$ (see Fig. 5). Let's consider a mobile u at a distance r_u from its serving BS. In the fluid model, each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{BS}zdzd\theta$ BS which contribute to function g . Their contribution to the sum over j is $\rho_{BS}zdzd\theta z^{-\eta}$ ($\eta > 2$). We approximate the integration surface by a ring with centre u , inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$ (see Fig. 6):

$$\begin{aligned} g(r_u, \eta) &= \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} z^{-\eta} z dz d\theta, \\ &= \frac{2\pi\rho_{BS}}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \end{aligned} \tag{22}$$

If the network is large, i.e., R_{nw} is big in front of R_c , g can be further approximated by (dropping subscript u):

$$g(r, \eta) = \frac{2\pi\rho_{BS}}{\eta - 2}(2R_c - r)^{2-\eta}. \quad (23)$$

Competing Interests

The authors declare that they have no competing interest.

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Figure 1: Hexagonal network and main parameters of the study: the cell range R , the inter-site distance $2R_c$ and the network size R_{nw} .

Figure 2: Influence of the shadowing standard deviation σ (dB) with $r = 0.2$ km and $\eta = 3.0$.

Table 1: CDF difference in dB between Monte Carlo simulations (SIM) on the one hand and CLCFM and FWBM on the other hand at 5, 50 and 90% ($\sigma = 3, 4, 6$ dB, * means greater than 3 dB).

σ (dB)	η	r [km]	CLCF (dB)	FW (dB)
3.0	2.7	0.2	0.8, 0.6, 0.3	0.8, 0.5, 0.2
3.0	2.7	0.5	0.2, 0.1, 0.1	0.2, 0.0, 0.3
3.0	2.7	0.9	0.1, 0.1, 0.4	0.2, 0.2, 0.9
3.0	3.0	0.2	2.0, 1.2, 1.3	2.0, 1.1, 1.0
3.0	3.0	0.5	0.2, 0.6, 0.6	0.2, 0.5, 0.1
3.0	3.0	0.9	0.7, 0.6, 1.1	0.7, 0.2, 0.2
3.0	3.3	0.2	2.1, 1.9, 1.5	2.1, 1.7, 1.1
3.0	3.3	0.5	0.3, 0.8, 1.3	0.2, 0.6, 0.6
3.0	3.3	0.9	1.2, 0.8, 1.6	1.2, 0.3, 0.2
3.0	3.5	0.2	2.1, 1.6, 1.6	2.1, 1.3, 1.0
3.0	3.5	0.5	0.9, 1.4, 2.0	0.9, 1.1, 1.1
3.0	3.5	0.9	0.7, 0.6, 1.6	0.7, 0.0, 0.1
4.0	2.7	0.2	0.2, 0.5, 0.4	0.1, 0.4, 0.1
4.0	2.7	0.5	1.0, 0.3, 0.1	1.1, 0.5, 0.5
4.0	2.7	0.9	0.5, 0.1, 0.0	0.4, 0.2, 0.7
4.0	3.0	0.2	1.4, 1.3, 0.6	1.4, 1.1, 0.1
4.0	3.0	0.5	0.2, 0.6, 1.3	0.3, 0.3, 0.6
4.0	3.0	0.9	0.2, 0.3, 1.3	0.3, 0.2, 0.0
4.0	3.3	0.2	1.0, 1.6, 1.9	1.0, 1.3, 1.2
4.0	3.3	0.5	1.3, 1.6, 1.8	1.2, 1.2, 0.7
4.0	3.3	0.9	0.8, 1.0, 2.6	0.7, 0.2, 0.4
4.0	3.5	0.2	1.5, 1.8, 2.5	1.5, 1.4, 1.6
4.0	3.5	0.5	2.2, 1.7, 2.6	2.1, 1.2, 1.3
4.0	3.5	0.9	0.4, 1.8, 3.0	0.3, 0.8, 0.4
6.0	2.7	0.2	0.9, 0.8, 1.0	0.8, 0.4, 0.3
6.0	2.7	0.5	0.9, 0.7, 0.5	1.0, 0.1, 0.7
6.0	2.7	0.9	0.1, 0.6, 2.2	0.1, 0.4, 0.5
6.0	3.0	0.2	2.7, 1.6, 2.4	2.6, 0.9, 0.9
6.0	3.0	0.5	1.7, 1.5, 2.7	1.5, 0.5, 0.3
6.0	3.0	0.9	1.0, 1.9, *	0.7, 0.0, 0.0
6.0	3.3	0.2	1.3, 2.8, *	1.1, 1.8, 1.5
6.0	3.3	0.5	0.6, 2.3, *	0.3, 0.8, 1.3
6.0	3.3	0.9	0.5, *, *	0.1, 0.8, 0.7
6.0	3.5	0.2	1.8, *, *	1.6, 2.0, 1.9
6.0	3.5	0.5	2.6, *, *	2.3, 1.9, 1.7
6.0	3.5	0.9	1.7, *, *	1.1, 0.7, 0.7

Figure 3: Influence of the distance to the serving BS r [km] with $\sigma = 4$ dB and $\eta = 3.0$.

