# Outage Probability for Joint Processing Coordinated Multi-Point (JP-CoMP) Performance Analysis in LTE-Advanced Networks

Marceau Coupechoux*
* TELECOM ParisTech, INFRES/RMS
joint work with D. Ben Cheikh (Orange Labs/TPT), J.M. Kelif (Orange Labs) and Ph. Godlewski (TPT)

Indian Institute of Science, ECE Seminar

09 Mar. 2011

# Cellular Networks Evolution (3GPP) I

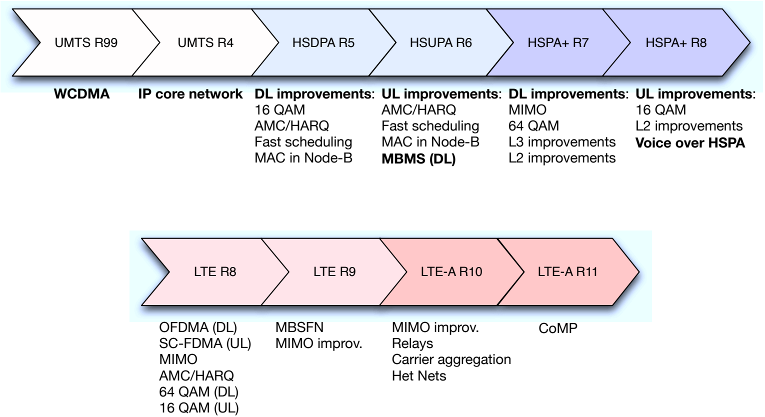
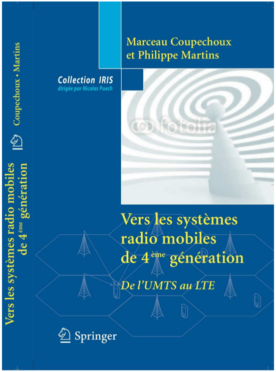



**Figure:** Evolution of the 3GPP Standards.

# Cellular Networks Evolution (3GPP) II




M. Coupechoux and Ph. Martins, "Vers la quatrième génération" (Towards Fourth Generation), Springer Verlag, 315 pp, to appear (hopefully) mid 2012.

# LTE

- LTE Requirements:
  - Peak data rates: 300 (DL) / 75 Mbps (UL)
  - Peak spectrum efficiency: 15 (DL) / 3.75 bps/Hz (UL)
  - Average spectrum efficiency: 2.67 (4x4 DL) / 0.74 (UL 1x2) bps/Hz/cell
  - U-plane latency $< 5$ ms
  - Spectrum flexibility: from 1.4 to 20 MHz
- LTE Key Features:
  - OFDMA (DL) / SC-FDMA (UL)
  - Adaptive Modulation (up to 64-QAM) and Coding / HARQ
  - MIMO (up to 4x4): Spatial Multiplexing, Transmit Diversity, Beamforming, MU-MIMO
  - Frequency selective fast scheduling
  - IP-based flat architecture

# LTE-Advanced Release 10

- LTE-Advanced Requirements:
  - Peak data rates: 1 Gbps (DL) / 500 Mbps (UL)
  - Peak spectrum efficiency: 30 (DL) / 15 bps/Hz (UL)
  - Average spectrum efficiency: 3.7 (4x4 DL) / 2.0 (UL 2x4) bps/Hz/cell
- LTE-Advanced Rel. 10 Features:
  - Carrier aggregation (up to 100 MHz)
  - Clustered SC-FDMA (UL)
  - MIMO: 8x8 (DL), 4x8 (UL), enhanced MU-MIMO
  - Relaying
  - Heterogeneous Networks (HetNets)

# Coordinated Multi-Point for LTE-A Rel. 11

- Goal: coordinate BS transmissions in order to reduce interference and improve cell-edge throughput.
- CS/CB: scheduling and beamforming are coordinated among clustered BS.
- JP: data is transmitted from several BS to the same UE *coherently* (tight synchronization is needed) or *non-coherently* (gain obtained from power boost).

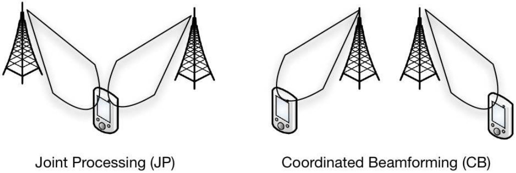



**Figure:** Coordinated Multi-Point (CoMP).

# Outage Probability and SIR CDF

SINR/SIR CDF is a crucial input for cellular network dimensioning:

- Coverage studies: $P[\gamma < \gamma_{th}]$, where $\gamma_{th}$ is the the common control channel SIR threshold provides coverage; $\gamma_{th}$ s.t. $P[\gamma < \gamma_{th}] = 10\%$ provides cell-edge throughput.
- Capacity studies: channel state distribution + M/G/1-PS (or other Markov chain [25]) approach provides capacity figures.

