AN ANALYTIC MODEL FOR EVALUATING OUTAGE AND HANDOVER PROBABILITY OF CELLULAR WIRELESS NETWORKS

T.T. VU, L. DÉCRESSEFOND AND P. MARTINS

ABSTRACT. We consider stochastic cellular networks where base stations locations form a homogeneous Poisson point process and each mobile is attached to the base station that provides the best mean signal power. The mobile is in outage if the SINR falls below some threshold. The handover decision has to be made if the mobile is in outage during several time slots. The outage probability and the handover probabilities are evaluated taking into account the effect of path loss, shadowing, Rayleigh fast fading, frequency factor reuse and conventional beamforming. The main assumption is that the Rayleigh fast fading changes at each time slot while other network components remain static during the period of study.

1. Introduction

In wireless networks, relative locations of both customers and resources play a crucial role in the performance of the whole system. Base stations (BS) are often assumed to be deployed according to an hexagonal pattern.

Figure 1. 2G and 3G base stations locations in Paris. Source: cartoradio.fr

As shown by Figure 1, this approach fails to capture the irregularity and randomness of a real network. Recently, stochastic model of nodes gained some interest. The most prevalent stochastic process in use is the Poisson point process since it allows some flexibility in the parameter choice and usually leads to elegant closed formulas. A configuration $\omega$ in a bounded domain $C$ of the plane which represents for instance a district or a town, is a finite set of points $\omega = \{x_1, \cdots, x_n\}$. Let
the set of configuration on \( C \), for a positive real \( \lambda \), the Poisson distribution of intensity \( \lambda \) over \( \Gamma_C \) is the probability measure \( \mathbb{P} \) such that: 1) for any \( A \subset C \), the integer-valued random variable \( \omega(A) \) which represents the number of points of \( \omega \) which fall into \( A \) is Poisson distributed with parameter \( \lambda \text{area}(A) \), 2) if \( A \) and \( B \) are two disjoint subsets of \( C \), the random variables \( \omega(A) \) and \( \omega(B) \) are independent. See [2, 3] for properties and applications of this model.

back to wireless network modeling, it is frequently assumed that a mobile once active in the network is served by the nearest BS. This is justified if we consider a path loss exponent model of radio propagation and if we remove the effect of fading. This assumption implies that cells (i.e. region of the plane served by each BS) are Poisson-Voronoi cells (for example, [2], page 63). We here consider that the mobile is served by the BS that provides the strongest mean signal power in time (best server). The mean signal power depends both on path loss and slow fading. This choice of serving BS can be made either by the mobile or the operator. Thus, our model can be thought as a generalization of the Poisson-Voronoi cell model.

Once the mobile is attached to a particular BS, the signal received from this BS is the useful signal whereas the cumulative signal received from other BS using the same frequency is considered as interference. It is not longer true when we consider advanced systems in which base stations are cooperative; however, our model covers almost all existing cellular networks. To model the frequency reuse, we add an independent mark to the atoms of our Poisson point process of BSs. A BS interferes with the other BSs that have the same mark. That is to say that we put a label on each position, thus the configurations become sets of the form \( \{(x_1, m_1), \ldots, (x_n, m_n)\} \). Marks are said to be independent if the random choice of \( m_j \) depends only on \( x_j \) and not on the other atoms (see again [2] for details).

In addition to the interference, the local noise can intervene. For a mobile to communicate with a BS, the signal-to-noise-plus-interference ratio (SINR) at this mobile location must exceed some threshold; in this case the mobile is covered, otherwise, it is said to be in outage. If the mobile is in outage during a period of time, i.e. for several consecutive time slots, a handover decision has to be made. It can be made by the mobile, the served BS, the network system or even by a neighboring BS. In this paper, we are interested in the calculation of the outage probability and the handover probability in explicit forms. Since we assume a homogeneous Poisson point process of BSs, but not fixed patterns, these results does not depend on the position of the mobile and can be considered as global, i.e. valid for all MSs.

In [9], Haenggi showed that the path loss fading process is a Poisson point process on the real line in the case of path loss exponent model. In [1], [2] and [3], Baccelli and al. established closed form expressions for outage probability of networks where each node tries to connect with a destination at fixed distance or with the nearest node in case of Rayleigh fading. In [6], Kelif et al. found an expression of the outage probability for cellular networks by mean of the so-called fluid model. In [5], Ganti et al. gave some interesting results about temporal and spatial correlation of wireless networks. In [10] and [11], outage probability of regular hexagonal cellular networks with reuse factor and adaptive beamforming was studied by simulation.

