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Progress Report

N-party Hardy Nonlocality Paradox for Symmetric States

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I. INTRODUCTION

Quantum mechanics has been known as a "controversial" theory mainly because of nonlocality. The famous 1935 paper by Einstein, Podolsky and Rosen [1] questions the validity of quantum mechanics because of the seemingly impossible nonlocal behavior in the theory. From that time on, our understanding of nonlocality evolved both on the theoretical [2] and experimental [3] fronts. With the arrival of quantum information, nonlocality has really found its use in a wide range of fields, ranging from winning games to the protection of privacy [4] [5] [6] [7].

The first application of nonlocality is communication complexity. Nonlocality helps reduce the information transfer needed to compute certain distributed functions [4]. Consider the following example: Alice and Bob each receive an n-bit input x and y, respectively. They are promised that either $x = y$ or x and y differ in exactly half the positions (obviously for this purpose we suppose n is an even number). They want to find out which case they are in with minimum communication. The optimum classical result, based on a combinatorial result, gives $0.007n$ bit as the lower bound of the communication needed to perfectly distinguish the two cases. However, using shared quantum entanglement they only need to communicate $\log_2 n$ bits.

Another aspect of quantum nonlocality closely related to communication complexity and showing a clear advantage in the quantum setting is quantum games, where "winning the game" means successful transfer of information or successful computation of some distributed task. In this context, nonlocality helps to give a higher winning probability than what is classically possible [7]. Consider the following simple game: Alice Bob receive one bit each, and after they have received their bit, they are not allowed to communicate with each other. They win if they can produce one output bit each, whose XOR is equal to the AND of the input bits. Classically they only have a winning probability of $\frac{3}{4}$, no matter what strategy they use. Using quantum nonlocality, they can achieve a winning probability of $cos^2(\frac{\pi}{8}) \approx 0.853$. As an even more striking example, consider another simple game: Alice, Bob and Charlie each receives an input bit, after this they are forbidden from communicating with each other. They win if they can produce three output bits, whose XOR is 0 if all three input bits are 0, and 1 if two out of the three input bits are 1. Classically, the best they can do is to achieve a winning probability of $\frac{3}{4}$, but with the help of nonlocality, they can achieve perfect winning probability!

Lastly, a new trend in cryptographic systems is so called "device independence" [8]. Traditionally in cryptography, the device used to encrypt/decrypt has to be trusted by all parties involved. This usually means that the device is manufactured by a well-known company and/or has passed some internationally recognized standard test to ensured its effectiveness. There is nothing to stop an eavesdropper, at least in theory, to tamper with the device without other people noticing it. With the help of nonlocality, the assumption made on these devices can be removed, leaving only laws of nature to guarantee the privacy of the communication [9] [10].

All the results above are relatively new (some very new) and could lead to important applications. Motivated by this, we want to explore some new features of nonlocality. Our main goal is to use nonlocality to classify entanglement. We want to use the structure of entanglement, which is a topic better understood, to help us discover the structure of nonlocality. The results in this report are a first step towards this goal.

II. TWO-PARTY HARDY PARADOX AND THE MAJORANA REPRESENTATION

Locality, as commonly understood in quantum theory, states that experimental results from separate locations do not interfere with each other. Another related concept is *realism*, which states that all physical properties that can be measured have predefined values. They both seem to be reasonable assumptions about a physical theory, and taken together, they are known as *local realism*. To add a mathematical flavor to it, we often suppose that the seemingly random results obtained by quantum measurements are actually decided by a random variable shared by all parties participating in the measurement, known as the *local hidden variable (LHV)*. In other words, what seems random in quantum theory is not random at all, there is an unknown function giving all the measurement results. Different "paradoxes" and "inequalities" exist to show that local realism (and LHV) is in conflict with how the world works in the quantum level [11] [12].