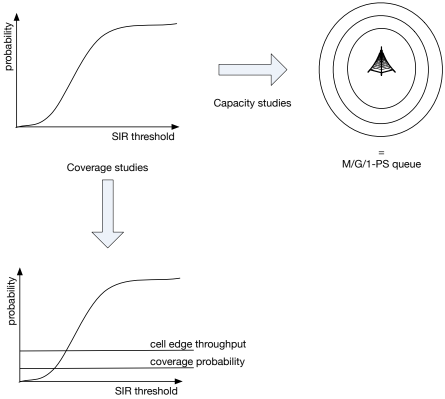



**Figure:** Outage probability and dimensioning issues.

# Outlines

- Related Work
- Maximum Ratio Transmission
- System Model
- Outage Probability
- Numerical Results

# Related Work I

- Performance of CoMP has been asserted in several works:
  - The JP strategy offers larger performance gain than the CS/CB [1, 2].
  - Multicell cooperation attain larger capacity than an isolated cell (measurement study [3]).
  - A field trial [4] confirms the throughput enhancement introduced by JP-CoMP.
  - In [5], different JP schemes are proposed and compared numerically.
  - In [6], zero forcing (ZF) and maximum ratio transmission (MRT) schemes are studied for femto cell coordination. Two power allocation algorithms are proposed and compared.
  - In [3, 4, 5, 6], authors performed simulation, measurement or field study but **no theoretical studies** were conducted.

# Related Work II

- There are few analytical works:
  - In [7], an analytical expression of the capacity outage probability was derived for an open-loop Alamouti-like CoMP downlink transmission in Rayleigh fading. The proposed SINR expression can only be achieved when using a distributed Alamouti for two cooperating BSs.
  - In [8], an analytical study of a multi-cell multi-antenna cooperative MRT/MRC scheme was conducted. An analytical expression of the PDF of the SIR was derived considering path-loss, shadowing and Rayleigh fading. Limitations of the study are:
    - User is supposed to be at equal distances from cooperative BSs,
    - Shadowing is modeled with a Gamma distribution,
    - Transmitters are supposed to be Poisson distributed,
    - Significant differences bw. simulations and theoretical results.
- Several works on feedback constraints.
- Several works on BS selection algorithms.

# Maximum Ratio Transmission I

Assume $n_t$ Tx antennas and $n_r$ Rx antennas without interference, then the received signal is:

$$y = Hws + n,$$

where

- $H$ is the $n_r \times n_t$ channel gain matrix,
- $w$ is the $n_t \times 1$ precoding vector with $||w||^2 \leq 1$,
- $s$ is the information symbol, $n$ is AWGN.

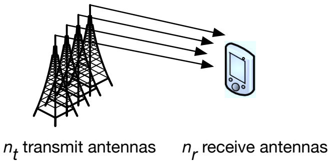



**Figure:** MRT Model.

# Maximum Ratio Transmission II

This equation can be interpreted as an AWGN channel with SNR:

$$\gamma = \frac{||Hw||^2}{\sigma_n^2} E[|s|^2].$$

The scheme that maximizes $||Hw||^2$ under the constraint $||w||^2 \leq 1$ is called MRT.

In general:

$$\frac{||Hw||^2}{||w||^2} \leq \lambda_{max}(H^H H),$$

and the optimal precoder is the normalized eigenvector associated to the largest eigenvalue of $H^H H$.

# Maximum Ratio Transmission III

If $n_r = 1$, then $H = h$ and using Cauchy-Schwartz inequality:

$$|hw|^2 \leq ||h||^2||w||^2 = ||h||^2,$$

with equality if $w$ is proportional to $h^*$.

The MRT precoder can thus be written:

$$w_{MRT} = \frac{h^*}{||h||}.$$

MRT maximizes the received signal power.

Drawbacks of the scheme:

- Interference is not taken into account.
- Perfect channel information is required at transmitter.

# System Model I

We consider the following system model:

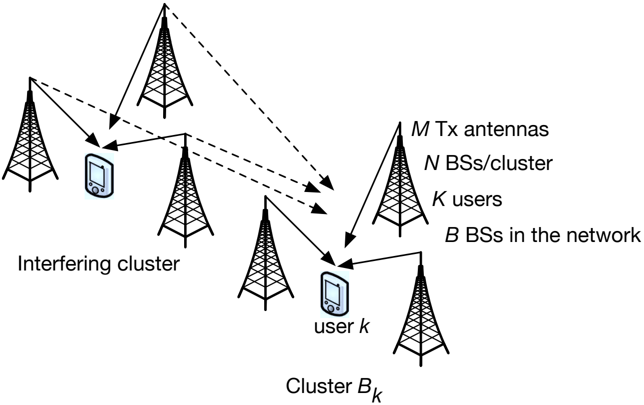



**Figure:** System Model.

# System Model II

We assume that JP-CoMP is based on MRT [14].

