A Background Extraction Technique for Bandwidth Mismatch Error in a Two-channel Time-Interleaved ADC

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Abstract—Bandwidth Mismatch between channels sampling circuits in a time-interleaved analog-to-digital data converter (TI ADC) causes undesirable distortions in the output spectrum. This error interferes in gain and phase with the static gain and clock skew errors respectively, which makes its extraction much difficult. In this paper, an online extraction technique of the bandwidth mismatch error is proposed. Furthermore, a correction solution is presented to validate this extraction.

Index Terms—Time-interleaved ADC, bandwidth mismatch, error extraction, polynomial approximation.

I. INTRODUCTION

The evolution of wireless communications moves towards higher data rate analog to digital converters (ADCs). To break this bottleneck, time interleaving is an efficient way to increase the speed while maintaining a good accuracy [1]. Its main idea consists of distributing signal samples between M ADCs operating each at \(\omega_o/M\) (\(\omega_o\) is the sampling frequency). The benefit of this approach is to increase the conversion rate while maintaining the same operation frequency of the analog blocks. An important condition to achieve a good reconstruction of the signal at the digital back-end in time-interleaved architecture is that all channels must be identical. Unfortunately, due to process and environment variations, each channel has a different behavior. These mismatches among channels generate undesired spectral components and can significantly degrade the linearity and the resolution of the system. These distortions are created by periodic mismatch factors such as offset, gain, timing and bandwidth [2, 3, 4] which increase with the number of ADCs. Much work has been proposed to compensate the mismatches in a digital part preferably to analog correction due to reliability and flexibility. Moreover, the evolution of technology improves power consumption of such techniques. Static gain and offset can be easily corrected using arithmetic operations [4, 5]. The problem of clock skew is more complex to correct; time varying filters are usually used to uniformly reconstruct the samples [4]-[7]. Finally, bandwidth mismatch error is less studied. In [8]-[11] the authors propose digital solutions by designing adaptive filters either on each channel or in the output of the TIADC. The first solution in [8] uses an inverse Fourier transform to compute the updated coefficients of the filters. The implementation of such adaptive filters that have non constant coefficients has a high complexity. The second solution, proposed in [9] is based on polynomial time varying filter structures where the bandwidth mismatch is compensated iteratively. Beyond the correction, all these proposed solutions assume that the error is already extracted. In fact, very few extraction methods of the bandwidth mismatch are proposed. One can refer to [12] where polynomial approximation is used to estimate a global error, meaning all existing errors. The inconvenient of this method is that analogue correction cannot be used since extracted information is global and hence it becomes difficult to correct the errors separately. Assuming that bandwidth mismatch error combines frequency dependent and non-linear gain and phase components, its extraction, taking into account the phase leads to retrieving clock skew error additionally which is more complex to deal with. Concerning the gain contribution, this is combined to the static gain error which mainly comes from the reference difference between channels. Nevertheless, this static gain error can be solved by using one single reference module for all channels and thus enables to deal with only the bandwidth mismatch contribution.

In this paper, we propose a background extraction method of the bandwidth mismatch error. In section II, we expose the system model by highlighting its two components and the impact of considering all the errors. Then the proposed extraction technique is detailed in section III. An overall architecture combining estimation and correction is presented in section IV and validated in section V by simulations. Finally we conclude in section VI.

II. SYSTEM MODEL

Along this paper, we consider a two-channel time-interleaved ADC and suppose all errors, except bandwidth

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mismatch are already corrected. During the sampling phase of the i-th channel, the sampling network behaves as a low pass filter imposed by the equivalent $R_{on}$ of the switches and the equivalent capacitor [3]. For simplicity, we consider in our study a first order low pass input filter.

Let us consider the system model shown in Fig. 1 which includes bandwidth mismatch error between the two channels’ input filters denoted by $H_0$ and $H_1$. Each channel is filtered then sampled at $(kM+i)T$, where $i$ is equal to zero for the top channel and one for the bottom one. $M$ is the number of channels (2 in this case) and $k$ is the discrete time index. The overall sampling frequency is denoted by $1/T$ and the input signal is bandlimited to $\pi/T$, so that the Nyquist criterion is fulfilled.