This paper is organized as follows. In Section 2 we describe our model. In Section 3, we calculate the outage probability. In Section 4, we calculate the handover
probability. Section 5 shows the numerical results and the difference between our model and the traditional hexagonal model.

2. System model

Given a BS (base station) located at \( y \), of transmission power \( P \), and an MS (mobile station) located at \( x \), the mobile’s received signal has average power \( L(y-x)P \) where \( L \) is the path loss function. The most used path loss function is the path loss exponent law \( L(z) = K|z|^{-\gamma} \) where \( |z| \) refers to the Euclid norm of \( z \). The parameter \( K \) depends on the characteristics of the antenna and the path loss exponent \( \gamma \), typically in the range (2, 4), characterizes the environment under study. Actually, this path loss model gives closed formulas but is not correct for small distances as it implies an almost infinite power close to the BS. It is thus often preferable to consider the modified path loss exponent model \( L(z) = K(\max\{R_0, |z|\})^{-\gamma} \) where \( R_0 \) is a reference distance. In addition to the deterministic large scale effect, there are two random factors that have to be considered. The first one, called shadowing, represents the signal attenuation caused by large obstacles such as buildings. The second, called fast fading, represents the impact of multi-path. The shadowing can be considered as constant during a period of communication of a mobile while the fast fading changes at each time slot. If no beamforming technique is used, the received signal power from the BS \( y \) to the MS \( x \) at the time slot \( l \) is \( P_{yx}[l] = r_{yx}[l]h_{yx}L(y-x)P \), where \( \{h_{yx}\}_{x,y \in \mathbb{R}^2} \) are copies of a random variable \( H \) while \( \{r_{yx}[l]\} \) are independent copies of \( R \) which is an exponential random variable of mean \( 1/\mu \). We suppose that for each \( x \), the random variables \( \{h_{yx}, y \in \mathbb{R}^2\} \) are independent, and \( p_H \) (resp. \( F_H \)) denotes their PDF (resp. complementary CDF).

The most used shadowing random model is log-normal shadowing, for which \( H \) is a log-normal random variable. In this case, we can write \( H \sim 10^G/10 \) where \( G \sim \mathcal{N}(0,\sigma^2) \). We now consider the conventional beamforming technique with \( n_t \) antennas. The power radiation pattern for a conventional beam-former is the product of the array factor times the radiation pattern of a single antenna. If \( \phi \) is the direction towards which the beam is steered, the array gain in the direction \( \theta \) is given by (\cite{11}, \cite{10}):

\[
\frac{\sin^2(n_t \xi (\sin(\theta) - \sin(\phi)))}{n_t \sin^2(\xi (\sin(\theta) - \sin(\phi)))} g(\theta),
\]

where \( g(\theta) \) is the gain in the direction \( \theta \) with one antenna. For simplicity we assume that the BS always steers to the direction of the served MS and the gain \( g(\theta) \) is positive constant on \((-\pi/2, \pi/2)\) and 0 otherwise (zero front-to-back power ratio).

Hence, the interference signal power from a BS to an MS attached to another BS using the same frequency, in the direction \( \theta \), will be reduced by a factor of:

\[
a(\theta) = 1_{\{\theta \in (-\pi/2, \pi/2)\}} \frac{\sin^2(n_t \xi (\sin(\theta)))}{n_t^2 \sin^2(\xi (\sin(\theta)))}.
\]

If the beamforming technique is not used \( a(\theta) = 1 \). We assume that the bandwidth is split in \( k \) non interfering sub-bands.

Thus, for a mobile at position \( x \), any BS is characterized by three quantities: its position \( y \), the sub-band in which it operates \( e \) and \( \xi^{-1} = h_{xy}L(y-x)P \). Once being in the network, the mobile \( x \) is attached to (or served by) the BS that provides the best average signal strength in time: it is attached to the BS which has the
minimal $\xi$. We denote by $y_0$ the position of the chosen BS and by $(y_n, n \geq 1)$ the positions of the other BSs. Sub-bands and $\xi$’s are indexed accordingly.