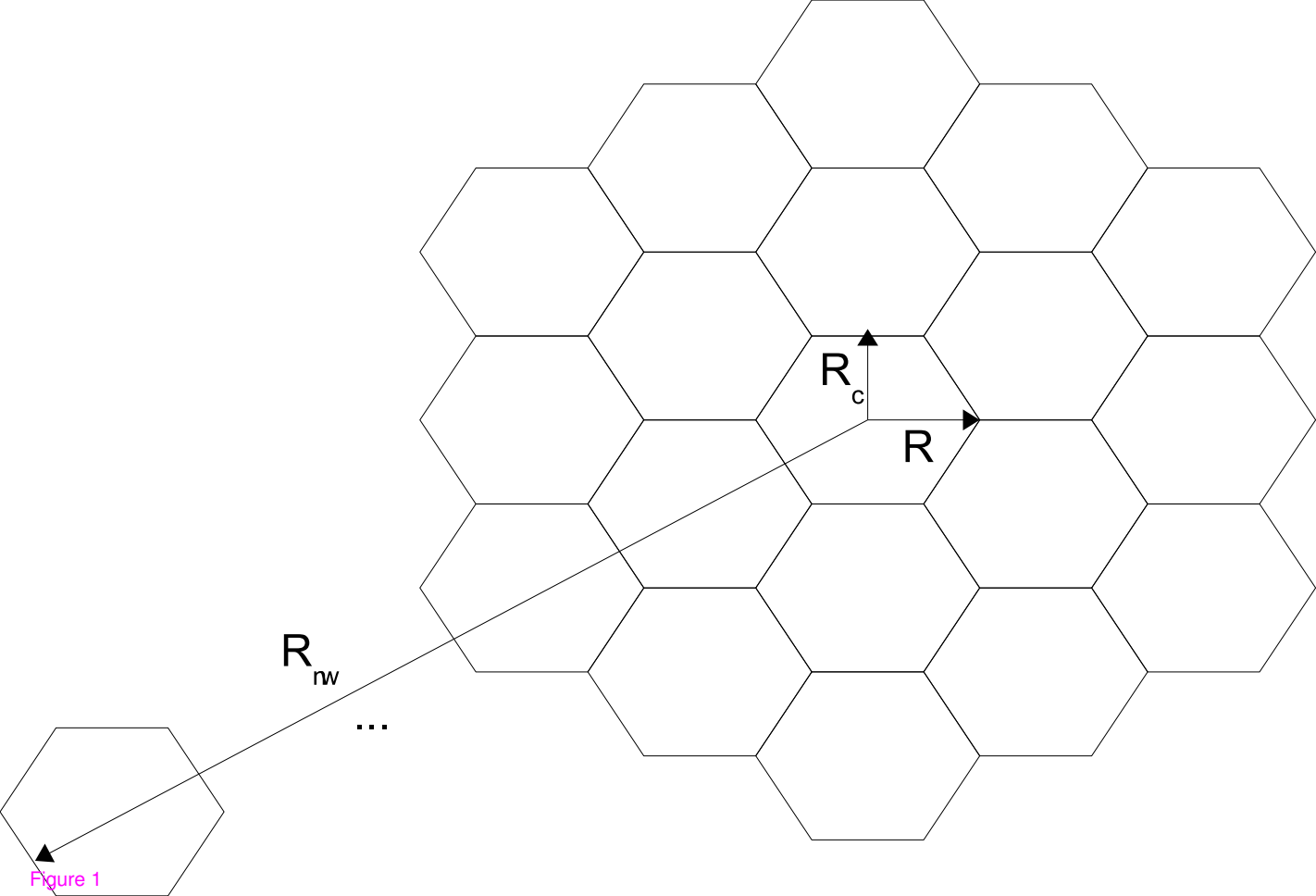
Figure 4: Influence of path-loss exponent η with $\sigma = 4$ dB and $r = 0.2$ km.

Table 2: CDF difference in dB between Monte Carlo Simulations (SIM) on the one hand and CLCFM and FWBM on the other hand at 5, 50 and 90% ($\sigma = 7$ and 8 dB, * means greater than 3 dB).