Suppose Alice and Bob each has a measurement device. The device can measure either one of two properties: color and shape. If one chooses to measure color, then the result will be either red or blue. If one chooses to measure shape, then the result will be either $\star$

or $\circ$. Alice and Bob can freely choose which property to measure, but their results will be correlated in the following way:

1. If Alice and Bob both choose to measure color, then sometimes they both get red.

2. If Alice and Bob both choose to measure shape, then they never both get $\circ$.

3. If Alice measures color and gets red, then if Bob measures shape he never gets $\star$.

4. If Alice measures shape and gets $\star$, then if Bob measures color he never gets red.

Consider one particular run of the experiment: Alice and Bob both choose to measure color and they both get red. This is possible because of (1). According to (3), should Bob chose to measure shape instead of color in this run, he will get $\circ$ with certainty. Similarly, according to (4), because Bob measured color and got red, should Alice chose to measure shape instead of color in this run, she would not get $\star$, she would get $\circ$ with certainty. Taking both conclusions together, should Alice and Bob chose to measure shape instead of color in this run of the experiment, they will both get $\circ$ with certainty, which will be in contradiction with (2), hence the paradox.

In a local hidden variable model, Alice and Bob share a hidden variable λ, which satisfies local realism:

$$P(A, B|a, b) = \int P(A|a, \lambda)P(B|b, \lambda)\rho(\lambda)d\lambda \tag{1}$$

where a, b denote the property to be measured chosen by Alice and Bob, A, B denote the results obtained by Alice and Bob, respectively. This is the mathematical statement of the principles of the first paragraph of this section. The correlations above translate to:

$$P(red, red|color, color) > 0 \tag{2}$$

$$P(\circ, \circ|shape, shape) = 0 \tag{3}$$

$$P(red, \star|color, shape) = 0 \tag{4}$$

$$P(\star, red|shape, color) = 0 \tag{5}$$

The paradox arises because a particular value of the hidden variable λ governs a particular run of the experiment. Following the reasoning above we get $P(\circ, \circ|shape, shape) = 1$ from (4) and (5), which contradicts (3).

Moving away from the paradox temporarily, we will introduce the swiss army knife that allows us to find suitable measurement basis to demonstrate the paradox and extend it to n-party: the Majorana representation [13].

An elegant geometrical way to visualize symmetric states using the Bloch sphere, the Majorana representation has been used to give a simple geometrical proof of the Kochen-Specker Theorem [14] [15] and to classify entanglement in symmetric states [16] [17] [18] [19] [20].

Permutation symmetric states are multi-party states invariant under the permutation of any two parties:$|\psi_{sym}\rangle = P_{ij}|\psi_{sym}\rangle$. Examples of permutation symmetric states includes states commonly used in quantum information processing like the GHZ and the W state. The Majorana representation decomposes an n-party permutation symmetric state into the sum of permutations of n qubits:

$$|\psi_{sym}\rangle = C(\sum_{perm} |\eta_1\rangle|\eta_2\rangle \dots |\eta_n\rangle)$$

where C is a normalization constant and the $|\eta_i\rangle$ are qubits, which we call *Majorana Points (MPs)*. This decomposition allows the state to be visualized by a set of n unordered points on the Bloch sphere. Different points can lie on the same spot on the sphere. When this happens, the points that lie on the same spot are called *degenerate*. If all points lie on the same spot, then the state is a product state: no longer entangled, but still permutation symmetric.

Fig. 1 is an example: the GHZ state in Majorana representation. The three Majorana Points, which all lie on the equator, are :

$$\begin{aligned}
|\eta_1\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
|\eta_2\rangle &= (\cos\frac{\pi}{4})|0\rangle + (e^{\frac{2\pi i}{3}} \sin\frac{\pi}{4})|1\rangle \\
|\eta_2\rangle &= (\cos\frac{\pi}{4})|0\rangle + (e^{\frac{4\pi i}{3}} \sin\frac{\pi}{4})|1\rangle
\end{aligned}$$

