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Intensity noise measurement of strongly attenuated laser diode pulses in the time domain.

Quyên Dinh Xuân[1], Romain Alléaume[2], Liantuan Xiao[3,a], François Treussart[3], Bernard Journet[1], and Jean-François Roch[3]

[1] Laboratoire SATIE, École Normale Supérieure de Cachan/CNRS UMR 8029, 61 avenue du Président Wilson, 94235 Cachan cedex.
[2] Laboratoire Traitement et Communication de l'Information, École Nationale Supérieure des Télécommunications/CNRS UMR 5141, 46 rue Barrault, 75634 Paris cedex 13.
[3] Laboratoire de Photonique Quantique et Moléculaire, École Normale Supérieure de Cachan/CNRS UMR 8537, 61 avenue du Président Wilson, 94235 Cachan cedex.

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Abstract. Developing the ability to characterize photon statistics of light sources has been one of the important driving forces of Quantum Optics. Photon statistics is also a crucial parameter to evaluate of quantum key distribution security. As practical quantum cryptographic systems encode information on faint laser pulses, we present a simple method to measure and calibrate their intensity noise with respect to shotnoise reference. The technique is based on the record of photodetection timetags in the photon counting regime. Two different methods are considered to produce light pulse: first, direct pulsing of a laser diode driving current, second, chopping the CW laser beam emitted by a laser diode with an acousto-optical modulator. As predicted by basic Quantum Optics theory, levels of attenuation used in practical quantum key distribution systems lead in both cases to Poissonian photon number distribution in the generated light pulses.

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1 Introduction

Significant efforts have been expended recently in the development of practical quantum key distribution (QKD) systems which rely on encoding information on single quantum objects [1,2], namely single-photon light pulses [3]. Although true single-photon QKD has been demonstrated recently [4–6], most QKD practical realizations rely on the simple attenuation of laser pulses [7]. Weak coherent pulses (WCPs) with mean photon number smaller than unity are indeed a simple solution to approximate single-photon sources since they only require basic optical elements, like standard semiconductor lasers and calibrated linear attenuators.

Whereas basic QKD security proofs require pure single-photon transmission, the use of faint laser pulses associated to Poissonian photon number statistics is an open door to information leakage towards an eavesdropper [8]. There are nevertheless several ways to guarantee unconditional security for practical setups which employs WCPs instead of pure single-photon pulses. Increasing the attenuation on the quantum channel in order to limit the presence of pulses containing two photons or more is a simple solution. In practice, the information-encoded laser pulses should correspond to a mean number of photon per pulse well below unity, down to a value which scales linearly with the transmission T in intensity of the quantum channel. The secret key exchange rate then scales as T^2, putting a severe limitation on the maximum transmission distance for which unconditional security of the key can be guaranteed [9–12].

To improve security of long-distance QKD, refined protocols like "differential phase-shift" QKD [13] and "decoy-state" QKD [14,15], have been proposed. By tailoring and monitoring the mutual temporal coherence or the photon number statistics between successive WCPs, these schemes allow to retain unconditional security with a limit in propagation distance which scales as T and therefore compete with performance achieved by true single-photon QKD.

However, as the security of QKD systems is deeply conditioned by the photon number statistics and since any failure might jeopardize the security of the quan-

Send offprint requests to: bernard.journet@satie.ens-cachan.fr
[a] *Present address:* College of Physics and Electronics Engineering, Shanxi University; State Key Laboratory of Quantum Optics and Quantum Optics Devices, Taiyuan 030006, China.

tum key establishment protocol, it is important to measure and control the photon number distribution in the information-encoded light pulses transmitted through the quantum channel. Photon number statistics is usually inferred from the second-order correlation function $g^{(2)}(\tau)$ [16], where τ represents the time interval between two detected photons. A good approximation of $g^{(2)}(\tau)$ is given by the histogram of time interval between two *consecutively detected* photons. This histogram can be recorded in the photon-counting regime using a Start-Stop technique in the standard Hanbury Brown and Twiss setup. As explained in Refs. [17] and [18], the $g^{(2)}(\tau)$ peak area allows one to infer the probabilities $P(n)$ of having $n \geq 2$ photons in a WCP containing a mean photon number α much smaller than unity.