$d[\text{Km}]$	η	$\sigma[\text{dB}]$	CLCF [dB]	FW [dB]
7.0	2.7	0.2	0.2, 0.9, 2.0	0.3, 0.1, 0.2
7.0	2.7	0.5	0.1, 1.1, 2.8	0.4, 0.1, 0.1
7.0	2.7	0.9	0.1, 2.4, *	0.6, 0.0, 0.1
7.0	3.0	0.2	1.5, 2.6, *	1.2, 1.1, 1.0
7.0	3.0	0.5	0.8, 3.0, *	1.2, 0.8, 1.2
7.0	3.0	0.9	2.4, *, *	1.7, 1.0, 1.4
7.0	3.3	0.2	0.2, *, *	0.6, 1.3, 2.8
7.0	3.3	0.5	2.3, *, *	1.7, 1.6, 2.4
7.0	3.3	0.9	2.6, *, *	1.6, 1.5, 2.0
7.0	3.5	0.2	2.9, *, *	2.4, 1.8, 2.2
7.0	3.5	0.5	1.3, *, *	0.6, 1.5, 2.4
7.0	3.5	0.9	1.7, *, *	0.5, 2.2, 2.7
8.0	2.7	0.2	0.4, 2.9, *	0.9, 0.8, 0.6
8.0	2.7	0.5	2.6, *, *	1.9, 0.4, 0.3
8.0	2.7	0.9	1.0, *, *	0.0, 0.4, 1.3
8.0	3.0	0.2	1.3, *, *	0.6, 1.1, 1.7
8.0	3.0	0.5	2.7, *, *	1.6, 1.1, 2.2
8.0	3.0	0.9	*, *, *	2.0, 1.7, 2.3
8.0	3.3	0.2	2.7, *, *	1.7, 2.6, 3.0
8.0	3.3	0.5	2.1, *, *	0.7, 1.9, *
8.0	3.3	0.9	2.9, *, *	0.5, 2.4, *
8.0	3.5	0.2	*, *, *	2.1, 3.0, *
8.0	3.5	0.5	2.7, *, *	1.0, 2.2, *
8.0	3.5	0.9	*, *, *	1.2, 2.2, *

Figure 5: Network and cell of interest in the fluid model; the distance between two BS is $2R_c$ and the network is made of a continuum of base stations.

Figure 6: Integration limits for function g computation.



