Relay Placement in Cellular Networks

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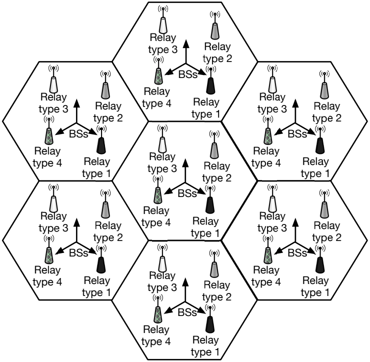
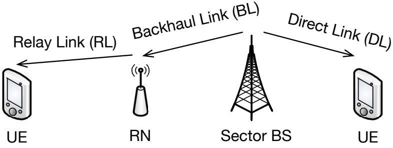
Introduction

- Relays are envisioned for: (a) coverage extension, (b) group mobility, (c) capacity boost
- Relay placement problem arises in various domains: WLAN, WiMAX, sensors, cellular, etc
- It is a sub-case of the "facility location problem" [Drezner04] with specificities: the objective function; the interdependence bw relays through interference
- Interference and traffic models lack of accuracy in the literature, see e.g. [Saleh12]

System Model I

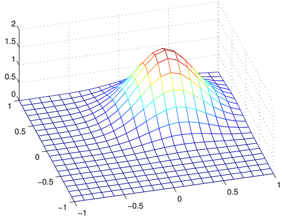
- Tri-sectorized sites, n relays per cell
- Out-of-band (infinite backhaul) and in-band relays
- *Best server* policy

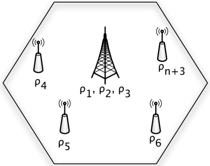




System Model II

- Every station is a M/G/1/PS queue with load $\bar{\rho}_i$
- $\omega(x) = \lambda(x)/\mu(x) = \bar{\omega}\phi(x)$ is the traffic density in x
- $\bar{\omega}$ is the average traffic density
- $\phi(x)$ is the traffic profile
- Every station is active with probability $P[\alpha_i = 1] = \min\{\bar{\rho}_i, 1\}$
- Physical data rate in x when served by i: $C(x, \boldsymbol{\alpha}_{-i})$





Performance of a Relay Placement I

Lemma

The load of a station i is expressed by:

$$\bar{\rho}_i = \frac{\bar{\omega}}{1-\tau(\bar{\omega})} \int_{\mathcal{S}_i} \phi(s) \mathbb{E}_{\boldsymbol{\alpha}} \left[\frac{1}{C(s, \boldsymbol{\alpha}_{-i})} \middle| \alpha_i = 1 \right] ds, \tag{1}$$

where $\mathcal{S}_i$ is the serving area of i.

The proof uses the assumptions that:

- the process $\boldsymbol{\alpha}(t)$ has a time-stationary limit $\boldsymbol{\alpha}$
- the shadowing changes *slowly* in time (and hence with respect to the realizations of $\boldsymbol{\alpha}(t)$)

Performance of a Relay Placement II

- Define $\boldsymbol{\rho} \triangleq (\bar{\rho}_1, ..., \bar{\rho}_{n+3})$
- Define $F(\boldsymbol{\rho}, \bar{\omega}) = (F_1(\boldsymbol{\rho}, \bar{\omega}), ..., F_{n+3}(\boldsymbol{\rho}, \bar{\omega}))$ as follows:

$$F_i(\boldsymbol{\rho}, \bar{\omega}) = \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{\mathcal{S}_i} \phi(s) \mathbb{E}_{\boldsymbol{\alpha}} \left[\frac{1}{C(s, \boldsymbol{\alpha}_{-i})} \middle| \alpha_i = 1 \right] ds \qquad (2)$$

- Define $\bar{\omega}_i^{max} \triangleq \frac{\bar{\omega}}{1-\tau(\bar{\omega})} \int_{\mathcal{S}_i} \frac{\phi(s)ds}{C(s,\mathbf{1})}$

Theorem

If $\boldsymbol{\alpha} \mapsto C(s, \boldsymbol{\alpha})$ is a continuous mapping and $1/C(s, \boldsymbol{\alpha})$ is non-decreasing in $\boldsymbol{\alpha}$, $F : \prod_{i=1}^{n+3} [0; \bar{\omega}_i^{max}] \to \prod_{i=1}^{n+3} [0; \bar{\omega}_i^{max}]$ has at least one fixed point.