Received signal can be written:

$$y_k = \sum_{b \in B_k} \sqrt{p_{b,k}} \mathbf{h}_{b,k} \mathbf{x}_{b,k} + \sum_{i=1, i \neq k}^{K} \sum_{j \in B_i} \sqrt{p_{j,k}} \mathbf{h}_{j,k} \mathbf{x}_{j,i} + n, \quad (1)$$

where

- $p_{b,k}$ is the power received by user $k$ from BS $b$, $n$ is the AWGN,
- $\mathbf{h}_{b,k} \in \mathbb{C}^{1 \times M}$ is the complex Gaussian channel bw. $k$ and $b$,
- $\mathbf{x}_{b,k} \in \mathbb{C}^{M \times 1}$ is the MRT data vector:

$$\mathbf{x}_{b,k} = \frac{\mathbf{h}_{b,k}^*}{\|\mathbf{h}_{b,k}\|} s_k. \quad (2)$$

# System Model III

The received power $p_{b,k}$ includes path-loss and shadowing:

$$p_{b,k} = P_{Tx} C r_{b,k}^{-\eta} 10^{\frac{\xi_{b,k}}{10}}, \quad (3)$$

where $P_{Tx}$ is the total transmit power of BS $b$, $C$ is a constant, $r_{b,k}$ is the distance bw. $k$ and $b$, $\eta$ is the path-loss exponent and $\xi_{b,k}$ is $\mathcal{N}(0,\sigma)$. Shadowing is now assumed to be constant.

The output SIR perceived by a user $k$ is given by:

$$\gamma_k = \frac{\left(\sum_{b\in B_k} \sqrt{p_{b,k}} \|\mathbf{h}_{b,k}\|\right)^2}{\sum_{i=1,i\neq k}^{K} \left|\sum_{j\in B_i} \sqrt{p_{j,k}} \mathbf{h}_{j,k} \frac{\mathbf{h}_{j,i}^*}{\|\mathbf{h}_{j,i}\|}\right|^2} = \frac{X}{Y}. \quad (4)$$

# Outage Probability I

## Useful signal CDF

We can write:

$$X = U^2, \quad U = \sum_{b \in B_k} \sqrt{p_{b,k}} \left\| \mathbf{h}_{b,k} \right\| . \tag{5}$$

$U$ can also be written as:

$$U = \sum_{b \in B_k} \sqrt{p_{b,k}} \sqrt{\sum_{i=1}^{M} |h_{b,k,i}|^2}. \tag{6}$$

Closed form expression of the PDF of $U$ is not known, we thus rely on the central limit theorem for causal functions.

# Outage Probability II

## Theorem (CLCF)

*Consider $n$ RV $X_i \geq 0$ with density $f_i$, means $\eta_i$ and variances $\sigma_i^2$. Let $X = X_1 + \cdots + X_n$ with density $f(t) = f_1(t) \star f_2(t) \cdots \star f_n(t)$. Denote*

$$\begin{aligned} \eta &= \eta_1 + \eta_2 + \cdots + \eta_n, \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2. \end{aligned}$$

*Under certain conditions, $f(t) \sim \frac{t^\alpha e^{-t/\beta}}{\beta^{\alpha+1}\Gamma(\alpha+1)}$ as $n \longrightarrow \infty$, where $\alpha + 1 = \frac{\eta^2}{\sigma^2}$ and $\beta = \frac{\sigma^2}{\eta}$.*

## Proof.

see [13]. □

# Outage Probability III

The use of the CLCF provides:

$$f_U(u) = \frac{u^{\nu-1} e^{-\frac{u}{\theta}}}{\Gamma(\nu)\theta^{\nu}}. \quad (7)$$

where $\nu = \frac{\mathrm{E}[U]^2}{\mathrm{var}(U)}$ and $\theta = \frac{\mathrm{var}(U)}{\mathrm{E}[U]}$. Denoting $V = \sqrt{\sum_{i=1}^{M} |h_{b,k,i}|^2}$, $V$ is a square root of a Gamma RV and thus we can write the PDF of $V$ as:

$$f_V(v) = \frac{2}{(M-1)!} v^{2M-1} e^{-v^2}, \quad (8)$$

$\mathrm{E}[V]$ can be derived using (8) and is given by:

$$\mathrm{E}[V] = \int_0^{\infty} \frac{2}{(M-1)!} v^{2M} e^{-v^2} dv = \frac{(2M-1)!!}{2^M (M-1)!} \sqrt{\pi}, \quad (9)$$

