A Markovian Approach for DEM Estimation From Multiple InSAR Data With Atmospheric Contributions

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***Abstract*—Accurate digital elevation model (DEM) estimation using synthetic aperture radar interferometry still remains a challenging problem in the geographical information science community, particularly in dealing with a high noise rate and atmospheric disturbances. Such task suffers from the lack of efficient and reliable methods to overcome these artifacts. This work provides a method that aims to solve this problem through a Bayesian formulation with the Markovian energy minimization framework. The DEM is generated from a set of multifrequency/multibaseline interferograms using a multichannel phase unwrapping algorithm combined with an estimation method of the atmospheric artifacts. A set of experimental results illustrates the effectiveness and robustness of the proposed approach.**

***Index Terms*—Atmospheric contributions, graph cuts, multichannel phase unwrapping (PU) (MCPU), synthetic aperture radar (SAR) interferometry (InSAR).**

I. Introduction

SYNTHETIC aperture radar (SAR) interferometry (InSAR) is a well-known technique to retrieve accurate digital elevation models (DEM) from multiple SAR images of the same surface, using the phase difference information, called *interferogram*. Such DEMs are of extensive use in SAR applications, particularly for differential interferometry to precisely measure ground displacements caused by earthquakes or volcanoes for instance. Therefore, accurate estimation of height maps is a challenging problem for the geographical information science community.

A necessary and critical step in the InSAR processing chain for the DEM estimation task is the recovery of 2π-multiple ambiguity from the measured phase, which is called the phase unwrapping (PU) operation. Because of the presence of noise and other artifacts in real data, the aliasing phenomenon for instance, robust PU algorithms are needed. Moreover, the high dimensionality of real data requires efficient PU algorithms in order to solve the problem with reasonable time and memory complexity.

The simplest unwrapping algorithm in [1] fails to reconstruct the absolute phase from real interferograms, if the *Itoh conditions* are violated, which are a low noise rate, absence of aliasing, and a continuity of the original profile (i.e., the highest true absolute phase discontinuity is less than π). We shall observe that such conditions are usually violated, and the unwrapping problem becomes an ill-posed inverse problem.

Local unwrapping approaches have been proposed in the last decades, referred to as path-following methods [2]. However, they remain always sensitive to the Itoh conditions. Ambiguous phase surfaces are usually incorrectly unwrapped. More interesting unwrapping methods have been proposed since, based on global approaches, mainly the minimum norm methods [3] and Bayesian-based estimation methods [4]. Nevertheless, correctly unwrapping the phase from one interferogram can be an impossible task. The high noise or aliasing combined with sudden true phase changes induces a loss of a lot of information to be able to recover the original profile. In such cases, the multichannel PU (MCPU) approach is a more appropriate method that may overcome these difficulties. In fact, the information coming from several interferograms with different baselines/frequencies increases the elevation ambiguity interval and thus reduces the PU ambiguity. This approach has been addressed with the Bayesian framework in different works [5], [6].

Multichannel interferograms are usually provided with a *repeat-pass* interferometry, i.e., the acquisition of SAR image pairs is not made simultaneously but with successive times of revisit to the same surface (35 days of time lags with the ERS satellite for instance). With this configuration, atmospheric conditions as well as surface may change. These changes present a real problem to the MCPU approaches, since the absolute phase would also change from one channel to another. An estimation of these contributions for each channel is necessary for a simultaneous use of multichannel data. Usually, this task is solved using a known DEM. Therefore, with no *a priori* knowledge about the surface topography, the problem of DEM estimation, together with the atmospheric phase estimation, becomes a complex problem.

In this letter, a Bayesian approach is proposed to solve the DEM estimation problem from InSAR data with atmospheric contributions. The plan of this letter is as follows. In Section II, related works to the interferometric phase correction from atmospheric effects are presented.Then, the proposed method is described through a novel probabilistic model and an efficient joint estimation algorithm of the absolute topographic and atmospheric phase data. In Section III, experiments on real InSAR data are performed showing accurate DEM mapping while removing the atmospheric artifacts.