While this photon-correlation measurement is well adapted to characterize time-independant photon statistics associated to stationnary processes, more direct measurement can be conceived of for pulsed feeble photon sources. Indeed, as first described in Ref. [19], the record of all photodetection events allows one to directly infer in the time-domain the corresponding photons statistics and to assess the value of the Mandel parameter related to the second moment of the photon number statistical distribution [20, 21]. Using similar detection setup, we reported the direct measurement of the photon statistics associated to a triggered single-photon source relying on the control of the fluorescence of a single dye molecule [22,23].

In this paper, we show how the same simple and direct scheme can be used to calibrate the photon statistics of strongly attenuated light pulses generated from a commercial laser diode. Using a balanced homodyne detection, we first measure the intensity noise in a CW emission regime. We calibrate its excess of classical intensity noise compared to the shotnoise reference associated to Poissonian photon statistics [16]. We then strongly attenuate the beam intensity with neutral density filters and we consider a pulsed regime, obtained either by pulsing the laser diode driving current or by chopping the CW-emitted laser beam with an acousto-optical modulator (AOM). Measurement results confirm that after strong attenuation levels similar to those used in practical QKD systems, eveny noisy classical pulsed light sources lead to WCPs with Poissonian photon number fluctuations.

2 Laser diode intensity noise measurement in CW mode

The laser diode is a single-mode GaAlAs laser diode (Hitachi, HL7851G) with a multi-quantum well structure, operating at 785 nm emission wavelength. The free-running diode has a threshold current $I_{\rm th}$ of 40 mA and a differential quantum efficiency (slope above threshold) of 66%. As shown in figure 1, we first measure the laser diode intensity noise at RF frequencies with a spectrum analyzer by means of a balanced homodyne detection [24]. It consists in two identical detectors based on a 1 mm^2 area PIN photodiode (Centronix, BPX 65) with a load resistor of 2 kΩ, followed by an ac-coupled home-made amplifier based on an ultra-low noise amplifier circuit (Optical Electronics Inc., AH0013) [25]. The quantum efficiency of the photodiodes used for this experiment is approximately 0.8 at the laser diode laser diode emission wavelength. The measured amplifier gain is 15 dB and its bandwidth 6 MHz.

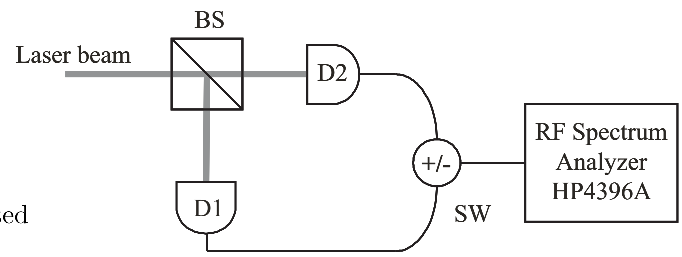

Fig. 1. Experimental set-up for homodyne detection aimed to measure the intensity noise of the laser beam emitted in a CW regime and compare it to the corresponding shotnoise reference. BS: 50/50 beamsplitter. SW: switch +/− corresponds to adding or subtracting respectively the two amplified photocurrents delivered by photodetectors D1 and D2.

Figure 2 shows the noise power spectra corresponding to a driving current intensity of 97 mA ($I/I_{\rm th} = 2.35$). The excess of intensity noise relative to the shotnoise reference is defined by:

$$N_{\rm excess} \equiv N_{\rm d} - N_{\rm s} \tag{1}$$

where $N_{\rm d}$ is the detected noise power spectral density and $N_{\rm s}$ is the corresponding shotnoise level. Note that we neglect any contribution of electronic noise as its level is more than 10 dB lower than all considered shotnoise references (see figure 2). As predicted by basic laser theory, we observe that the excess of intensity noise decreases with an increasing value of the driving current: for a driving current above the laser threshold larger than $2.8 \times I_{\rm th}$, the laser diode intensity noise becomes shotnoise limited.