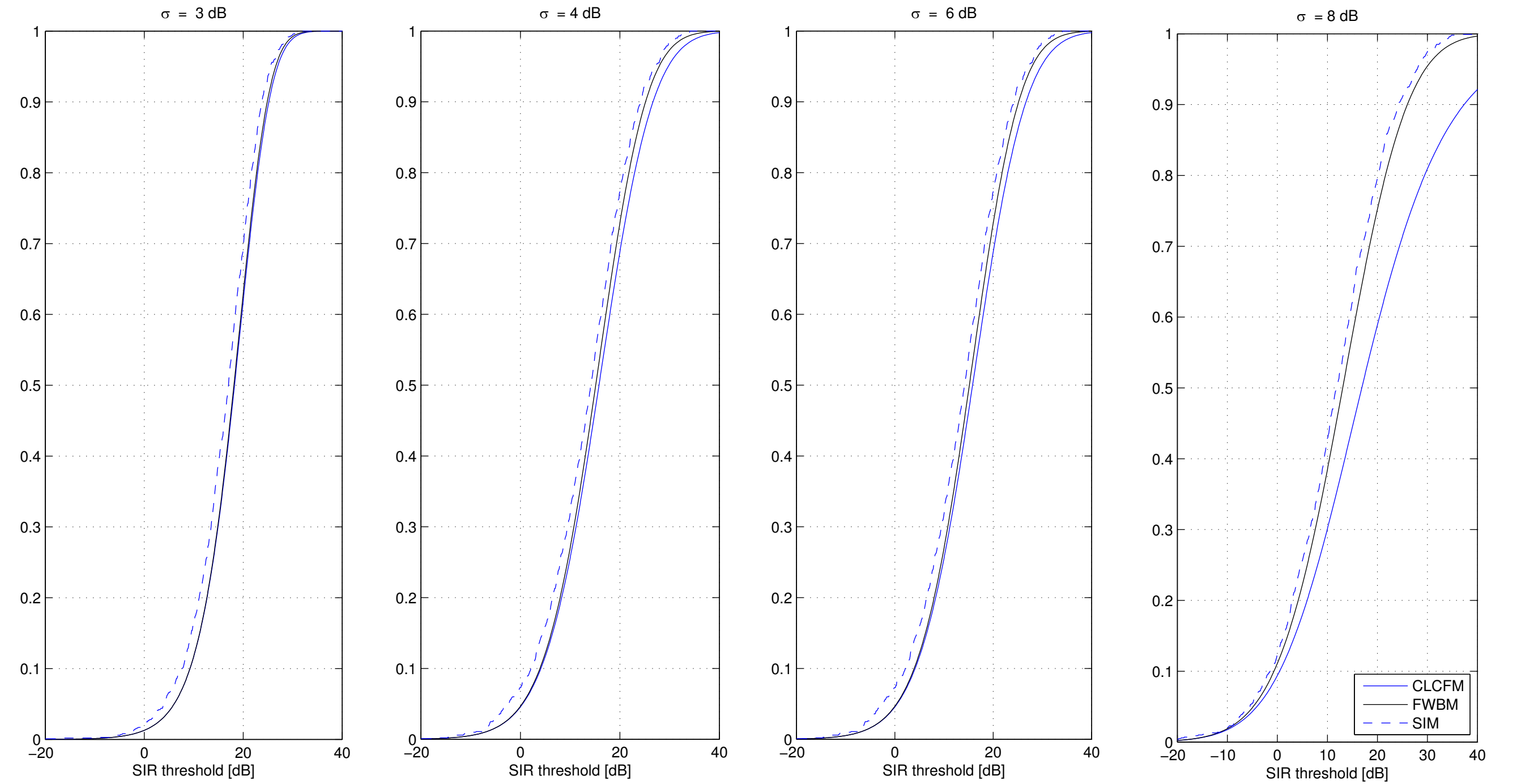


Figure 2

