Impact on Coverage and Capacity of Reduced Transmit Power in Cellular Networks

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Outlines

- Interference Model
- Outage Probabilities
- Interference Factor Analysis
- Noise and Power Analysis
- Applications
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Interference Model: SINR

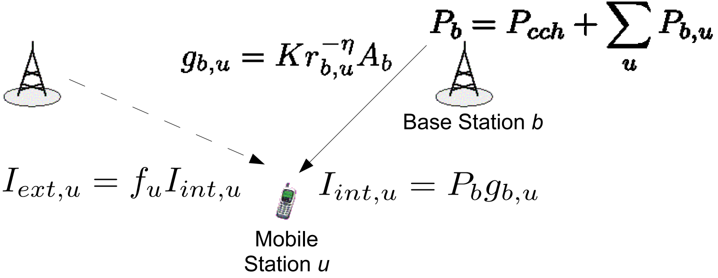


Figure: Interference model.

- SINR is a central parameter for performance evaluation:

$$\gamma_u^* = \frac{S_u}{\alpha(I_{int,u} - S_u) + I_{ext,u} + N_{th}}.$$

Interference Model: Output Power

- Other-cell interference factor: $f_u = I_{ext,u}/I_{int,u}$

$$f_u = \frac{\sum_{j \neq b} P_j g_{j,u}}{P_b g_{b,u}}$$

- Transmitted power for mobile u: $P_{b,u} = S_u/g_{b,u}$

$$P_{b,u} = \frac{\gamma_u^*}{1 + \alpha\gamma_u^*}(\alpha P_b + f_u P_b + N_{th}/g_{b,u}).$$

- Total BS output power:

$$P_b = \frac{P_{cch} + \sum_u \frac{\gamma_u^*}{1+\alpha\gamma_u^*}\frac{N_{th}}{g_{b,u}}}{1 - \sum_u \frac{\gamma_u^*}{1+\alpha\gamma_u^*}(\alpha + f_u)}.$$

Outage Probabilities: Generic Expression

- For n MS with a single service:

$$P_{out}^{(n)} = Pr\left[\sum_{u=0}^{n-1} T_u > \frac{1-\varphi}{\beta} - n\alpha\right],$$

- where $\varphi = P_{cch}/P_{max}$, $\beta = \gamma^*/(1+\alpha\gamma^*)$ and

$$T_u = f_u + h_u.$$

- Two terms appear:
  - The OCIF: f_u and
  - A noise factor: $h_u = \frac{N_{th}}{P_{max}g_{b,u}}$.

Outage Probabilities: Without Shadowing

- The outage probability is now (Gaussian approx.):

$$P_{out}^{(n)} = Q\left(\frac{\frac{1-\varphi}{\beta} - n\mu_T - n\alpha}{\sqrt{n}\sigma_T}\right).$$

- where:

$$\begin{aligned}\mu_T &= \mu_{f_0} + \mu_{h_0} \\ \sigma_T^2 &= \sigma_{f_0}^2 + \sigma_{h_0}^2 + 2\mathbb{E}[f_0 h_0] - 2\mu_{f_0}\mu_{h_0}\end{aligned}$$

- Means and standard deviations are taken over the uniform distribution of MS on the cell area.

Outage Probabilities: With Shadowing

- The outage probability is now:

$$P_{out}^{(n)} = Q\left(\frac{\frac{1-\varphi}{\beta} - nM_T - n\alpha}{\sqrt{n}S_T}\right),$$

- where:

$$\begin{aligned} M_T &= M_f + M_h, \\ S_T^2 &= S_f^2 + S_h^2 + 2\mathbb{E}[f_u h_u] - 2M_f M_h, \end{aligned}$$

- Means and standard deviations are taken both over the shadowing variations and mobile location.

Interference Analysis: Fluid Model

- Interfering BS are approximated by a continuum of BS.
- Each elementary surface $zdzd\theta$ at distance z from u contains $\rho_{BS}zdzd\theta$ BS and contributes with $\rho_{BS}zdzd\theta P_b K z^{-\eta}$ to the interference.