# Outage Probability IV

So that:

$$\mathrm{E}[U] = \frac{(2M-1)!!}{2^M(M-1)!}\sqrt{\pi}\sum_{b\in B_k}\sqrt{p_{b,k}}. \quad (10)$$

On the other hand:

$$\begin{aligned}\mathrm{var}(U) &= \sum_{b\in B_k} p_{b,k}\mathrm{E}[\|\mathbf{h}_{b,k}\|^2] - \sum_{b\in B_k} p_{b,k}\mathrm{E}[\|\mathbf{h}_{b,k}\|]^2 \\ &= \left(M - \pi\left(\frac{(2M-1)!!}{2^M(M-1)!}\right)^2\right)\sum_{b\in B_k} p_{b,k}. \quad (11)\end{aligned}$$

Note: $(2N)!! = 2 \times 4 \times \cdots \times (2N)$ and $(2N+1)!! = 3 \times 5 \times \cdots \times (2N+1)$.

# Outage Probability V

The parameters $\nu$ and $\theta$ are thus given by:

$$\nu = \frac{(2M-1)!!^2}{M2^{2M}(M-1)!^2 - \pi(2M-1)!!^2} \frac{\left(\sum_{b \in B_k} \sqrt{p_{b,k}}\right)^2}{\sum_{b \in B_k} p_{b,k}}, \quad (12)$$

$$\theta = \frac{M2^{2M}(M-1)!^2 - \pi(2M-1)!!^2}{2^M(M-1)!(2M-1)!!\sqrt{\pi}} \frac{\sum_{b \in B_k} p_{b,k}}{\sum_{b \in B_k} \sqrt{p_{b,k}}}. \quad (13)$$

From (10) and (11):

- Mean useful signal power increases with $N$.
- Variability also increases with $N$.
- It can be shown that mean and variance increase with $M$.

# Outage Probability VI

Using the Gamma approximation of the PDF of $U$, we can derive the CDF of $X$ as follows:

$$F_X(v) = \int_0^{\sqrt{v}} \frac{u^{\nu-1}e^{-\frac{u}{\theta}}}{\Gamma(\nu)\theta^{\nu}}du, \quad (14)$$

$$= \frac{\gamma(\nu, \frac{\sqrt{v}}{\theta})}{\Gamma(\nu)}, \quad (15)$$

where $\gamma(.,.)$ is the lower incomplete gamma function:

$$\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt.$$

# Outage Probability VII

## Interference PDF

We can write now:

$$Y = \sum_{i=1, i \neq k}^{K} \left| \sum_{j \in B_i} \sqrt{p_{j,k}} g_{j,k,i} \right|^2, \tag{16}$$

where $g_{j,k,i} = \mathbf{h}_{j,k} \frac{\mathbf{h}_{j,i}^*}{\|\mathbf{h}_{j,i}\|}$.

$g_{j,k,i}$ is complex Gaussian independent of $\mathbf{h}_{j,i}$.

# Outage Probability VIII

## Claim ([15])

*Consider zero-mean complex Gaussian vectors* $\mathbf{h}_0 = [h_{1,0}, h_{2,0}, \cdots, h_{N,0}]^H$ *and* $\mathbf{h}_j = [h_{1,j}, h_{2,j}, \cdots, h_{N,j}]^H$*. Let* $g_j = \frac{\mathbf{h}_0^H \mathbf{h}_j}{\|\mathbf{h}_0\|}$*.*
*Then,* $g_j$ *is independent on* $\mathbf{h}_0$*.*

## Proof.

$g_j|\mathbf{h}_0$ is zero-mean complex Gaussian and:

$$E[g_j|\mathbf{h}_0] = \frac{\mathbf{h}_0^H}{\|\mathbf{h}_0\|} E[\mathbf{h}_j] = 0$$

$$E[|g_j|^2|\mathbf{h}_0] = \frac{\mathbf{h}_0^H E[\mathbf{h}_j \mathbf{h}_j^H] \mathbf{h}_0}{\|\mathbf{h}_0\|^2} = \frac{\mathbf{h}_0^H I_N \mathbf{h}_0}{\|\mathbf{h}_0\|^2} = 1$$