Assume that each BS using frequency $e_0$ is always serving an MS, and denote by $\theta_i$ the argument of the segment $[x, y_i]$. The SINR at time slot $t$ is given by:

$$s_x[t] = \frac{r_{y_0x}[t]^{\xi^{-1}}}{N + \sum_{i \neq 0} 1_{\{e_i = e_0\}}a(\theta_i)r_{y_i,x}[t]^{\xi^{-1}}}$$

where $N$ is the noise power, assumed to be constant. The term

$$I_x = \sum_{i \neq b(x)} 1_{\{e_i = e_0\}}a(\theta_i)r_{y_i,x}[t]^{\xi^{-1}}$$

is the sum of all interference. In order to communicate with the attached BS, the SINR must not fall below some threshold $T$.

We assume that the base stations are distributed in the plane according to a Poisson point process $\Pi_B$ of intensity $\lambda_B$. The frequency $e_i$ at which operates $y_i$ is chosen uniformly in $\{1, \cdots, k\}$ where $k$ is the frequency reuse factor. The BSs that have the same mark interfere between themselves. Our reuse model can be considered as a worst case scenario since the sub-bands are distributed at random, in contrast with planned network patterns where frequencies are attributed to BSs in order to minimize interferences. The subsequent computations rely mainly on the following theorem.

**Theorem 1.** The family of random variables $\Xi = \{(h_{y,x}L(y - x)P)^{-1}, y \in \Pi_B\}$ is a Poisson point process on $\mathbb{R}^+$ with intensity $d\Lambda(t) = \lambda_B B'(t)dt$ where $B(\beta) = \int_{\mathbb{R}^2} F_H((L(z)P\beta)^{-1})dz$.

**Proof.** Define the marked point process $\Pi^\ast = \{y_i, h_{y_i,x}\}_{i=0}^\infty$. It is a Poisson point process of intensity $\lambda_B dy \otimes f_H(t)dt$ because the marks are independent and identically distributed. Consider the probability kernel $p((z, t), A) = 1_{\{L(z)P^{-1}, (A)\}}$ for all sets $A \subset \mathbb{R}^+$ and apply the displacement theorem [2, Theorem 1.3.9], to obtain that $\Xi$ is a Poisson point process whose intensity measure we denote by $\Lambda$.

Moreover, for any $\beta$

$$\Lambda([0, \beta]) = \lambda_B \int_{\mathbb{R}^2 \otimes \mathbb{R}} 1_{\{t \geq (\beta P L(z)P)^{-1}\}}p_H(t)dzdt$$

$$= \lambda_B \int_{\mathbb{R}^2} F_H((\beta P L(z)P)^{-1})dz = \lambda_B B(\beta).$$

This concludes the proof. \qed

By straightforward quadratures, we get the following proposition.

**Proposition 1.** If $L(z) = K(\max\{R_0, |z|\})^{-\gamma}$ then:

$$B(\beta) = C_1 \beta^{\frac{\gamma}{2}} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \theta_{\frac{\pi}{2}} p_H(t)dt,$$

where $C_1 = \pi(PK)^{\frac{\gamma}{2}}$. For log-normal shadowing $H \sim 10^{G/10}$ where $G \sim N(0, \sigma^2)$ and we have:

$$B(\beta) = C_1 \beta^{\frac{\gamma}{2}} e^{(\frac{2\sigma_1}{\sigma_1})^2} Q\left(\frac{-\ln \beta - \ln(PKR_0^{-\gamma})}{\sigma_1} - \frac{2\sigma_1}{\gamma}\right)$$

where $Q(u) = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} e^{-u^2/2}du$ and $\sigma_1 = \frac{\sigma \ln 10}{10}$. 


For the exponent pathloss model, it is sufficient to put \( R_0 = 0 \) in the above formulas. This particular result could be derived from [9]. We observe that the distribution of the point process \( \Xi \) does depend only on \( E(H^{\frac{2}{\gamma}}) \) but not on the distribution of \( H \) itself. This phenomenon can be explained as in [4, Page 159].

3. Outage analysis

The mobile at \( x \) suffers an outage at time slot \( l \) whenever its SINR falls below a threshold \( T \) at this slot. For the sake of notations, in this Section, we drop the index \( l \) as it is fixed.