The i-th channel output can then be expressed as:

$$y_i[k] = \left(1 + g(\epsilon_{t_i})\right)x[(kM + i)T + f(\epsilon_{t_i})]$$  \hspace{1cm} (1)

where, $1 + g(\epsilon_{t_i})$ and $f(\epsilon_{t_i})$ are respectively the gain and the phase, $T$ the overall sampling period and, $\epsilon_{t_i}$ the relative constant time error ($\epsilon_{t_i} = (\tau_i - \tau_r)/\tau_r$).

For a first order input filter, one has:

$$1 + g(\epsilon_{t_i}) = \frac{1}{\sqrt{1 + (\omega \tau_r (1 + \epsilon_{t_i}))^2}}$$

$$f(\epsilon_{t_i}) = -\tan(\omega \tau_r (1 + \epsilon_{t_i}))$$

in which, $\omega$ refers to the input frequency and $\tau_r$ denotes the reference.

According to Equation (2), one notices that the gain and the phase are frequency dependent and nonlinear, which makes the error extraction more complicated. Furthermore, by combining the gain and the phase contributions, bandwidth mismatch can be divided into two components as shown in Equation (4). The first component is in phase with the input signal when the second one is observed by shifting the input by $\pi/2$. When including all other errors, these components are combined to the static gain and the clock skew errors respectively. Consequently, the errors’ separation becomes unfeasible.

### III. THE PROPOSED BANDWIDTH MISMATCH EXTRACTION

Based on the previous system model, we now propose the bandwidth extraction block diagram shown in Fig. 2.

The squared expression of the i-th channel output is given by:

$$y_i^2[k] = \frac{1}{2(1 + (\omega \tau_i)^2)(1 + \cos(2\omega T(kM + i) - 2\tan(\omega \tau_i)))}, i \in \{0, 1\} \hspace{1cm} (5)$$

The first term represents the DC value and depends on the time constant of the channel. This DC term can be extracted by averaging thanks to the sine wave expression of the second term. In fact, in order to retrieve the bandwidth mismatch error, we feed an integrator with the following loss function.

$$loss[k] = y_0^2[k] - y_1^2[k] \hspace{1cm} (6)$$

#### A. Integrator

The output of the integrator is characterized by the slope and the trace functions; this is described in the following: The slope corresponds to the difference between the squared channels’ DC values. From Equation (5), we can express the slope as:

$$Slope = \frac{1}{2} \left( \frac{1}{1 + (\omega \tau_0)^2} - \frac{1}{1 + (\omega \tau_1)^2} \right).$$

To make the channels perfectly matched, it is clear that the slope must be equal to zero. To this end, analog correction by tuning the input filters can be considered. However this is tricky to reach the precision of 0.01% of mismatch in CMOS technologies. Nevertheless, an alternative solution is proposed in a digital domain using Farrow filters [9], [12] which will be explained in section IV.
The trace of the loss function is the difference between the second terms in Equation (5) of the two channels, and it is function of the mismatch error and the input frequency. The thickness of this trace gives the minimum threshold value that must be chosen for the integrator which defines the convergence time of the estimator.

B. Notch at $\omega_s/4$

It is important to notice that at the $\omega_s/4$ input frequency, it is impossible to distinguish the fundamental from the spurious tone which makes the detection unfeasible. Moreover, at this frequency, the value of the trace thickness can limit the performances of the integrator. In fact, at this input frequency, $\omega_m T = \pi/2$ which leads to $\sin(2\omega_m T(kM+i))=0$ and $\cos(2\omega_m T(kM+i))=\pm 1$ depending on whether $i$ refers to channel 0 or 1. According to this, the trace is maximal and the threshold needs to be very high. To overcome this problem, the $\omega_s/4$ frequency can be filtered using a $1+z^{-2}$ filter [4], [6]. This so-called “Notch at $\omega_s/4$” represented in Fig. 2 slightly attenuates the adjacent frequencies but this can be improved if necessary.

IV. THE PROPOSED OVERALL BANDWIDTH MISMATCH COMPENSATION ARCHITECTURE

In this section, we expose a global structure to validate the proposed extraction method. Fig. 3 shows the block diagram of the digital calibration.