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II. DEM AND ATMOSPHERIC ABSOLUTE PHASE ESTIMATION

A. Atmospheric Contributions

Atmospheric effects in SAR imaging are the result of additional time delays when microwave radar pulses pass through the troposphere (with a changing refractive index). Thus, additional phase quantities are merged with the topographic phase and lead to interferograms with mixed atmospheric and topographic fringes. Therefore, topography estimation from the measured interferograms requires atmospheric correction to retrieve the correct height and also to dispose coherent multichannel phase data for further processing.

A number of works addressed the atmospheric correction problem. Some approaches exploit meteorological parameters such as the temperature, the pressure, and the water vapor content to eliminate these atmospheric artifacts [7]. However, such parameters are not always provided with the data. Thus, the authors in [8] propose alternative methods using permanent scatterers (PSs) from the time series of interferograms. As results would be highly dependent to the density of PSs, other works develop methods directly based on the interferograms. For instance, in [9], the correlation between interferograms having a common acquisition is used to remove the atmospheric disturbances. In [10], the authors provide a two-step method that first estimates global-scale atmospheric contributions given a DEM and then refines the obtained results by retrieving local atmospheric artifacts. However, both of these two approaches require a common master SAR image in order to exploit some correlation from interferograms. Furthermore, a large number of interferograms are required for accurate estimation. Therefore, providing an approach that overcomes such limiting conditions is necessary. The method that we propose in this letter achieves the DEM estimation jointly with the atmospheric effect correction lying on the power of the Bayesian framework.

We note that a similar structure of the proposed method was introduced in [11], where the authors aim to estimate InSAR phase offsets from a set of interferograms in order to make the Bayesian-based MCPU approach possible. Our approach extends this idea to deal with general global-scale tropospheric artifacts in a set of InSAR data, while providing much more efficient algorithm using the advances in discrete optimization tools in the computer vision field.

B. Multichannel MAP-MRF Energy Model

The multichannel approach for DEM estimation consists in combining two or more interferograms of the same scene to reduce phase ambiguity in the unwrapping step. These data are obtained either with a multifrequency or multibaseline configuration. The first one requires sensors operating at different frequencies to provide statistical independent interferograms. The second one, which is the most usual, provides SAR images with slightly different view angles. For the rest of this letter, without loss of generality, we will consider only notations with the multibaseline configuration.

Let us consider a surface imaged with a SAR sensor with the multibaseline technique and assume that no ground displacements took place in the time period during which the series of InSAR data were acquired. The wrapped measured phase ψ_c of the cth channel can be modeled as follows:

$$\psi_c = \left\langle \phi_c^{\mathrm{top}} + \phi_c^{\mathrm{atm}} + \phi_c^{\mathrm{noise}} \right\rangle_{2\pi}, \qquad c \in \{1 \ldots M\} \tag{1}$$

with M as the total number of multibaseline interferograms, ϕ^{noise} as the interferometric noise, ϕ^{atm} as the atmospheric phase contribution, and $\phi_c^{\mathrm{top}} = b_c h$ as the topographic phase which is geometrically related to the altitude h through the current channel acquisition parameter $b_c = (4\pi B_c^{\perp}/\lambda R \sin(\theta))$ ($B_c^{\perp}$ is the orthogonal baseline, λ is the wavelength, R is the distance from the first antenna to the center of the surface, and θ is the view angle).

Some works proposed a simplified model to the absolute atmospheric phase relying on empirical results [10]. They show an affine relationship between such information and the topography, if local atmospheric heterogeneities are neglected. Thereby, the following global model will be considered in the current work:

$$\phi_c^{\mathrm{atm}} = \alpha_c h + \beta_c, \qquad c \in \{1 \ldots M\} \tag{2}$$

with two static parameters for each channel: the slope α_c and the offset β_c.

Retrieving the height information from the noisy wrapped multichannel data Ψ with the atmospheric contributions is an ill-posed inverse problem. The Bayesian framework with the maximum *a posteriori* (MAP) estimator together with the Markov random fields (MRFs) is a powerful tool to deal with such inverse problems, if statistical models are provided.

This problem of DEM reconstruction from multichannel InSAR data was addressed in several works. A simple and accurate model was proposed in [6], showing good and robust reconstruction results from highly ambiguous phase data. However, the problem of atmospheric effects was not addressed. So, in the following, we extend the already proposed model to deal with such contributions.