It can be easily verified that

$$\begin{aligned}
|GHZ\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\
&= \sum_{perm}(|\eta_1\rangle|\eta_2\rangle|\eta_3\rangle)
\end{aligned}$$

When the Majorana points only occur at North and South poles, the state is called a *Dicke state* [21]. We use $|S(n,k)\rangle$ to denote Dicke states, where n is the number of qubits and

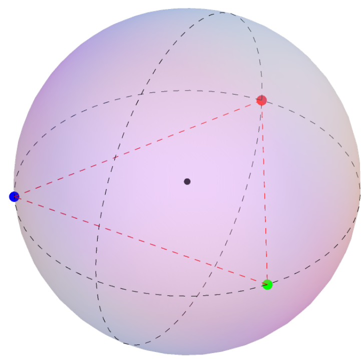


FIG. 1: The GHZ state in Majorana representation

k is the number of $|1\rangle$s.Dicke states are special because they have rotational symmetry along the z axis, and it is very difficult to break this symmetry. This makes the entanglement in Dickes states somewhat more resilient, but difficult to show nonlocal behavior via violation of an inequality that uses the Hardy paradox (though other methods exist [22] [23]). We will treat Dicke states as special cases later. Fig. 2 gives two examples of Dicke states, the first one is the W state $|S(3,1)\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$, the second one is the state $|S(2,2)\rangle = \frac{1}{\sqrt{6}}(|0011\rangle + |1100\rangle + |1010\rangle + |0101\rangle + |1001\rangle + |0110\rangle)$.

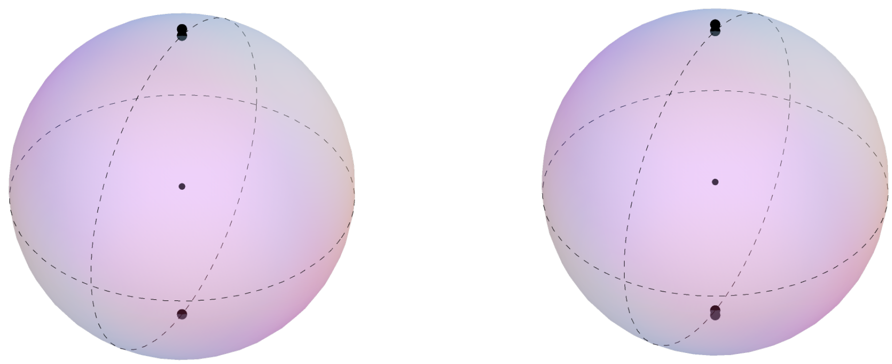
The Majorana representation has some interesting properties. Here we mention a few that will become useful later:

Property 1. *Local unitaries correspond to rotations of the sphere:*
$U \otimes \ldots \otimes U|\psi\rangle = C \sum_{perm} U|\eta_1\rangle U|\eta_2\rangle \ldots U|\eta_n\rangle$

Property 2. *It is easy to find product states orthogonal to the permutation symmetric state:*
${}^{n\otimes}\langle\phi|\psi\rangle = 0$, *if and only if* $|\phi\rangle = |\eta_i^{\perp}\rangle, \forall i = 1 \ldots n$, *with* $\langle\eta_i^{\perp}|\eta_i\rangle = 0$

Property 3. *After a projective measurement on one party, the remaining state is still permutation symmetric, but (usually) with different MPs:*
$\langle a|\psi\rangle = C' \sum_{perm}(|\gamma_1\rangle|\gamma_2\rangle \ldots |\gamma_{(n-1)}\rangle)$

It should be noted that the word "usually" in Property 3 refers to the exception of one



(a) W state (b) $|S(2,2)\rangle$

FIG. 2: Examples of Dicke states: W state($|S(3,1)\rangle$) on the left, $|S(2,2)\rangle$ on the right

special case: Dicke states. When a Dicke state is measured in the $\{|0\rangle, |1\rangle\}$ basis, which is also the basis of all its MPs, the remaining state is still a Dicke state, but with one less point in the north or south pole, depending on the measurement result. For all other permutation symmetric states, at least one MP will be different after a projective measurement on one party. This makes Dicke states the only permutation symmetric states on which our n-party Hardy paradox and inequality do not work.