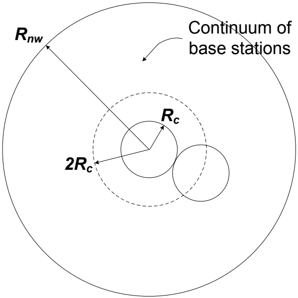


Figure: Cellular network approximation.

Interference Analysis: Fluid Model

- Discrete sum is approximated by an integral:

$$I_{ext,u} = \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} P_b K z^{-\eta} z dz d\theta \quad (1)$$

- If network size is large:

$$f_0 = \frac{2\pi \rho_{BS} r_u^{\eta}}{\eta - 2} (2R_c - r_u)^{2-\eta}. \quad (2)$$

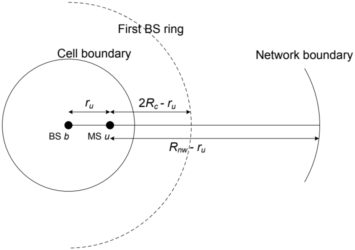


Figure: Integration limits for interference computation.

Interference Analysis: Fluid Model

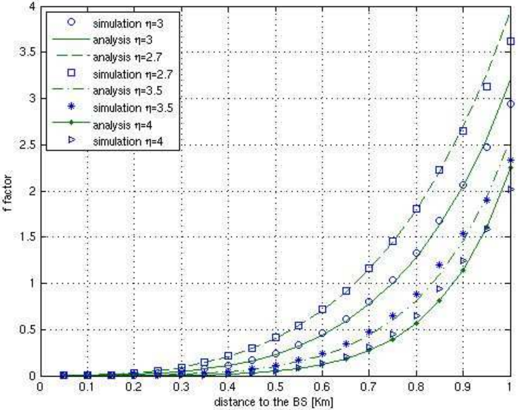


Figure: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on an hexagonal network with $\eta = 2.7$, 3, 3.5, and 4.

Interference Analysis: Without Shadowing

- OCIF is obtained from the fluid model:

$$f_0 = \frac{2\pi\rho_{BS} r^\eta}{\eta - 2}(2R_c - r)^{2-\eta}.$$

- We integrate over the cell area:

$$\mu_{f_0} = \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta^2 - 4}\left(\frac{R_e}{R_c}\right)^\eta {}_2F_1(\eta - 2, \eta + 2, \eta + 3, R_e/2R_c).$$

  where ${}_2F_1(a, b, c, z)$ is the hypergeometric function, whose integral form is given by:

$${}_2F_1(a, b, c, z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}dt,$$

- The same for σ_{f_0} (can be expressed in closed-form using ${}_2F_1$).

Interference Analysis: With Shadowing

- At a distance r_u, f_u can be approx. by a log-normal RV with Fenton-Wilkinson $\longrightarrow m_f$ and σ_f.
- We then integrate RV f_u over the cell area:

$$
\begin{aligned}
M_f &= \int_0^{R_e} \mathbb{E}[f_u|r]p_r(r)dr = \int_0^{R_e} f_0(r)J(r,\sigma)e^{a^2 s_f^2/2}\frac{2r}{R_e^2}dr, \\
\mathbb{E}\left[f_u^2\right] &= \int_0^{R_e} \mathbb{E}\left[f_u^2|r\right]p_r(r)dr = \int_0^{R_e}(f_0(r)J(r,\sigma))^2 e^{2a^2 s_f^2}\frac{2r}{R_e^2}dr.
\end{aligned}
$$

- where $J(r_u,\sigma) = e^{a^2\sigma^2/2}\left(L(r_u,\eta)(e^{a^2\sigma^2}-1)+1\right)^{-\frac{1}{2}}$ and $L(r_u,\eta) = \frac{f_0(r_u,2\eta)}{f_0(r_u,\eta)^2}$