Assuming the affine correlation between the atmospheric phase and the topography, and also the conditional independence between the multichannel data sites, the statistical likelihood distribution function of multichannel InSAR data could be expressed as follows:

$$P(\Psi | h, \alpha, \beta) = \prod_{p=1}^{N} \prod_{c=1}^{M} P(\psi_{p,c} | h_p, \alpha_c, \beta_c) \tag{3}$$

where $\Psi = [\psi_1, \ldots, \psi_M]^{\mathrm{T}}$ denotes the collection of the multichannel phase data, N is the size of one image, α and β are the two vectors collecting the multichannel parameters of the global-scale atmospheric model, and $P(\psi_{p,c} | h_p, \alpha_c, \beta_c)$ denotes the single channel likelihood function given by (4), shown at the bottom of the next page, with γ_c as the absolute coherence of the cth channel.

To model the estimated altitude h, we consider an MRF with pairwise interactions. The weighted total variation prior energy is considered as a prior model. The latter has recently shown good results for DEM estimation for both natural and urban areas. It preserves both discontinuities and smooth surfaces, while checking useful properties for an efficient MAP-MRF estimation task [6].

- **Initialization:** $\widetilde{h}^{(0)} = 0, \widetilde{\alpha}^{(0)} = \widetilde{\beta}^{(0)} = 0$
- **Repeat until convergence**

  – **DEM –Step (MAP-MRF):**
  $$\widetilde{h}^{(i)} = \underset{h}{\operatorname{argmin}}\, E\left(h \middle| \Psi, \gamma, \widetilde{\alpha}^{(i\ 1)}, \widetilde{\beta}^{(i\ 1)}\right)$$
  – **Atmospheric-Step (ML):**
  $$\left(\widetilde{\alpha}^{(i)}, \widetilde{\beta}^{(i)}\right) = \underset{\alpha,\beta}{\operatorname{argmin}}\, E\left(\alpha, \beta \middle| \Psi, \gamma, \widetilde{h}^{(i)}\right)$$

Fig. 1. Joint DEM/global atmospheric parameter estimation algorithm.

The total energy for the joint estimation problem of DEM and atmospheric contributions is formulated as follows:

$$E(h,\alpha,\beta) = -\log\left(P(\Psi|h,\alpha,\beta)\right) + \sum_{(p,q)} w_{p,q}|h_p - h_q| \tag{5}$$

where (p, q) denotes a couple of neighboring pixels and $\{w_{p,q}\}_{(p,q)}$ denotes the local hyperparameters for the prior set as described in [12].

Finally, performing a minimization algorithm of the energy (5) gives an optimal joint solution of the DEM and the atmospheric parameters. However, because of the nonconvexity of the energy function and the high dimensionality of data, an efficient minimization algorithm is required for such inference problem.

Several discrete optimization algorithms have recently been proposed in the computer vision community, allowing efficient minimization of nonconvex Markovian energy functions with convex priors. The algorithm in [12] has recently shown accurate results for the multichannel InSAR PU problem. Therefore, it will be considered in this work as an intermediate step for the joint DEM/atmosphere estimation. The proposed method is detailed in the following sections.

C. Efficient Joint DEM/Atmosphere Estimation

The proposed two-step estimation algorithm is shown in Fig. 1. At the first step (DEM-Step), a DEM is generated without taking into account the atmospheric contributions. At the second step (Atmospheric-Step), more accurate estimation of these parameters is performed knowing an approximate estimation of the topography. The two steps are repeated until convergence to an optimal joint solution.