**Theorem 2.** The outage probability is given by

\[
(4) \quad p_o := \mathbb{P}(s_x < T) = 1 - \lambda_B \int_0^\infty B'(\beta)e^{-\lambda_B B(\beta) - NT_{ \mu} - \frac{\lambda_B}{2\pi k} D(\beta)} \, d\beta
\]

where \( D(\beta) = \int_{-\pi}^{\pi} d\theta \int_0^\infty B'(\xi)(1 + \xi/T_{\alpha}(\theta))^{-1} d\xi. \)

**Proof.** Since \( r_{y_0,x} \) is an exponential r.v. of mean 1/\( \mu \) we have:

\[
\mathbb{P}(s_x \geq T|\xi_0 = \beta) = \mathbb{P}(r_{y_0,x} \geq T_B(N + I_x(\beta)) | \xi_0 = \beta) = E(e^{-\mu B(\beta)(N + I_x(\beta))} | \xi_0 = \beta) = e^{-NT_{ \mu} B(\beta)} \mathcal{L}_{I_x(\beta)}(T_B \mu)
\]

where \( I_x(\beta) \) is the distribution of the random variable \( I_x \) given \( (\xi_0 = \beta) \) and \( \mathcal{L}_{I_x(\beta)} \) is its Laplace transform. Given \( (\xi_0 = \beta) \), according to strong Markov property, the point process \( \{\xi_1\}_{i \geq 0} \) is a Poisson point process on \( (\beta, \infty) \) with intensity \( \lambda_B B'(\xi)d\xi \).

By thinning, the point process \( \{\xi_i\}_{i \geq 0, c_1 = c_0} \) is a Poisson point process on \( (\beta, \infty) \) with intensity \( k^{-1}\lambda_B B'(\xi)d\xi \). Hence, \( \mathcal{L}_{I_x(\beta)} \) can be calculated as follows (see [2]):

\[
\mathcal{L}_{I_x(\beta)}(u) = \exp\left(-\int_\beta^\infty \frac{\lambda_B}{2\pi k} B'(\xi)(1 - E(e^{-a(\theta)u\xi^{-1}}))d\xi\right)
\]

\[
= \exp\left(-\frac{\lambda_B}{2\pi k} \int_\beta^\infty B'(\xi)d\xi \int_0^\pi dr \int_{-\pi}^\pi d\theta e^{-ur}(1 - e^{-a(\theta)ur\xi^{-1}})d\theta\right)
\]

\[
= \exp\left(-\frac{\lambda_B}{2\pi k} \int_{-\pi}^{\pi} d\theta \int_\beta^\infty B'(\xi) \frac{d\xi}{1 + \xi(ua(\theta)^{-1})^{-1}}\right).
\]

Thus, we get

\[
\mathbb{P}(s_x \geq T|\xi_0 = \beta) = \exp\left(-NT_{ \mu} B(\beta) - \frac{\lambda_B}{2\pi k} D(\beta)\right).
\]

Since the distribution density of \( \xi_0 \) is \( \lambda_B B'(\beta)e^{- \lambda_B B(\beta)} \), by averaging over all \( \xi_0 \) we obtain (4). \( \square \)

**Proposition 2.** In the interference-limited regime \( (N = 0) \), we have

\[
(6) \quad p_o(T) = 1 - \lambda_B \int_0^\infty B'(\beta)e^{-\lambda_B B(\beta) - \frac{\lambda_B}{2\pi k} D(\beta)} \, d\beta.
\]

If \( L(\gamma) = K|z|^{-\gamma} \) we have:

\[
(7) \quad p_o(T) = 1 - \int_0^\infty e^{-M_{\alpha} - G_\alpha \gamma} \, d\alpha
\]
where $M := M(k, T, \gamma) = 1 + \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta \int_{1}^{\infty} \frac{du}{1+(T.a(\theta))^{-1}u^2}$ and $G = NT\mu(\lambda_B C)^{-\frac{1}{2}}$.

If $L(z) = K|z|^{-\gamma}$ and $N = 0$ we have:

\begin{equation}
(8) \quad p_o(T) = 1 - \frac{1}{M}.
\end{equation}

Some interesting facts are observed from these results:

- Rewrite the expression of SINR as

  \begin{equation}
  s_x[l] = \frac{\tau_{yo,x}[l]|\xi_0^{-1}}{\mu N + \sum_{\lambda \neq 0} 1\{e_i = e_0\}a(\theta_i)\tau_{yi,x}[l]|\xi_1^{-1}}
  \end{equation}

  where $\tau_{yo,x}[l] = \mu r_{yo,x}[l]$. Since $r_{yo,x}[l]$ is an exponential random variable of mean $1/\mu$, $\tau_{yo,x}[l]$ is an exponential random variable of mean 1. Hence by the above equation it is expected that the outage probability depends on the product $\mu N$ but not directly on $\mu$ and $N$. It is an increasing function of $N\mu$ which is confirmed by (4). The fact that the outage probability is an increasing function of $\mu$ and $N$ is quite natural, increasing of the noise of the fast fading influence always deteriorates the system performances.