□

# Outage Probability IX

Let $c_{k,i} = \sum_{j \in B_i} \sqrt{p_{j,k}} g_{j,k,i}$, it is the sum of independent complex Gaussian random variables. $Y$ can be written as:

$$Y = \sum_{i=1, i \neq k}^{K} |c_{k,i}|^2 , \tag{17}$$

$\{c_{k,i}\}_{(i=1...K, i \neq k)}$ being independent zero-mean complex Gaussian elements with variances:

$$\lambda_{i,k} = \mathrm{var}(c_{k,i}) = \sum_{j \in B_i} p_{j,k}. \tag{18}$$

# Outage Probability X

The PDF of $Y$ is, hence, given by [16]:

$$f_Y(y) = \sum_{i=1, i \neq k}^{K} \frac{\Pi_i}{\lambda_{i,k}} \exp(-\frac{y}{\lambda_{i,k}}), \quad (19)$$

where $\Pi_i = \prod_{p=1..K, p \neq i, p \neq k} \frac{\lambda_{p,k}}{\lambda_{p,k} - \lambda_{i,k}}$.
Having the PDF of $X$ and the PDF of $Y$, and since they are independent RV, we derive the outage probability as follows:

$$\begin{aligned} P(\gamma_k < \gamma_{th}) &= \int F_X(\gamma_{th} y) f_Y(y) dy, & (20) \\ &= \sum_{i=1, i \neq k}^{K} \frac{\Pi_i}{\lambda_{i,k} \Gamma(\nu)} \int \exp(-\frac{y}{\lambda_i}) \gamma(\nu, \frac{\sqrt{\gamma_{th} y}}{\theta}) dy. & (21) \end{aligned}$$

# Outage Probability XI

The outage probability is given by [17] [18]:

$$P(\gamma_k < \gamma_{th}) = \sum_{i=1, i \neq k}^{K} \Pi_i \left( \frac{\sqrt{\gamma_{th} \lambda_{i,k}}}{2\theta} \right)^{\nu} U \left( \frac{\nu}{2}, \frac{1}{2}, \frac{\gamma_{th} \lambda_{i,k}}{4\theta^2} \right) \text{ (CoMP)}$$

where $U(.,.,.)$ is the confluent hypergeometric function of second kind:

$$U(a, b, z) = \frac{1}{\Gamma(a)} \int_0^{\infty} e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

# Numerical Results I

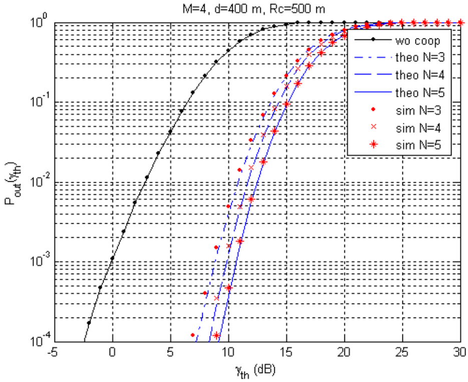



**Figure:** Comparison between simulated and analytical results of the outage probability without CoMP and with CoMP MRT strategy for 3, 4 or 5 cooperating BSs.

# Numerical Results II

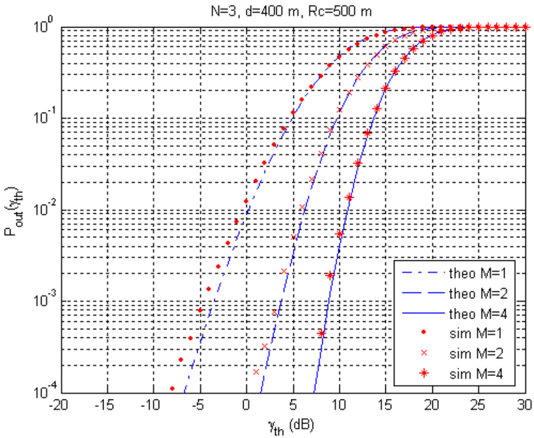



**Figure:** Influence of the number of transmit antennas.

# Numerical Results III

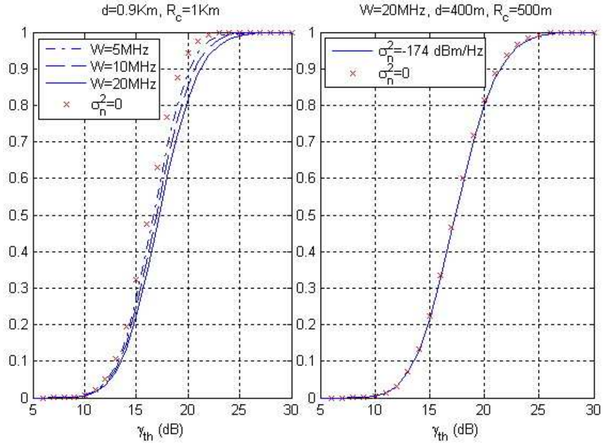



**Figure:** Impact of the noise power on the outage probability.

# Numerical Results IV

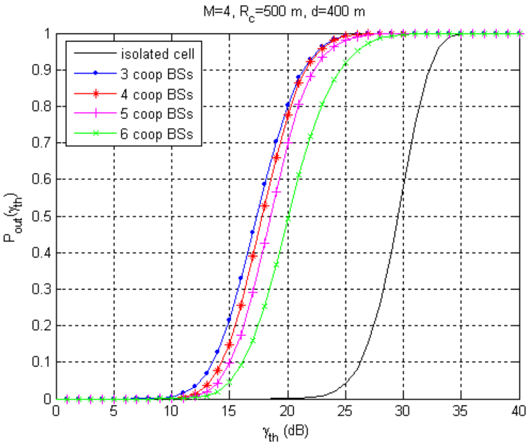



**Figure:** Comparison with an isolated cell.

# Other Obtained Results I

OFDMA Cellular networks [19]:

$$\mathbb{P}(SIR_{eff} < \delta) = 1 - Q\left[\frac{\log_2(1+\delta) - \mu_{MIC}}{\sigma_{MIC}}\right]. \quad \text{(OFDMA)}$$

$$\mu_{MIC} = E[C_n], \tag{22}$$

$$\sigma^2_{MIC} = \frac{1}{N_{sc}}(E[C_n^2] - E[C_n]^2). \tag{23}$$

$$E[C_n] = \int\int (1 - Q_f(x,t))\, e^{-x} dxdt, \tag{24}$$

$$E[C_n^2] = \int\int 2t\,(1 - Q_f(x,t))\, e^{-x} dxdt. \tag{25}$$

$$Q_f(x,t) = Q\left[\frac{10\log_{10}(\frac{x}{2^t-1}) - m_f}{s_f}\right]. \tag{26}$$

# Other Obtained Results II

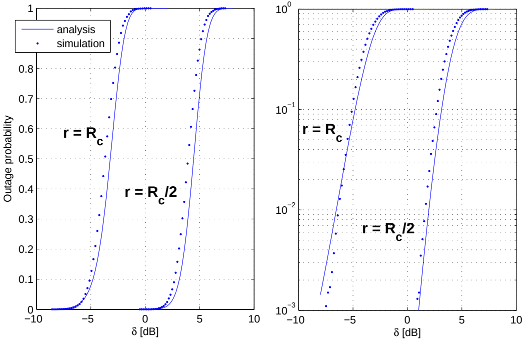



**Figure:** Outage probability at cell edge ($r = R_c$) and inside the cell ($r = R_c/2$); comparison between analysis (solid curves) and simulations (dotted curves) on a hexagonal network ($N_{sc} = 48$, $\eta = 3$, $\sigma = 6$ dB).

# Other Obtained Results III

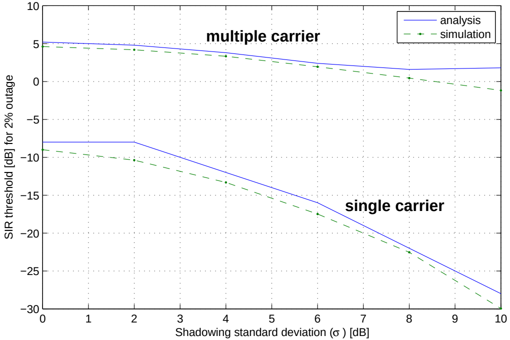



**Figure:** SIR threshold $\delta$ in the cell ($r = R_c/2$) for 2% outage probability as a function of the shadowing standard deviation; comparison between analysis (solid curves) and simulations (dotted curves) on a hexagonal network ($N_{sc} = 48$, $\eta = 3$).

# Other Obtained Results IV

MIMO Alamouti transmission [20]:

$$P_{out}(\gamma_{th}) = 1-\left(\frac{p_0}{2\gamma_{th}\beta+p_0}\right)^{\alpha}\left(1+\frac{2\gamma_{th}\beta}{2\gamma_{th}\beta+p_0}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)}\right). \text{(ALAM)}$$

$$\alpha = \frac{2}{1+\rho_z}\frac{(\sum_{j=1}^{B} p_j)^2}{\sum_{j=1}^{B} p_j^2}, \tag{27}$$

$$\beta = \frac{1+\rho_z}{2}\frac{\sum_{j=1}^{B} p_j^2}{\sum_{j=1}^{B} p_j}. \tag{28}$$

$$\rho_z = 0.0167 \tag{29}$$

# Other Obtained Results V

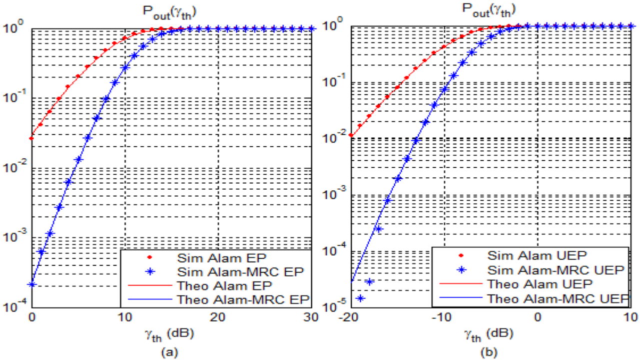



**Figure:** $P_{out}$ versus SINR threshold for $2 \times 1$ MISO Alamouti and $2 \times 2$ MIMO Alamouti with MRC receiver systems in case of equal (a) and unequal (b) received interference power (EP and UEP resp. in legends).