Armed with the Majorana representation, we will use it to find suitable measurement basis to satisfy the probabilistic statements in the beginning of this section. First of all, a 2-party permutation symmetric state $|\psi\rangle$ can be rewritten as:

$$|\psi\rangle = K(|\eta_1\rangle|\eta_2\rangle + |\eta_2\rangle|\eta_1\rangle) = K(|\eta_1\rangle|\gamma\rangle + \langle\eta_1^\perp|\eta_2\rangle|\eta_1^\perp\rangle|\eta_1\rangle)$$

We can use the table below to translate from the *color* basis to $\{|\gamma\rangle, |\gamma^\perp\rangle\}$ basis, and from the *shape* basis to $\{|\eta_1\rangle, |\eta_1^\perp\rangle\}$ basis:

Because $P(\alpha, \beta|\psi) = |\langle\alpha|\langle\beta|\psi\rangle|^2$, translating the probabilities we had earlier into ampli-

$\|\gamma\rangle = blue$	$\|\gamma^{\perp}\rangle = red$
$\|\eta_1\rangle = \star$	$\|\eta_1^{\perp}\rangle = \circ$

TABLE I: Basis Translation Table (2-party)

tudes using the table above, we have:

$$\langle\gamma^{\perp}|\langle\gamma^{\perp}|\psi\rangle \neq 0 \tag{6}$$

$$\langle\gamma^{\perp}|\langle\eta_1|\psi\rangle = 0 \tag{7}$$

$$\langle\eta_1|\langle\gamma^{\perp}|\psi\rangle = 0 \tag{8}$$

$$\langle\eta_1^{\perp}|\langle\eta_1^{\perp}|\psi\rangle = 0 \tag{9}$$

Note that the first condition is not alway satisfied by an arbitrary permutation symmetric state. It is satisfied when $|\gamma\rangle$ is neither $|\eta_1\rangle$ nor $|\eta_2\rangle$. From the discussion above we know that Dicke states do not satisfy this condition. In 2-party case, Dicke states correspond to the maximally entangled states, which means that the paradox (and the associated inequality) works for all 2-qubit entangled states except maximally entangled states, a result shown by Hardy in his original paper [24]. Since all 2-qubit states can be brought to a symmetric state by local unitary (i.e. Schmidt basis), the method given above can be used on all 2-qubit states except the maximally entangled states [24].

III. N-PARTY HARDY PARADOX AND THE MAJORANA REPRESENTATION

Now we are in a position to generalize the 2-party paradox to n-party. The basic idea remains the same: there is a positive probability that everyone gets red when measuring the *color* basis, but assuming local realism, we can show this is impossible from other probabilistic statements. Using the same conventions as in the 2-party case, the probabilistic statements of the n-party paradox are (r = *red*, S = *shape*, C = *color*):

$$P(r, r, \dots, r|C, C, \dots, C) > 0 \tag{10}$$

$$P(\star, r, \dots, r|S, C, \dots, C) = 0 \tag{11}$$

$$P(r, \star, \dots, r|C, S, \dots, C) = 0 \tag{12}$$

$$\vdots$$

$$P(r, r, \dots, \star|C, C, \dots, S) = 0 \tag{13}$$

$$P(\circ, \circ, \dots, \circ|S, S, \dots, S) = 0 \tag{14}$$

For an arbitrary n-qubit entangled pure state, there is no guarantee that we know of that the two sets of measurement basis which give the probabilities above exist. For permutation symmetric states however, we can use the Majorana representation to systematically find them. For this purpose we need the last two of the three properties of the Majorana representation listed in the last section:

- *It is easy to find a product state orthogonal to a permutation symmetric state.*

- *After a projective measurement on one party, the remaining state is still a permutation symmetric state, but (usually) with different MPs.*