- It is also expected that in the interference limited case ($N = 0$) the outage probability does not depend on $\mu$. It is confirmed by (6). Physically it means that in the absence of noise, the fast fading modifies the channel (from the MS to each BS) characteristics by the same factor, thus the SINR does not change.

- In the interference limited scenario with the exponent pathloss model, the outage probability does not depend neither on $\mu$, nor on the BS density $\lambda_B$ and nor on the distribution of shadowing $H$. It is due to the scaling properties of the pathloss function and of the Poisson point process. The outage probability is a decreasing function of the pathloss exponent $\gamma$, reflecting the fact that bad propagation environment deteriorates the received SINR.

- In the presence of noise $N > 0$ and still for the exponent pathloss model, the outage probability is an increasing function of $\lambda_B$. Hence, it can be thought that the more an operator installs BSs, the better the network is. In addition, if the density of BSs goes to infinity then outage will never occur. However it is not true. In fact, if the density of BSs is very high, the distances between a MS and BSs tend to be relatively small. Hence, the exponent pathloss model is no longer valid since it is not accurate at small distances. If the modified exponent pathloss is used, the outage probability must converge to 0. The outage probability is also an increasing function of $E(H^{\frac{\gamma}{2}})$, and if the shadowing $H$ follows log-normal distribution then the outage probability will be an increasing function of $\sigma$. We recover an other well known fact: the increasing of uncertainty of the radio channel deteriorates the performance of the network.

4. HANDOVER ANALYSIS

If the MS is in outage for $n$ consecutive time slots, a handover decision has to be made. Keep in mind that only the Rayleigh fast fading changes at each time slot. Let $A_l$ be the event that the mobile is in outage in the time slot $l$, and $A^c_l$ its complement and observe that in fact $\mathbb{P}(\cap_{i=1}^n A^c_{j_i}) = \mathbb{P}(\cap_{i=1}^n A_{j_i})$. By definition
Proceeding as for Theorem 2, we get

\[ p_{ho} := \mathbb{P}(\cap_{i=1}^{n} A_{i+i-1}) = \mathbb{P}(\cap_{i=1}^{n} A_i). \]

We have

\[ p_{ho} = 1 + \sum_{m=1}^{n} (-1)^m \sum_{j_1 \neq \ldots \neq j_m \in \{1, \ldots, n\}} \mathbb{P}(\cap_{i=1}^{m} A_{j_i}). \]

\[ = 1 + \sum_{m=1}^{n} \frac{(-1)^m n!}{m!(n-m)!} \mathbb{P}(\cap_{i=1}^{m} A_i^c). \]

**Theorem 3.** The handover probability is given by:

\[ p_{ho} = 1 + \sum_{m=1}^{n} \frac{(-1)^m n!}{m!(n-m)!} q_m, \]

where \( q_m = \mathbb{P}(\cap_{i=1}^{m} A_i^c) \) is given by:

\[ q_m = \int_{0}^{\infty} \lambda_B B'(\beta) e^{-\lambda_B B(\beta) - NT \mu \beta - \frac{m}{mNT} D_m(\beta)} d\beta, \]

and \( D_m(\beta) = \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} B'(\xi)(1 - \frac{1}{1 + mTN(\beta \theta \xi)^m})^m d\xi. \)

**Proof.** We need to calculate the probability \( \mathbb{P}(\cap_{i=1}^{m} A_i^c) \) that is the probability that the mobile is covered in \( m \) different time slots.

\[ \mathbb{P}(\cap_{i=1}^{m} A_i^c | \xi_0 = \beta) = \mathbb{P}(s_x[1] \geq T, \ldots, s_x[m] \geq T | \xi_0 = \beta) \]

\[ = \mathbb{P}(r_{y,x}[i] \geq \beta(TN + I_x[i]), i = 1 \ldots m | \xi_0 = \beta) \]

\[ = E(e^{-\mu(mTN + \sum_{i=1}^{m} I_x(\beta)[i])} | \xi_0 = \beta) \]

\[ = e^{-mNT \mu \beta} L_{\sum_{i=1}^{m} I_x(\beta)[i]}(T \mu \beta) \]

where \( I_x(\beta)[i] \) is the distribution of the random variable \( I_x[i] \) given \( (\xi_0 = \beta) \). We have :

\[ \sum_{i=1}^{m} I_x(\beta)[i] = \sum_{j=1}^{\infty} 1_{(e_i = e_0)} \xi_i^{-1} a(\theta_i)(\sum_{i=1}^{m} r_{y,x}[i]). \]