a) *DEM-Step*: Assuming fixed atmospheric parameters, the DEM estimation task corresponds to the MCPU problem addressed in [12] using the MRF energy minimization framework with efficient graph-cut optimization tools. The authors proposed a new optimization algorithm based on a mixed optimization strategy. First, a discrete optimization algorithm with the graph-cut technique is performed. The surface height values are considered within a quantized label set $\mathcal{L}$. Starting from a uniformly initialized height image $h^{(0)}$, several partition moves that change the solution are achieved iteratively, until convergence toward an optimal configuration (a local minimum of the energy function). The considered partition moves are large and multilabel, allowing a large number of pixels to change their current labels to a set of new labels in an optimal way. These moves are carried out efficiently, owing to the graph-cut technique. In fact, at each iteration, a layered graph is constructed such that a minimum cut on this graph gives the optimal configuration within the set of all possible configurations. Owing to the polynomial time minimum-cut algorithm in [13], the algorithm efficiently converges toward a local minimum of the energy. More details on the graph topography and convergence properties are presented in [12]. Starting from the discrete optimum, which is assumed to be a correct reconstructed absolute phase, local hyperparameters of the prior energy are estimated to conserve discontinuities, followed by a continuous optimization algorithm (the gradient descent for instance). A continuous surface height is finally reconstructed. This algorithm has shown better DEM estimation results compared to the state-of-the-art approaches. Usually, global optima of the energy function are reached, owing to the optimal large and multilabel move-making technique. Efficiency is guaranteed using a fast optimization scheme with the graph-cut technique.

b) *Atmospheric-Step*: Starting from the generated DEM at the previous step, the M atmospheric couple of parameters (α_c, β_c) is approximated by the maximum likelihood (ML) estimator as follows:

$$(\hat{\alpha}, \hat{\beta}) = \underset{\alpha,\beta}{\operatorname{argmin}}\, E(\alpha, \beta|\Psi, \gamma, h) \tag{6}$$

which is a simple estimation task, where the deterministic atmospheric parameters are retrieved iteratively by maximizing the likelihood function (4) with a fixed height.

It is easy to see that inaccurate atmospheric parameters will be obtained at the first steps of the proposed method, since the estimation is mainly dependent on the accuracy of the input DEM. More precisely, according to the global-scale atmospheric model, surface with high topography will be highly sensitive to the errors of DEM estimation. However, we note that, for low topographic regions, the multichannel approach

$$P(\psi_{p,c}|h_p,\alpha_c,\beta_c) = \frac{1-|\gamma_{p,c}|}{2\pi\left(1-|\gamma_{p,c}|^2\cos\left(\psi_{p,c}-(b_c+\alpha_c)h_p-\beta_c\right)^2\right)} \times \left(1+\frac{|\gamma_{p,c}|\cos\left(\psi_{p,c}-(b_c+\alpha_c)h_p-\beta_c\right) cos^{-1}\left(-|\gamma_{p,c}|\cos\left(\psi_{p,c}-(b_c+\alpha_c)h_p-\beta_c\right)\right)}{\left(1-|\gamma_{p,c}|^2\cos\left(\psi_{p,c}-(b_c+\alpha_c)h_p-\beta_c\right)^2\right)^{1/2}}\right) \tag{4}$$