Suppose the n parties share a permutation symmetric state $|\psi\rangle$. We will start by looking at the last probabilistic statement. It is obvious we can use Property 1 to achieve this probability, if we take $|\eta_i^\perp\rangle = \circ$. This also implies that $|\eta_i\rangle = \star$. The n statements in the middle are actually the same because of the symmetry of $|\psi\rangle$. For simplicity, we will only look at the first one, $P(\star, r, \dots, r|S, C, \dots, C) = 0$. The first $\star$ corresponds a projection to $|\eta_i\rangle$. According to Property 2, the remaining $(n-1)$-party state is still permutation symmetric, with MPs $\{|\gamma_1\rangle, |\gamma_2\rangle, \dots, |\gamma_{n-1}\rangle\}$. To make this probability zero, all we have to do is to set $r = |\gamma_j^\perp\rangle$. The choice of $|\eta_i\rangle$ and $|\gamma_j\rangle$ is very flexible but not arbitrary. Because the first statement stipulates that $|\gamma_j\rangle$ is not a MP of $|\psi\rangle$, otherwise the probability is zero. For all permutation symmetric states except Dicke states, we can always choose a $|\gamma_j\rangle$ different from all the $|\eta_i\rangle$s so that all the probabilistic statement above are satisfied. Similar to the 2-party case above, the only permutation symmetric states for which it is impossible to find acceptable $|\gamma_j\rangle$s for all $|\eta_i\rangle$s are the Dicke states. Because the $|\eta_i\rangle$s are either $|0\rangle$ or $|1\rangle$, and after a projection to $|0\rangle$ or $|1\rangle$, the state is still a Dicke state with only $|0\rangle$ and $|1\rangle$ as MPs, this makes all choices of $|\gamma_j\rangle$ equal to at least one $|\eta_i\rangle$.

The two tables below lists explicitly the translation of basis and the associated amplitudes to achieve the probabilities above.

$\lvert\gamma_j\rangle = b$	$\lvert\gamma_j^\perp\rangle = r$
$\lvert\eta_i\rangle = \star$	$\lvert\eta_i^\perp\rangle = \circ$

TABLE II: Basis Translation Table (n-party)

$$({}^{n\otimes}\langle\gamma_j^\perp|)|\psi\rangle \neq 0 \tag{15}$$

$$\langle\eta_i|({}^{(n-1)\otimes}\langle\gamma_j^\perp|)|\psi\rangle = 0 \tag{16}$$

$$\langle\gamma_j^\perp|\langle\eta_i|({}^{(n-2)\otimes}\langle\gamma_j^\perp|)|\psi\rangle = 0 \tag{17}$$

$$\vdots$$

$$({}^{(n-1)\otimes}\langle\gamma_j^\perp|)\langle\eta_i|\psi\rangle = 0 \tag{18}$$

$$({}^{n\otimes}\langle\eta_i^\perp|)|\psi\rangle = 0 \tag{19}$$

The probabilistic statements make a logical paradox, but not an experimentally testable prediction. In order to do experimental tests, we need to put these probabilistic statements into an inequality with an upper or lower bound derived from local realism, then show that quantum mechanics can violate this bound by using the right state/measurement combination. In the 2-party case, Hardy showed that the CH-inequality can be violated by almost all entangled states. Here we give a similar inequality for n-party, prove its classical upper bound (i.e. the bound achieved by assuming local realism or LHV theory), then we will show that this inequality is violated for all permutation symmetric states excluding the Dicke states.

The inequality is given below. For simplicity, we have omitted the choice of basis in the probabilities, assuming this is evident given the measurement result.