3 Predicted value of the Mandel parameter for WCP

In order to quantify the fluctuations of the number n of *detected* photons per pulse, we use the Mandel parameter expressed as:

$$Q \equiv \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle} - 1. \tag{2}$$

In Eq. (2), $\langle n^2\rangle - \langle n\rangle^2$ is the variance of the number of detected photons per pulse and $\langle n\rangle$ is its mean value. In case of *perfect detection efficiency*, Poissonian photon statistics would yield $Q = 0$ whereas sub-poissonian and super-poissonian statistics correspond respectively to negative and positive value of the Q parameter [20]. However, a bias is introduced in the measured photocounts statistics by detection deadtime due to recovery time of computer

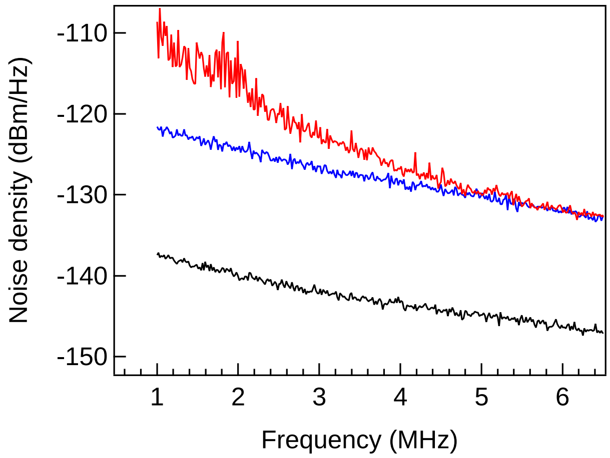

Fig. 2. Noise power spectral densities. Upper curve: intensity noise power density for a laser diode driving current intensity of 97 mA ($I/I_{\rm th} = 2.35$). Middle curve: corresponding shotnoise reference obtained by the difference between the two photodetectors outputs of the balanced homodyne detection setup. Lower curve: electronic noise. The decrease of all noise levels with increasing frequency is due to photodetector amplifier finite bandpass.

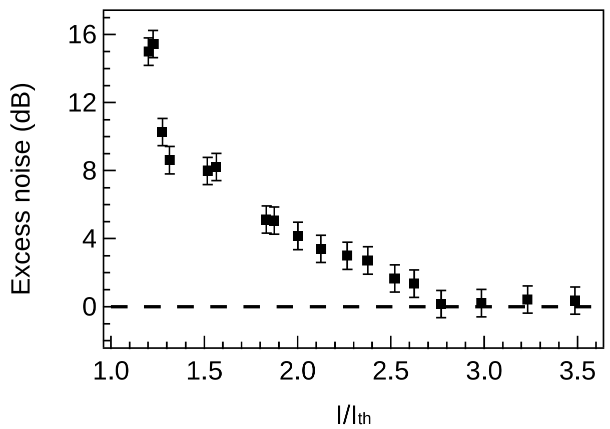

Fig. 3. Excess of noise $N_{\rm excess}$ measured at a frequency of 5 MHz, as a function of the laser driving current intensity $I/I_{\rm th}$ with $I_{\rm th} = 40$ mA. Above a value of $\simeq 2.8 \times I_{\rm th}$, the laser intensity becomes shotnoise limited.

board electronics after the record of a photocount timetag (around 250 ns deadtime per channel, see section 4.2).

Let us consider a WCP associated to a Poissonian photon-number distribution with a mean number of photon α. We assume that α takes into account the attenuation due to optical transmission and detector quantum efficiencies which are smaller from unity. This procedure is justified by the fact that under linear attenuation, a Poissonian photon number distribution remains Poissonian [16].