Noise and Power Analysis: Without Shadowing

- This is a simple case since: $h_u = h_0 = \frac{N_{th}}{P_{max} K r_u^{-\eta}}$
- Mean and standard deviation over MS locations:

$$\mu_{h_0} = \frac{2 N_{th} R_e^{\eta}}{P_{max} K (\eta + 2)}$$

$$\sigma_{h_0} = \frac{R_e^{2\eta}}{\eta + 1} \left(\frac{N_{th}}{P_{max} K} \right)^2 .$$

- $\mathbb{E}[f_0 h_0]$ involves an hypergeometric function but can be computed with:

$$\mathbb{E}[f_0 h_0] = \frac{2 N_{th} \pi \rho_{BS}}{P_{max} K (\eta - 2)} \int_0^{R_e} r^{2\eta} (2R_c - r)^{2-\eta} p_r(r) dr.$$

Noise and Power Analysis: With Shadowing

- Thermal noise factor is now: $h_u = h_0/A_b = h_0 10^{-\xi_b/10}$.
- And so: $M_h = \mu_{h_0}\mathbb{E}\left[10^{-\xi_b/10}\right] = \mu_{h_0} e^{a^2\sigma^2/2}$ (the same for S_h).
- $f_u h_u$ (both terms are not ind.) can be approx. at a given distance r by a log-normal RV using Fenton-Wilkinson.
- We then integrate over the cell area:

$$\begin{aligned}
\mathbb{E}[f_u h_u] &= \int_0^{R_e} \mathbb{E}[f_u h_u | r] p_r(r) dr \\
&= \frac{4\pi\rho_{BS} N_{th} e^{3a^2\sigma^2/2}}{P_{max} K R_e^2(\eta - 2)} \int_0^{R_e} r^{2\eta+1}(2R_c - r)^{2-\eta} dr.
\end{aligned}$$

- Again, this can be expressed in closed-form using ${}_2F_1$.

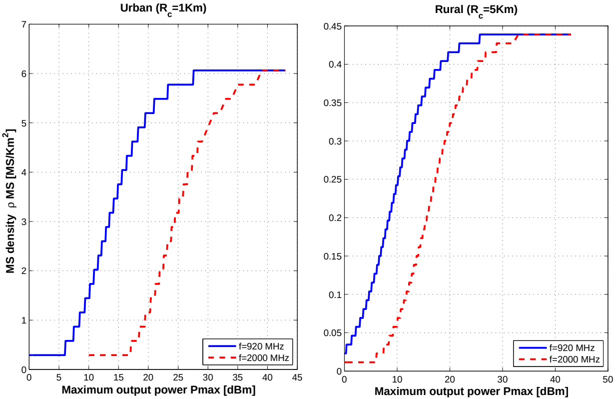
Applications: Scenarios

- Common parameters: CDMA network, $\gamma^* = -19$ dB, $W = 5$ MHz, $\alpha = 0.6$, $\varphi = 0.2$, $N_0 = -174$ dBm/Hz.
- Urban and rural scenarios:

Table: Propagation parameters

	K (2 GHz)	K (920 MHz)	σ (dB)	t	η	R_c
Urban	$4.95\ 10^{-4}$	$6.24\ 10^{-3}$	6	0.5	3.41	1 Km
Rural	0.88	4.51	4	0.5	3.41	5 Km

Applications: Capacity

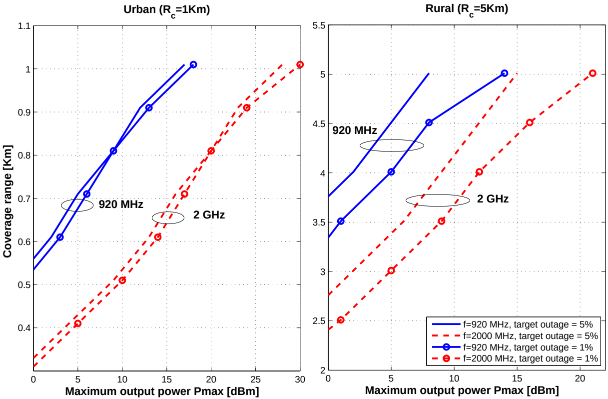


- We set $P^*_{out} = 5\%$
- For a given P_{max}, n_{MS} is the max nb. of MS such that $P_{out} < P^*_{out}$
- $\rho_{MS} = n_{MS}/\pi R_e^2$