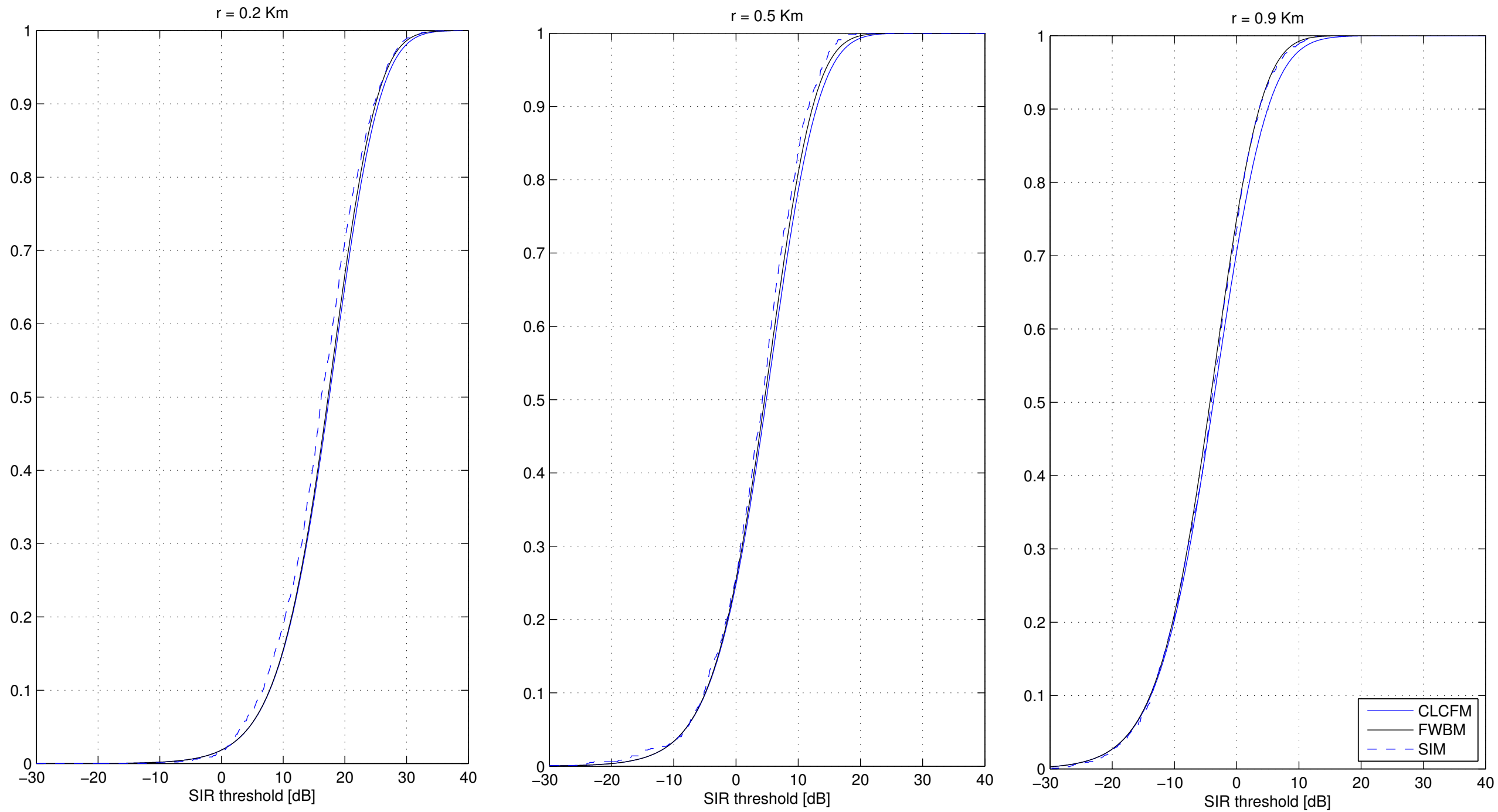


Figure 3