As the random variables \( r_{y,x}[i] \) are independent copies of the exponential random variable \( R \), the random variables \( \sum_{i=1}^{m} r_{y,x}[i] \) are also i.i.d and the common Laplace transform of the latter is :

\[ L_{\sum_{i=1}^{m} r_{y,x}[i]}(u) = (L_R(u))^m = \left( \frac{\mu}{\mu + u} \right)^m. \]

The Laplace transform of \( \sum_{i=1}^{m} I_x(\beta)[i] \) is now:

\[ L_{\sum_{i=1}^{m} I_x(\beta)[i]}(u) = e^{-\lambda_B B'(\beta) e^{-\lambda_B B(\beta) - NT \mu \beta - \frac{m}{mNT} D_m(\beta)}} \]

Proceeding as for Theorem 2, we get

\[ q_m = \int_{0}^{\infty} \lambda_B B'(\beta) e^{-\lambda_B B(\beta) - NT \mu \beta - \frac{m}{mNT} D_m(\beta)} d\beta. \]

The result follows. \( \square \)

We can obtain more closed expression for \( q_m \) in some special cases.
Proposition 3. In the interference limited regime $N = 0$, we have:

$$q_m = \int_0^\infty \lambda B B'(\beta) e^{-\lambda B(\beta)} - \frac{\lambda B}{2\pi} D_m(\beta) d\beta.$$ 

If $L(z) = K|z|^{-\gamma}$ then:

$$q_m = \int_0^\infty e^{-M_m + \gamma^2/2} d\alpha$$

where $M_m = 1 + \frac{1}{2\pi} \int_\pi^\pi d\theta \int_1^\infty (1 - \frac{1}{1 + T_a(\theta) u})^m) du$.

If $N = 0$ and $L(z) = K|z|^{-\gamma}$ we have: $q_m = 1/M_m$.

From these computations, the same kind of conclusions as for outage probability can be drawn.

5. Numerical results and comparison to the hexagonal model

We place a MS at the origin $o$ and consider a region $B(o, R_g)$ where $R_g = 10,000(m)$. The BSs are distributed according to a Poisson point process in this region. The path loss exponent model is considered. The default values of the model parameters are $K = -20$ dB, $P = 0$ dB, $n_t = 8$ and $\mu = 1$.

In the literature, the hexagonal model is widely used and studied so we would like to compare the two models. For a fair comparison, the density of BSs must be chosen to be the same, i.e the area of an hexagonal cell must be $1/\lambda B$. Unlike the Poisson model where each BS is randomly assigned a frequency, in the hexagonal model, the frequencies are well assigned so that an interfering BS is far from the transmitting BS and BSs of different frequency are grouped in reuse patterns. The reuse factor $k$ in the hexagonal model is determined by $k = i^2 + j^2 + ij$ where integers $i$, $j$ are the relative location of co-channel cell.

Figure 2 shows the outage probability versus the SINR threshold of the Poisson model and the hexagonal model in the case $k = 7$. As we can see, the outage probability in the case of Poisson model is always greater than that of hexagonal model as expected. The difference is about 8 (dB) in the case $\gamma = 4$ and 6(dB) in the case $\gamma = 3$.

In Figure 4, we can see that the outage probability is a decreasing function of $\gamma$ as theoretically observed. In Figure 5, we see if the reuse factor $k$ increases, the MS has to do less handover. Thus, increasing the reuse factor has a positive effect on the system performance not only in term of outage but also in term of handover.

6. Conclusion

In this paper we have investigated the outage and handover probabilities of wireless cellular networks taking into account the reuse factor, the beamforming, the path loss, the slow fading and the fast fading. We valid our model by simulation and compare numerical results to that of hexagonal model. The closed form expressions derived in the this paper can be considered as an upper bound for a real system.

References

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$\lambda = 1.9099 \times 10^{-6}$, $\sigma = 6$, $\mu = 1$, $N = -174 (\text{dBm})$, $k = 7$

Figure 2. Outage probability vs SINR threshold

Figure 3. Handover probability vs SINR threshold
Figure 4. Outage probability vs path loss exponent $\gamma$, Poisson model

Figure 5. Handover probability vs path loss exponent $\gamma$, Poisson model, $n = 3$

Institut Mines-Telecom, Telecom ParisTech, LTCI, Paris, FRANCE
E-mail address: {vu, decreuse, martins}@telecom-paristech.fr