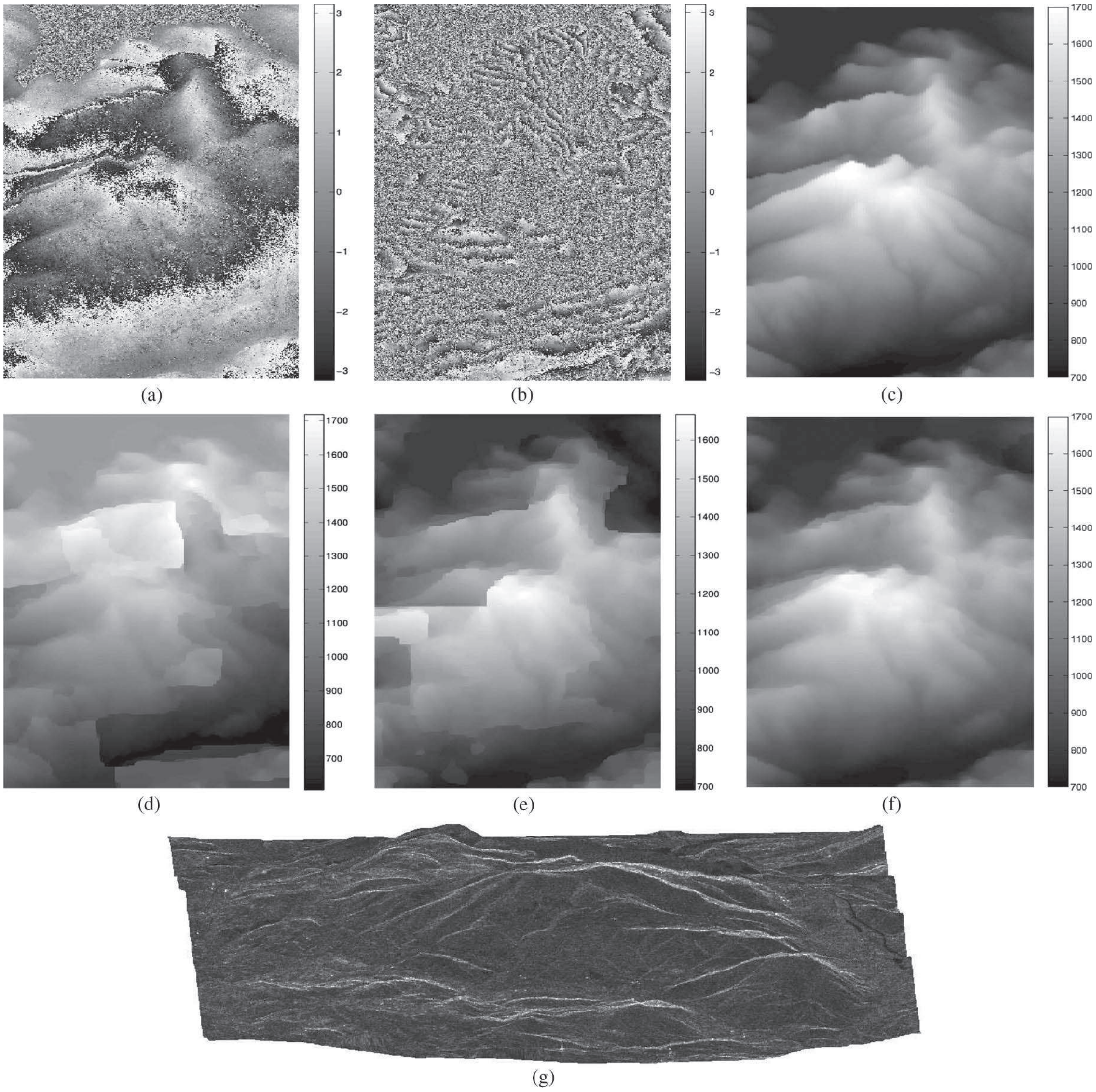

Fig. 2. (a) First (least ambiguous) interferogram. (b) Last (most ambiguous) interferogram. (c) SRTM DEM. (d) Estimated DEM without atmospheric correction using the algorithm in [12]. (e) Estimated DEM with only the multichannel offsets' estimation. (f) New estimated DEM with atmospheric correction. (g) Three-dimensional view of the obtained DEM with the SAR amplitude superposed.

leads to a correct DEM estimation if the offsets β_c are well retrieved at the first Atmospheric-Step. This is usually the case, as observed in different experiments. Then, the atmospheric parameters, together with the DEM, are further improved iteratively until convergence to an optimal joint solution.

Furthermore, similar to [11], only coherent pixels are considered in the ML estimation, in order to reduce errors due to the low coherent regions. Since the coherence maps are provided as input information to the estimation problem, a threshold is fixed to select the most coherent pixels.

In the next section, the experiments conducted to highlight the effectiveness of the proposed two-step algorithm are presented, and results are compared to previous works.

III. Experimental Results

The proposed approach has been first tested on simulated data generated with different noise levels and atmospheric perturbations. Then, it has been validated on real InSAR data. In this section, only the DEM of real data is generated and compared to other approaches.

We dispose of a set of six C-band ERS InSAR tandem data (interferograms and coherence maps) acquired on a mountain area of Serre-Ponçon (southeast of France). The smallest orthogonal baseline is about 15 m, and the biggest one is about 151 m. Interferograms related to these baselines are shown in Fig. 2(a) and (b).