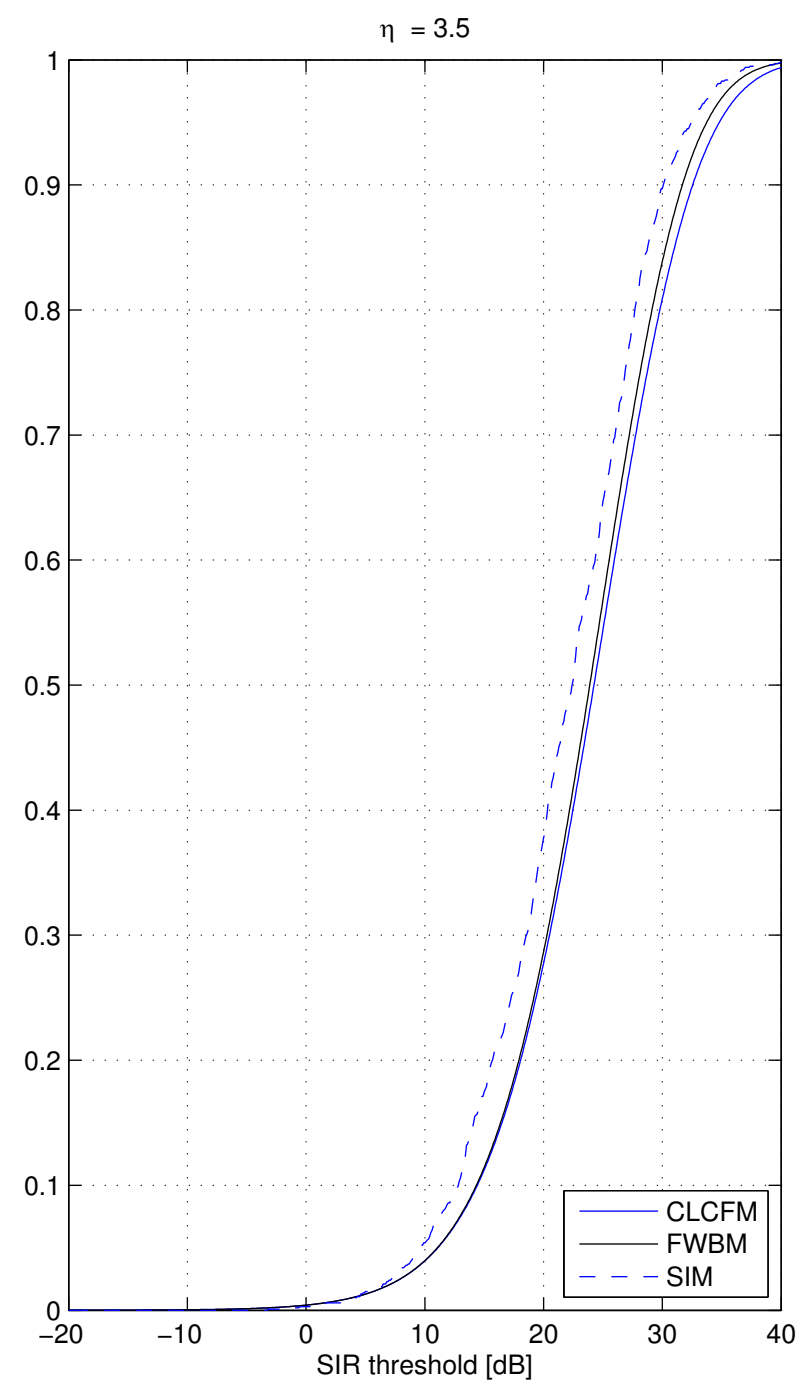
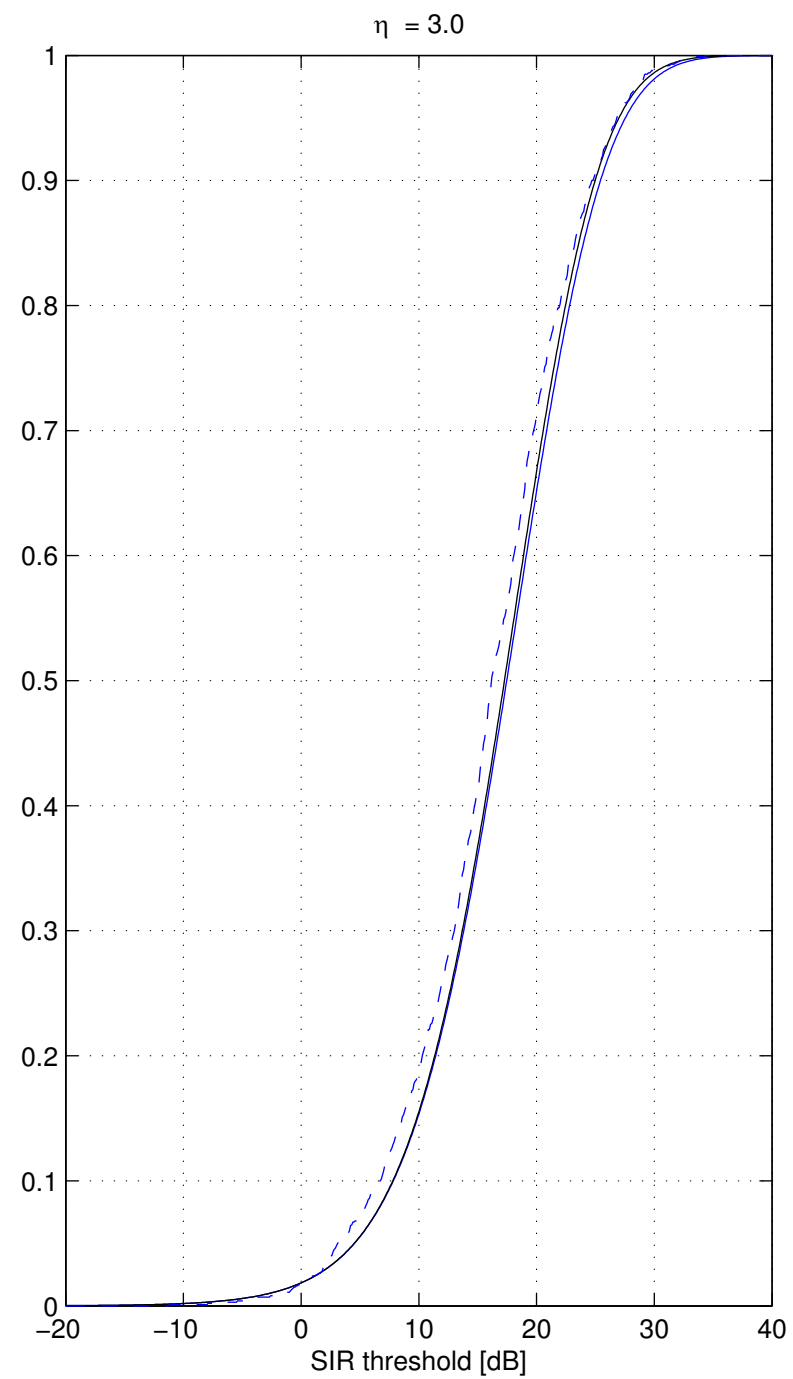