Performance of a Relay Placement III

Definition

The *capacity* of the cell is defined as

$$C_{cell} \triangleq \bar{\omega}^{max} A_c \; [bit/sec/Hz/cell], \tag{3}$$

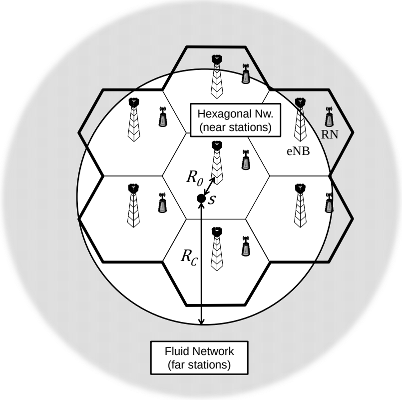
where A_c is the cell surface and $\bar{\omega}^{max}$ is the maximum average traffic density that can be supported by a relay placement (without any station being saturated):

$$\bar{\omega}^{max} = \max\{\bar{\omega} \in \mathbb{R}_+ : F(\boldsymbol{\rho}, \bar{\omega}) = \boldsymbol{\rho} \text{ and } \boldsymbol{\rho} \in [0,1)^{n+3}\} \tag{4}$$

- For every $\bar{\omega}$, $\boldsymbol{\rho}$ is obtained by fixed point equation
- If $\forall i,\ \rho_i < 1$, $\bar{\omega}$ can be supported.

Performance of a Relay Placement IV

- We use the fluid model to reduce computational complexity
- Outside the first ring, BSs and relays are replaced by a continuum of stations



Optimal Placement I

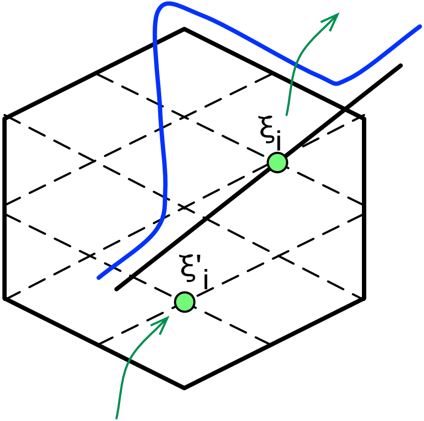
- The facility location problem is known to be NP-hard
- We rely on Simulated Annealing with acceptance rate:

$$\Xi(x \to x') \;=\; \min\left(1,\; e^{-\frac{U(x')-U(x)}{T_m}} \cdot \frac{r(x' \to x)}{r(x \to x')}\right), \qquad (5)$$

- where x is a configuration, $U(x) = -C_{cell}(x)$ is the energy, T_m is the temperature and r is the proposal law (symmetric here).
- We use several "acceleration techniques".

Optimal Placement II

- "Restricted image spaces" [Robini99]: it is preferable to draw x' close to x. We adopt a symmetric gaussian proposal pdf. A candidate falling outside the cell is "periodized" inside the hexagon.