![Figure 3: Compensation model.](image)

The challenge of the digital filter $F(e^{j\omega})$ is to compensate the analog filter $H_s(j\omega)$ to match better with the reference filter $H_0(j\omega)$. To do this, we use a Farrow filter [9], [12] based on polynomial approximation. Assuming first order channel input filters, the frequency response of the compensation filter $F_s$ can be expressed as:

$$F(e^{j\omega}) = \frac{H_0(j\omega)}{H_s(j\omega)} = \frac{1 + j\omega \tau_0}{1 + j\omega \tau_1} \quad (7)$$

A Taylor approximation can be applied in Equation (7). This can be argued by the fact that the realistic values of bandwidth mismatch are around 1% which is small enough to consider a polynomial approximation [9], [13]. A third order development of $F(e^{j\omega})$ leads to:

$$F(e^{j\omega}) = 1 - \varepsilon_s \tau_0 (j\omega) + \varepsilon_s \tau_0^2 (j\omega)^2 - \varepsilon_s \tau_0^3 (j\omega)^3 \quad (8)$$

in which $\varepsilon_s$ is the mismatch error between the two channels.

From the above expression, the filter can be designed as a set of fixed differentiators $G_i = (j\omega)_i$, $i = \{1, 2, 3\}$, followed by the mismatch coefficients. These latter are comparable to the ones calculated in [9] with the difference of having $\varepsilon_i$ instead of $\tau_0$ in Equation (8).

The proposed overall architecture is shown in Fig. 4. One should note that, according to the slope value, the parameter $\varepsilon_s$ is adjusted in such a way that the slope converges to zero.

![Figure 4: Overall architecture: extraction and polynomial correction.](image)

V. SIMULATIONS AND RESULTS

In this section, we present the performances of the proposed extraction technique for a two-channel 16-bit time-interleaved converter. First we consider in Example 1 and 2 an ideal correction equivalent to tuning the analog filter. Static gain and clock skew are previously compensated. Bandwidth mismatch is set to 0.53% and the cut-off frequency of the 0-th channel input filter is given by $\omega_c = \omega_s/2$.

Regarding the polynomial compensation filter, and since we focus on the validation of the extraction method, the design optimization of each differentiator is beyond the scope of this paper. The impulse response coefficients of each filter are calculated using an ideal IDFT transforms. Example 3 shows its efficiency to validate the extraction technique where a 50-taps causal FIR filters are used and the reference cutoff frequency is set to $2\omega_c$.

Example 1:

The first example illustrates the case of a sinusoidal input at $\omega_0 = 0.2\omega_s$. The threshold is equal to three times the trace thickness in this example and the correction step is 0.1%. The filter coefficients updates change the slope value when this one exceeds the threshold to finally swing around zero after about 25000 samples as shown in Fig. 5. The mismatch error between the two channels reaches 0.03% equivalent to a 95dB of SFDR (Fig. 6).

![Figure 5: Convergence of the extracted slope.](image)
Figure 6: Improvement of the SFDR after Compensation from 70 dB to 95 dB.

Example 2:
The second example shows that the extraction is functional for a multi-sin input signal. A four patterns CDMA signal sampled at 184 Msps is used in this case. Fig. 7 (a) shows the CDMA signal after the notch at ω/4, the central frequency is set to 0.2ωs and the mismatch before compensation is 1.3%. We can observe in Fig. 7 (b) that the notch filter does not affect the extraction, and at the end SFDR is improved up to 95 dB like in Example 1.

Figure 7: CDMA signal input after the notch filter (a) before compensation (b) after compensation.

Example 3:
Finally, Fig. 8 shows the improvement of the SFDR using the polynomial filter. The convergence is fullfilled for εs=0.5% and it can be seen that for this value the SFDR is almost constant over the bandwidth. For an exact value of εs=0.53%, the SFDR is equal to 85 dB. This is due to the polynomial approximation and could be improved by using more optimized filters.

Figure 8: SFDR before and after polynomial filtering.

VI. CONCLUSION
In this paper, we have proposed an extraction technique of the bandwidth mismatch error in a two-channel time-interleaved ADC. Nevertheless, this is extendable to an M channels considering one of them as a reference. The solution is based on the minimization of the channel power difference average which is function of the mismatch error. A suited digital approximation filter was presented in an overall architecture to match with the proposed extraction technique by tuning only one parameter corresponding to the bandwidth mismatch error. Finally, the simulations have treated both single tone input signal and wideband signal to validate the proposed solution. In case an effective correction method is used, our extraction method achieves up to 95 dB SFDR.

REFERENCES