# Other Obtained Results VI

MU-MIMO Zero-Forcing/Beamforming [21]:

$$P_{out}(\gamma_{th}) = \frac{\Gamma(\nu+\alpha)}{\Gamma(\nu)\mu^{\nu}\alpha!} \frac{p_0^{\nu}(K\gamma_{th})^{\alpha}}{(\gamma_{th}K + \frac{p_0}{\mu})^{\nu+\alpha}} {}_2F_1(1,\nu+\alpha;\alpha+1;\frac{\gamma_{th}K}{\gamma_{th}K+\frac{p_0}{\mu}}) \quad \text{(ZF)}$$

where ${}_2F_1(.,.;.;.)$ is the Gauss' hypergeometric function [17],

$$\nu = (1-\frac{2(2\alpha-1)\theta^2-(\alpha-1)^2(\alpha-2)}{(5\alpha-4)\theta^2+2(\alpha-1)(\alpha-2)\theta})\frac{\left(\sum_{j=1}^{B} p_j\right)^2}{\sum_{j=1}^{B} p_j^2} \quad (30)$$

$$\mu = \frac{(5\alpha-4)\theta^2+2(\alpha-1)(\alpha-2)\theta}{(\alpha-1)(\alpha-2)\theta+(\alpha-1)^2(\alpha-2)}\frac{\sum_{j=1}^{B} p_j^2}{\sum_{j=1}^{B} p_j} \quad (31)$$

$$\alpha = M-K+1, \quad \theta = (1-\rho^2), \quad \rho = 0.154 \quad (32)$$

# Other Obtained Results VII

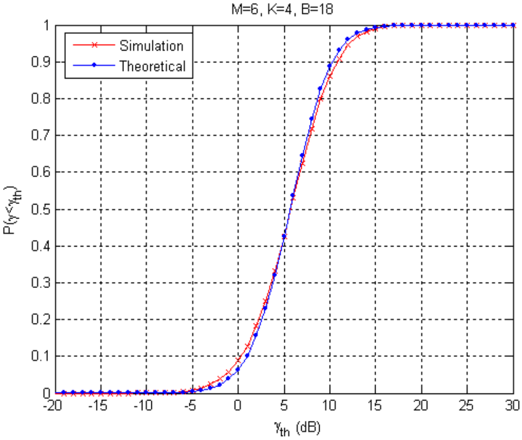



**Figure:** Outage probability of a zero forcing multicellular precoded system (4 users, 6 antennas at the BS).

# Conclusion

We have derived an approximate closed form expression of the SIR CDF.
Further work:

- Consider more realistic scenarios
- Compare with other beamforming schemes
- Consider dynamic studies

Outage probabilities can be efficiently used for solving various dimensioning issues:

- Effect of network densification [22]
- Limiting transmit power in green cellular networks [23]
- Perf. evaluation of frequency reuse schemes in OFDMA networks [24]
- Dynamic performance study of WiMAX networks [25]

# References I

[1] J. Li, E. Lu, and I-Tai Lu.
Performance Benchmark for Network MIMO Systems: A Unified Approach for MMSE Transceiver Design and Performance Analysis.
In *Proc. IEEE GLOBECOM*, pages 1–6, Dec. 2010.

[2] Y. Rui, M. Li, P. Chengo, Y. Luo, and A. Guo.
Achievable Rates of Coordinated Multi-Point Transmission Schemes under Imperfect CSI.
In *Proc. IEEE ICC*, pages 1–6, 2011.

[3] V. Jungnickel and al.
Capacity Measurements in a Cooperative MIMO Network.
*IEEE Trans. on Veh. Tech.*, 58(5):2392–2405, Jun. 2009.

# References II


[4] P. Marsch, M. Grieger, and G. Fettweis.
Large Scale Field Trial Results on Different Uplink Coordinated Multi-Point (CoMP) Concepts in an Urban Environment.
In *Proc. IEEE WCNC*, pages 1858–1863, Mar. 2011.

[5] H. Zhang, H. Dai, and Q. Zhou.
Base Station Cooperation for Multiuser MIMO: Joint Transmission and BS Selection.
In *Proc. Conf. Inform. Sciences and Sys.*, Mar. 2004.