Optimal Placement III

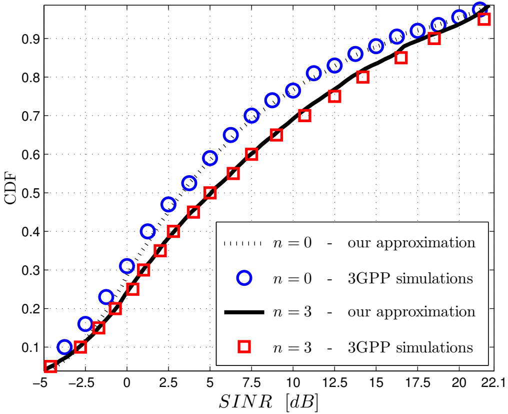
- "Adaptive temperature scheme": initial temperature is adjusted by imposing that the initial average acceptance ratio $\Xi_0 = \lessdot \Xi(x \to x') \gtrdot \in [a, b]$. A dichotomic-type update process is applied:

$$T_0 \leftarrow \beta \; T_0 \;\; \text{if} \;\; \Xi_0 > b \;\; \text{and} \;\; T_0 \leftarrow \frac{0.5}{\beta} \; T_0 \;\; \text{if} \;\; \Xi_0 < a.$$

- "Multi-scale implementation": SA is launched first on a coarse grid then on a finer grid. It allows, at lower level, to avoid spurious transitions between (high level) symmetry-invariant configurations, which can arise due to the complexity of the global energy landscape

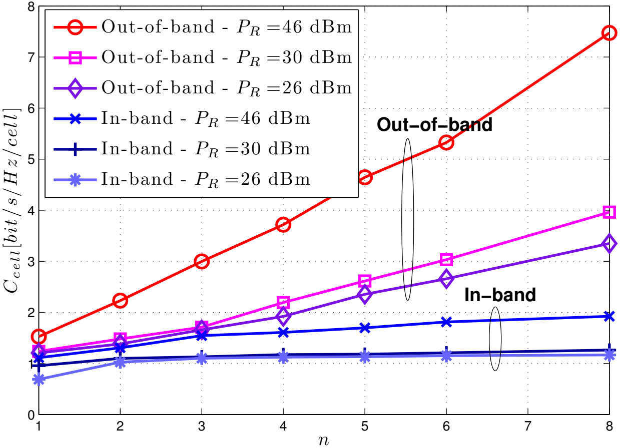
Numerical Work I

- Model validation



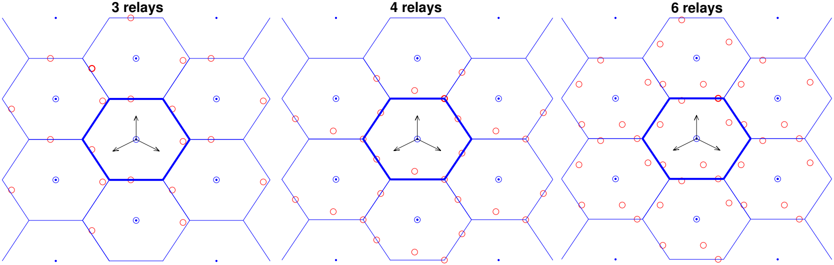
Numerical Work II

- Cell capacity increases even with in-band relays



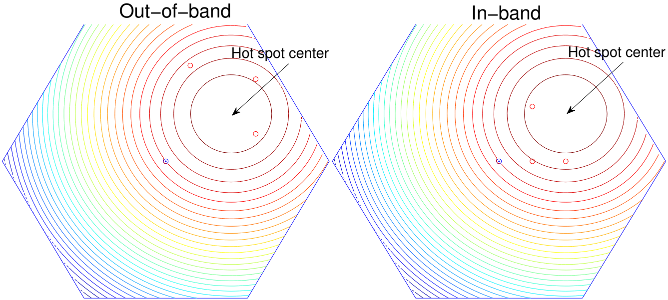
Numerical Work III

- We observe a "repulsion effect" when n or P_R increases
- Relays tend to organize in rings around the BS



Numerical Work IV

- Out-of-band relays offload the BS at cell edge
- In-band relays are closer to the BS because of the backhaul bottleneck



Conclusion

Thank you for your attention