Inequality 1. $P(r, r, \ldots, r) - P(\star, r, \ldots, r) - P(r, \star, \ldots, r) - \ldots - P(r, r, \ldots, \star) - P(\circ, \circ, \ldots, \circ)$ ≤ 0

Proof. Assuming LHV, joint probabilities are products of probabilities of each party (cf. Eq. 1): $P(r, r, \ldots, r) = P_1(r)P_2(r)\ldots P_n(r)$, $P(\circ, \circ, \ldots, \circ) = P_1(\circ)P_2(\circ)\ldots P_n(\circ), \ldots$, where the subscripts are added to show probabilities of obtaining the same result by different parties are independent.

The inequality above can be rewritten as:
$P_1(r)P_2(r)\ldots P_n(r) - P_1(\circ)P_2(\circ)\ldots P_n(\circ) - (1-P_1(\circ))P_2(r)\ldots P_n(r) - \ldots - P_1(r)\ldots P_{n-1}(r)(1-P_n(\circ)) \leq 0$, which can be further expanded as:

$$\begin{gathered}
P_1(r)P_2(r)\ldots P_n(r) - P_1(\circ)P_2(\circ)\ldots P_n(\circ) \\
+P_1(\circ)P_2(r)\ldots P_n(r) - P_2(r)P_3(r)\ldots P_n(r) \\
+P_1(r)P_2(\circ)\ldots P_n(r) - P_1(r)P_3(r)\ldots P_n(r) \\
\vdots \\
+P_1(r)P_2(r)\ldots P_n(\circ) - P_1(r)P_2(r)\ldots P_{n-1}(r) \leq 0
\end{gathered}$$

Focusing on the second and third term in the inequality above, we have: $P_1(\circ)(P_2(r)\ldots P_n(r) - P_2(\circ)\ldots P_n(\circ))$. If $P_2(r)\ldots P_n(r) \leq P_2(\circ)\ldots P_n(\circ)$, then the inequality above holds, and we have terminated our proof. Otherwise, $P_1(\circ)(P_2(r)\ldots P_n(r) - P_2(\circ)\ldots P_n(\circ)) \leq (P_2(r)\ldots P_n(r) - P_2(\circ)\ldots P_n(\circ))$, which means the inequality above $\leq$

$$\begin{gathered}
P_1(r)P_2(r)\ldots P_n(r) - P_2(\circ)P_3(\circ)\ldots P_n(\circ) \\
+P_1(r)P_2(\circ)\ldots P_n(r) - P_1(r)P_3(r)\ldots P_n(r) \\
\vdots \\
+P_1(r)P_2(r)\ldots P_n(\circ) - P_1(r)P_2(r)\ldots P_{n-1}(r) \leq 0
\end{gathered}$$

Repeat the same procedure n times, each time assuming the product of $P(r)$ is bigger than the product of $P(\circ)$ (otherwise the inequality holds and we can terminate the proof). What is left when this has been done is $P_1(r)P_2(r)\ldots P_n(r) - P_1(r)P_2(r)\ldots P_{n-1}(r) \leq 0$, which is clearly true. Thus Inequality 1 is true[1]. □

The inequality given in Inequality 1 is violated by all non-Dicke permutation symmetric states using the basis we found in Table II. The only positive term in the inequality corresponds to the positive probability we have earlier, and all other terms in the inequality are zero, following our probabilistic statements.

[1] An alternative proof is given in [25]

IV. CONCLUSIONS

We used the Majorana representation to extend 2-party Hardy's paradox to n-party, we also gave an inequality whose violation shows nonlocality, and used the Majorana representation to find measurement settings for almost all permutation symmetric states that will show this violation (thus is nonlocal).

A recent experiment [26] has been done to test the temporal version of the 2-party Hardy paradox with violation of the CH-inequality. It shows that experimental demonstration of the paradox by violation of the inequality is indeed possible. However, no experiment has been done to test the n-party version for $n > 2$. Difficulties in preparation of a multiparty entangled state may be the biggest obstacle to experimental work. Optical methods do exist to generate multiparty permutation symmetric states, from the Dicke state [27] to more general ones [28]. Hopefully experimental methods will advance enough in the near future that allow us to perform a real test of nonlocality by showing a violation of the inequality given here.

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