[6] S. Ben Halima, M. Helard, and D.T. Phan-Huy.
New Coordination and Resource Allocation Schemes for Uniform Rate in Femtocell Networks.
In *Proc. IEEE VTC-Spring*, pages 1–5, May 2011.

# References III


[7] V. Garcia, N. Lebedev, and J.-M. Gorce.
Capacity Outage Probability for Multi-Cell Processing Under Rayleigh Fading.
*IEEE Commun. Lett.*, 15(8):801–803, Aug. 2011.

[8] X. Ge, K. Huang, C-X. Wang, X. Hong, and X. Yang.
Capacity Analysis of a Multi-Cell Multi-Antenna Cooperative Cellular Network with Co-Channel Interference.
*IEEE Trans. on Wireless Commun.*, 10(10):3298–3309, Oct. 2011.

[9] J.-M. Kélif, M. Coupechoux, and Ph. Godlewski.
A fluid model for performance analysis in cellular networks.
*EURASIP Journal on Wireless Communications and Networking*, 2010, July 2010.

# References IV


[10] J. M. Kélif, M. Coupechoux, and Ph. Godlewski.
Effect of shadowing on outage probability in fluid cellular radio networks.
In *Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, WiOpt*, Berlin, Germany, March 2008.

[11] J. M. Kélif and M. Coupechoux.
Impact of topology and shadowing on the outage probability of cellular networks.
In *IEEE International Conference on Communications, ICC*, Dresden, Germany, June 2009.

# References V

[12] D. Ben Cheikh, J.-M. Kélif, M. Coupechoux, and Ph. Godlewski.
Sir distribution analysis in cellular networks considering the joint impact of path-loss, shadowing and fast fading.
*EURASIP Journal on Wireless Communications and Networking*, 2011(137), October 2011.

[13] A. Papoulis.
*The Fourier Integral and Its Applications*.
McGraw-Hill, New York, NY, 1962.

[14] Lo, T.K.Y.
Maximum Ratio Transmission.
In *Proc. IEEE ICC*, 1999.

# References VI

[15] A. Shah and A. Haimovich.
Performance Analysis of Maximum Ratio Combining and Comparison with Optimum Combining for Mobile Radio Communications with Cochannel Interference.
*IEEE Tran. on Veh. Tech.*, 49(4):1454–1463, Jul. 2000.

[16] D. Ben Cheikh Battikh, J.-M. Kelif, F.A. Abdallah, and Dinh-Thuy Phan-Huy.
Time Reversal Outage Probability for Wideband Indoor Wireless Communications.
In *Proc. IEEE PIMRC*, pages 999–1003, Sept. 2010.

[17] I.S. Gradshteyn and I.W. Ryzhik.
*Table of integrals series and products*.
Academic Press, Moscow, 1963.

# References VII

[18] D. Ben Cheikh, J.-M. Kélif, M. Coupechoux, and Ph. Godlewski.
Analytical Joint Processing Multi-Point Cooperation Performance in Rayleigh Fading.
*submitted*, 2012.

[19] J. M. Kélif, M. Coupechoux, and Ph. Godlewski.
On the Dimensioning of Cellular OFDMA Networks.
*Elsevier Physical Communication*, 2012 (in press).

[20] D. Ben Cheikh, J.-M. Kélif, M. Coupechoux, and Ph. Godlewski.
Outage probability in a multi-cellular network using alamouti scheme.
In *IEEE Sarnoff Symposium*, Princeton, USA, April 2010.

# References VIII

[21] D. Ben Cheikh, J.-M. Kélif, M. Coupechoux, and Ph. Godlewski.
Multicellular zero forcing precoding performance in rayleigh and shadow fading.
In *IEEE Vehicular Technology Conference, VTC Spring*, Budapest, Hungary, May 2011.

[22] J. M. Kélif and M. Coupechoux.
Cell breathing, sectorization and densification in cellular networks.
In *Intl. Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, WiOpt*, Seoul, Korea, June 2009.

[23] J.-M. Kélif, M. Coupechoux, and F. Marache.
Limiting power transmission of green cellular networks: Impact on coverage and capacity.
In *IEEE International Conference on Communications, ICC*, Cape Town, South Africa, May 2010.

# References IX

[24] M. Maqbool, Ph. Godlewski, M. Coupechoux, and J.-M. Kélif.
Analytical performance evaluation of various frequency reuse and scheduling schemes in cellular ofdma networks.
*Performance Evaluation*, 67(4):318–337, April 2010.

[25] S. Doirieux, B. Baynat, M. Maqbool, and M. Coupechoux.
An efficient analytical model for the dimensioning of wimax networks supporting multi-profile best effort traffic.
*Elsevier Computer Communications*, 33(10):1162–1179, June 2010.