- Effect of Freq. $\uparrow$: $K \downarrow$, f_u is unchanged, $h_u \uparrow$
- Effect of Rural deployment: $R_c \uparrow$ so more power is needed per MS but $K \uparrow$ and $\sigma \downarrow$. Cell range increase has a dominant influence.

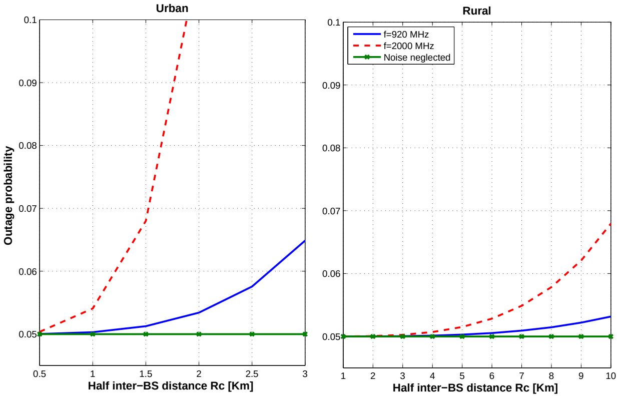
Applications: Coverage

- $P^*_{out} = 1$ or 5%
- ρ_{MS} is fixed for rural and urban
- Cov. range R_e is variable
- For given P_{max}, we look for R_e such that $P_{out} < P^*_{out}$



- When $P_{max} \downarrow$, $R_e \downarrow$ because less MS can be served and average power per MS should decrease.
- A small degradation of QoS allows an important power reduction in rural

Applications: Should we neglect noise ?

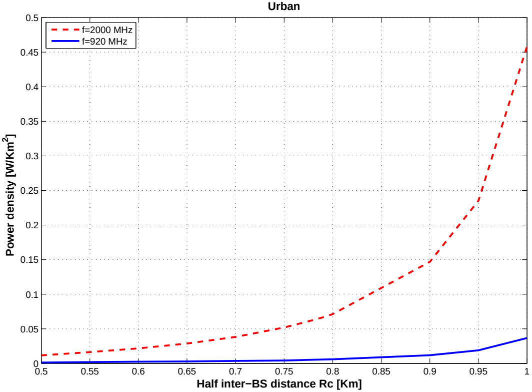


- $P^*_{out} = 5\%$
- $P_{max} = 43$ dBm
- n_{MS} is fixed such that $P_{out} = P^*_{out}$ when noise is neglected.
- We then compute P_{out} while considering noise.

- Noise neglected $\Rightarrow P_{out}$ doesn't depend on K, frequency, R_c (homothetic networks).
- Noise cannot be neglected for $R_c > 1$ Km in urban and $R_c > 7$ Km in rural at 2 GHz (if we accept 0.5% error).

Applications: Power Density and Densification

- $P^*_{out} = 5\%$
- MS density constant
- Full coverage is assumed
- For a given R_c, P_{max} is such that $P_{out} < P^*_{out}$
- Power density is $P_{max}/\pi R_e^2$



- At 2 GHz, 11% more BS means half power density.
- Deploying small and femto cells are good means of reducing electromagnetic pollution provided that transmission power is optimized.

Conclusion

- This work analyzes interference, noise and outpout power in cellular networks and their impact on outage.
- Fluid model provides a simple formula for the OCIF.
- Integrations are done both over shadowing variations and MS locations.
- Slight QoS degradation implies much lower output powers (rural).
- Slight increase of BS nbr implies much lower power densities (2GHz).