We also dispose of the Shuttle Radar Topography Mission (SRTM) DEM related to the same region, projected into the same radar geometry as the InSAR data Fig. 2(c). The SRTM DEM has 90-m spatial resolution and 16-m absolute vertical height accuracy, which is a lower resolution compared to the data that we are using (spatial resolution of 25 m).

As atmospheric effects on mountains are known to be very significant, and increase linearly with the altitude, we selected a mountain region of Serre-Ponçon surrounded by a lake in order

TABLE I
ESTIMATED GLOBAL-SCALE ATMOSPHERIC PARAMETERS

Channel	$B^{\perp}$ (m)	α	β
1	15	0.00	-1.22
2	29	0.01	1.41
3	52	0.00	-0.65
4	97	-0.01	-1.53
5	112	-0.01	0.02
6	151	-0.02	0.02

to confirm the robustness of the proposed method while dealing with complex structures and low coherent regions.

The estimated DEM without atmospheric correction is shown in Fig. 2(d) using the DEM-Step algorithm in [12]. We clearly see the necessity of atmosphere correction from the input InSAR images in order to retrieve the correct surface altitude. Assuming that only phase offsets have to be eliminated from the set of interferograms, as studied in [11] but with a different regularization model, the result shown in Fig. 2(e) is processed using our method while setting the slope parameters α_c to zero. We also note unwrapping errors that are mainly located in regions with high altitudes. Thus, atmospheric artifacts are somehow related to the height of the scene. A global linear model could be an approximate model to such contributions.

Note that a blocking effect appears in the reconstructed profile 2(e) that is a result of the unwrapping difficulties paired with total-variation-based regularization. This artifact could possibly be removed when applying other *a priori* models [11].

As we can see in Fig. 2(f), the reconstructed DEM with the proposed approach seems to be much more coherent and very similar to the SRTM one, with better accuracy. The obtained DEM is down sampled to the spatial accuracy of the SRTM DEM, giving a normalized mean square error of about 8.289×10^{-4}. The retrieved global-scale atmospheric parameters are depicted in Table I. We shall note that, even if low level of atmospheric slope parameters is recorded, they induce a height variation of about 40 m, leading to incoherent multibaseline channels for a surface reconstruction. We also observe different behaviors of the atmospheric disturbances on the data set. In some channels, only phase offsets are recorded, while the linear model is met in the others. This could be explained by the various meteorological conditions affecting the data set. The channels with only offsets are acquired in July and August, while the others are acquired in October, November, and March. The atmospheric phase contribution that is correlated with the height of the scene is likely to be well observed in case of worse meteorological conditions.

A 3-D view of the reconstructed region is also highlighted in Fig. 2(g) with a SAR amplitude image as a texture.

Computational results are also interesting, owing to the fast PU algorithm used for the DEM-Step. Three (DEM-Step + Atmospheric-Step) iterations of the algorithm were sufficient to converge toward an optimal joint solution; each one has required about 16-min running time to reconstruct the DEM image of size 650 × 300 pixels with 1000 height labels (an absolute height accuracy of 1 m), which is a competitive complexity result in the nonconvex MRF energy minimization framework. We have to note also that only one iteration of (Atmospheric-Step + DEM-Step) would be enough to reconstruct a DEM with the same quality if we take the SRTM DEM as an input image to first estimate the atmospheric parameters and then perform the DEM reconstruction.

IV. CONCLUSION

In this letter, the DEM estimation problem from a set of InSAR data corrupted by noise and atmospheric contributions has been addressed within the Bayesian MAP-MRF framework. Results show the effectiveness and robustness of the proposed approach to retrieve correct and precise scene altitude, compared to the SRTM DEM, even in the presence of a few of the multichannel interferograms. Accuracy will increase if more interferograms are provided, while keeping the same complexity, owing to the Bayesian framework and the efficient optimization algorithms. As only global-scale atmospheric contributions are considered in this work, which are observed in the current InSAR data, local heterogeneities can be considered as a perspective. However, a special effort needs to be focused on the efficiency of the joint estimation algorithm of local height and atmospheric contributions. Moreover, we would stress that the proposed method can be extended to jointly estimate the DEM and surface displacement, if a statistical model of the latter is provided.

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