We consider a Hanbury Brown and Twiss detection setup with two silicon avalanche photodiode in single-photon counting regime (see figure 4). Because the detection deatime in the photodetection chain is longer than the pulse duration considered, we can only detect a single photon per detector for each pulse. One can then readily calculate the *true* photocounts probability distribution $P(n)$ for $n = 0, 1, 2$ measured with the Hanbury Brown and Twiss setup [23]

$$P(0) = \mathrm{e}^{-\alpha}, \tag{3}$$
$$P(1) = 2\mathrm{e}^{-\alpha/2}(1 - \mathrm{e}^{-\alpha/2}), \tag{4}$$
$$P(2) = (1 - \mathrm{e}^{-\alpha/2})^2, \tag{5}$$

and $\langle n \rangle = 2(1 - \mathrm{e}^{-\alpha/2})$ is the mean number of *detected* photons per pulse. This value is smaller than α, as a consequence of detection saturation.

The predicted value of the measured Mandel parameter for WCP is then:

$$Q_{\rm WCP} = \mathrm{e}^{-\alpha/2} - 1 = -\frac{\langle n \rangle}{2}. \tag{6}$$

Note that the bias introduced in the photon number statistics by detection deadtime leads to a negative value for the Mandel parameter associated to WCP having a Poissonian photon number distribution. As an example, for $\langle n \rangle = 0.1$ (corresponding to $\alpha = 0.1026$), one predicts $P(0) = 0.9025$, $P(1) = 0.0950$, $P(2) = 0.0025$, and $Q_{\rm WCP} = -0.05$.

For comparison, let us consider a perfect triggered single-photon source (SPS). The Mandel parameter of such a noiseless source would be $Q_{\rm SPS} = -1$ in case of *perfect detection*. However, due to non-unit quantum efficiency of transmission and detection, the measured Mandel parameter is related to $Q_{\rm SPS}$, by $Q = \eta \times Q_{\rm SPS}$ [26], where η is the overall quantum efficiency. For a perfect single-photon emission process, this parameter is equal to the mean number $\langle n \rangle$ of photon per pulse. Therefore, $Q = -\langle n \rangle$ for a perfect SPS. We have already experimentally checked the validity of this prediction [22], giving evidence for the quantum reduction of intensity noise expected for such a regular stream of single photons.

4 Photon number statistics for strongly attenuated laser pulses

4.1 Production of laser pulses

The experimental setup for producing faint laser pulses and measuring the associated photon number statistics is depicted in Figure 4. The laser diode is either directly driven by a home-made electrical pulsed current source ($\simeq$ 10 ns pulse duration) or by a commercial CW laser diode current controler (Profile Opt. Sys., LDC 8002). In the later case, the output collimated laser beam is focused into an acousto-optical modulator (AA, MT110) driven by a pulsed generator. Short pulses of light ($\simeq$ 8 ns pulse duration) are then obtained in the first-order diffracted beam which is selected by a slit. The chopped laser beam is guided to the detection setup, through a single-mode optical fiber. At the output, neutral density filters attenuate the light intensity to a mean number of *detected* photon per pulse of $\simeq 0.1$. The detection setup consists of two identical single-photon-counting avalanche photodiodes (APD) arranged in a the Hanbury Brown and Twiss configuration, on both sides of a 50/50 non-polarizing beamsplitter. A glass short-pass filter in front of each photodetector suppresses optical cross-talk between them [27].