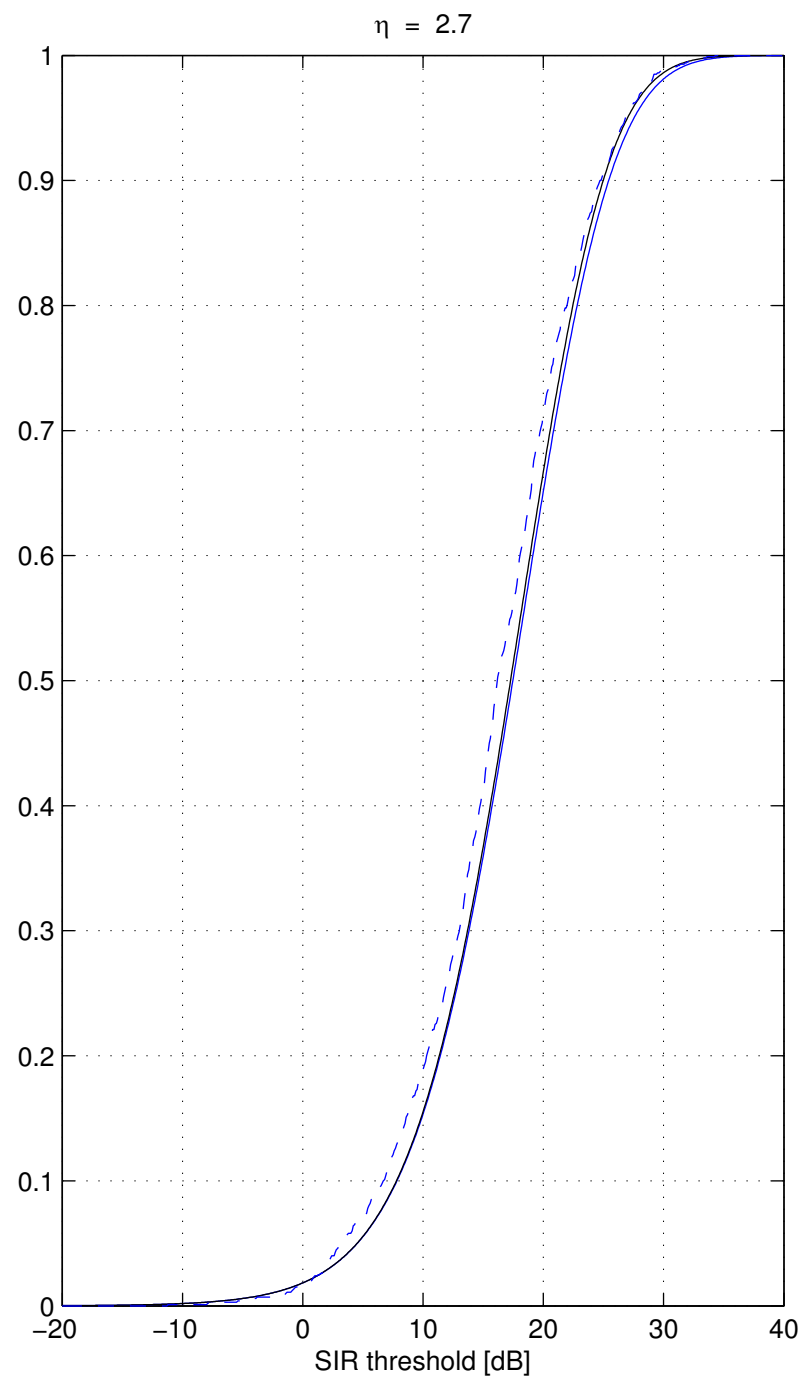


Figure 4

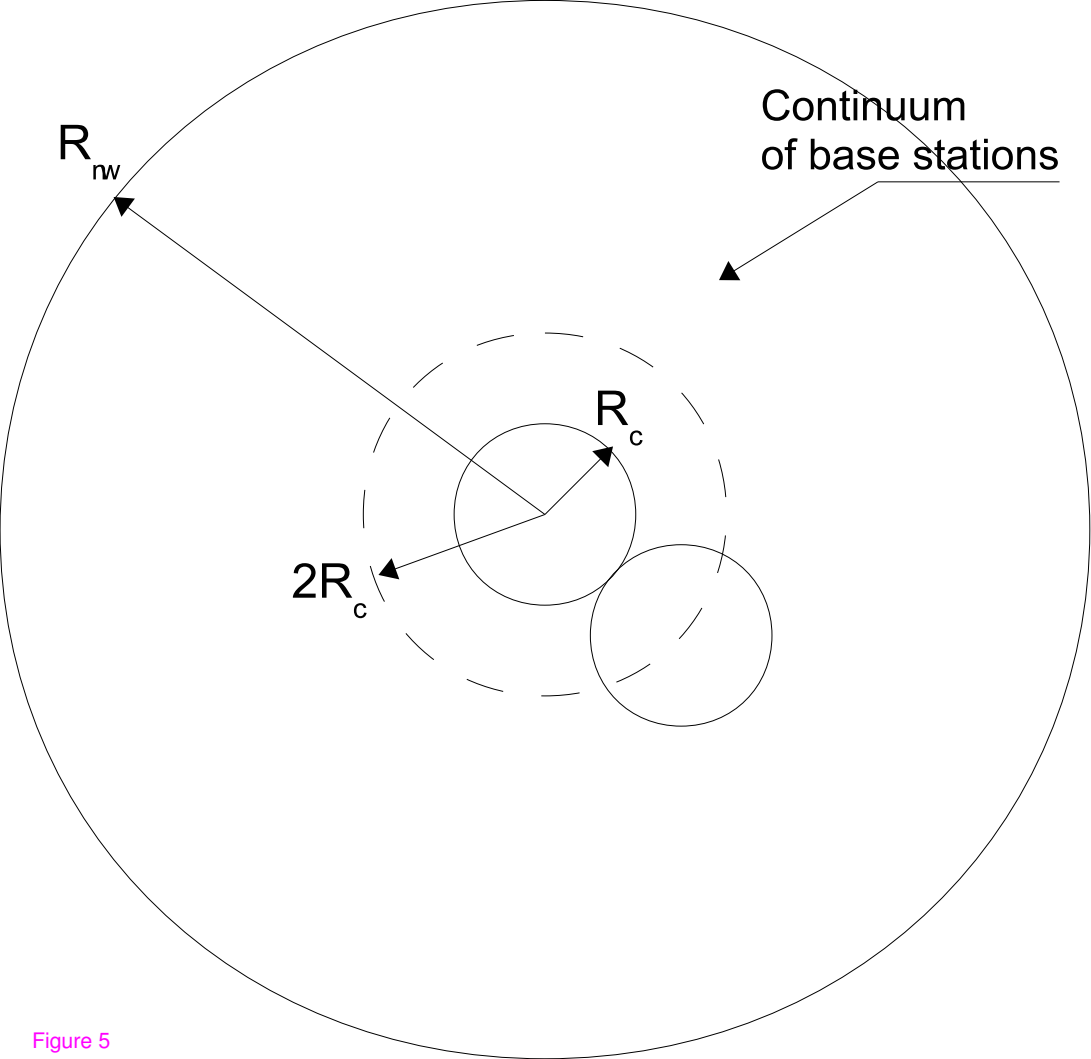


Figure 5

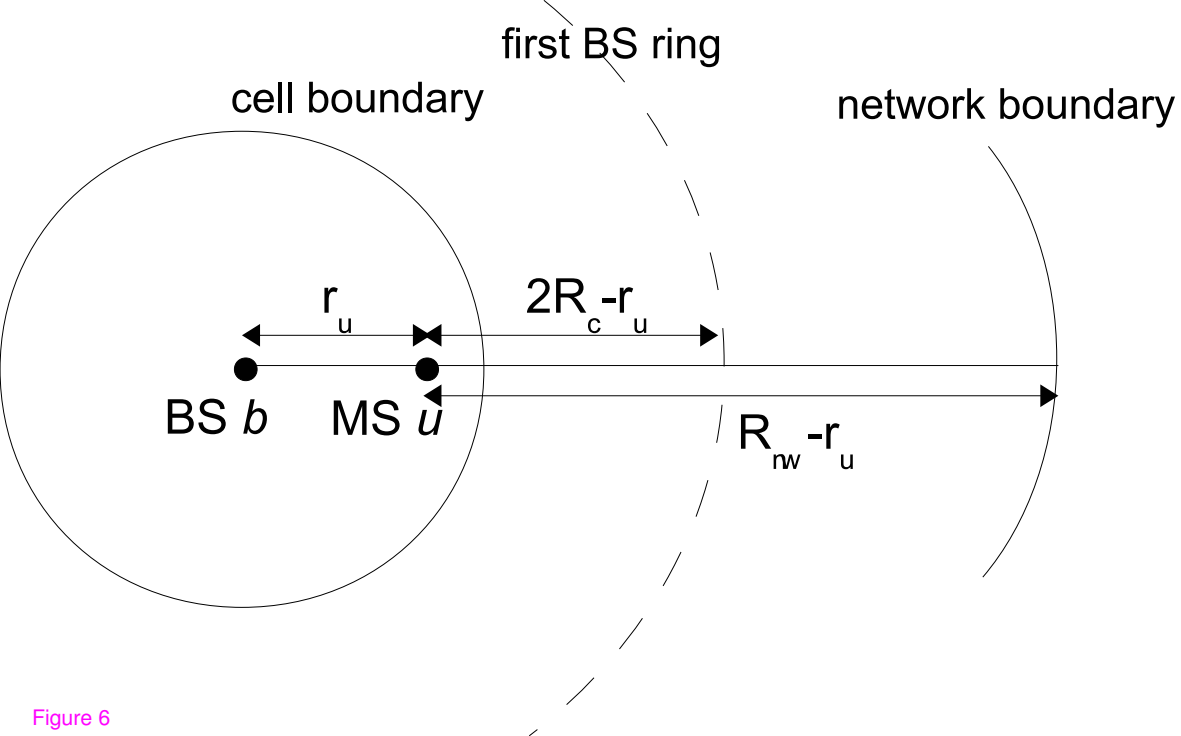


Figure 6