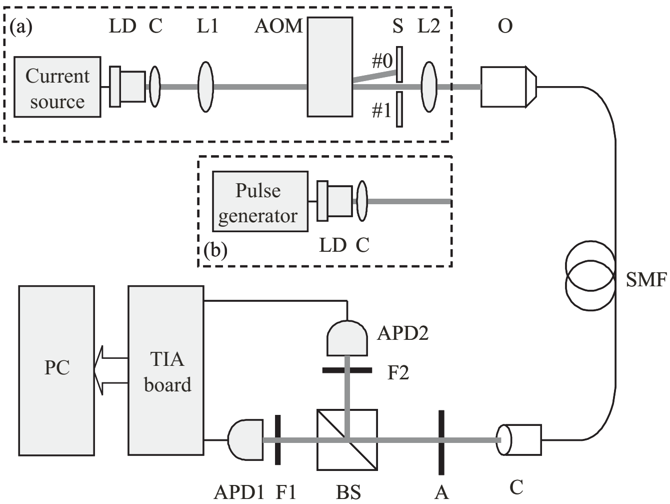

Fig. 4. Experimental setup to measure the photon statistics of strongly attenuated laser diode pulses. LD: laser diode; C: collimator; L: lens; AOM: acousto-optical modulator used for beam chopping; S: slit; O: microscope objective; SMF: single-mode optical fiber; A: attenuation with neutral density filters; BS: 50/50 non-polarizing beamsplitter; F: glass filter; APDs: avalanche photodiodes; TIA: time interval analyzer; PC: computer.

4.2 Intensity noise measurement in the pulsed regime

The timestamp of each photodetection event is recorded thanks to a Time Interval Analyzer (TIA, GuideTech model GT653) computer board. Note that its 75 ps time resolution exceeds by far the requirements of our experiment. The deadtime of each detection channel is 250 ns, limited by the TIA. The repetition rate is then set to a value of 2 MHz, corresponding to a period longer than the detection deadtime.

The home-made pulsed current driver and the AOM pulse driver are synchronized on internal clocks of frequency higher than the repetition period of excitation. The set of detected timestamps is synchronized on an excitation timebase after its acquisition, using a numerical procedure described in Ref. [23]. We then build the table $\{n_i\}$ of recorded number of photocounts $n_i = 0, 1, 2$ for each pulse i. Photons which are delayed by more than ten times the pulse duration are considered as dark counts of the two photodetectors, and are therefore rejected in the analysis procedure. Such post-gated detection allows us to remove almost all noise contribution from the two avalanche photodiodes used in the detection setup.

4.3 Intensity noise statistical analysis

Intensity noise measurements are summarized in Table 1, where the measured Mandel parameter Q is displayed for two different values of the laser diode driving current intensity. So that all measurements correspond to a mean number of detected photon per pulse of $\simeq 0.1$, strong attenuation of more 80 dB is introduced. Note that this attenuation exceeds by far the $\simeq 16\,\text{dB}$ excess of intensity noise previously measured when the laser diode is operated in the CW mode at I_{th}.

Corresponding values of the Mandel parameter are inferred from the $\{n_i\}$ table and Eq.2. Each set of photocounts consists of about 32000 timestamps, the number of which being limited by the TIA built-in memory depth. For example in a typical experiment, 31312 photons were recorded, resulting from 284397 excitation pulses. Among them, 29896 detection events were single-photon “clicks” and 708 were two-photons “clicks” corresponding to coincidences between the two paths of detection.

Table 1. Measurements of the photon statistics of strongly attenuated laser pulses (Mod.) generated either by beam chopping with an AOM (AO) or by pulsing the laser diode driving current (PE). Two different intensities of the laser diode driving current are considered. Attenuation factor (Att.) is chosen to yield a mean number $\langle n\rangle$ of detected photons per pulse of $\simeq 0.1$ as confirmed by the measured value of $\langle n\rangle$. Q is the *measured* Mandel parameter inferred from the set of timestamps. RE is the relative difference $(Q - Q_{\text{WCP}})/Q_{\text{WCP}}$ between Q and the predicted value $Q_{\text{WCP}} = -\langle n\rangle/2$ corresponding to equivalent weak coherent pulses (see Eq.(6)).

Mod.	I (mA)	I/I_{th}	Att. (dB)	$\langle n\rangle$	Q	RE (%)
AO	40	1	80	0.09875	-0.04928	- 0.2
AO	140	3.5	100	0.10466	-0.05286	1.0
PE	40	1	80	0.09973	-0.04974	-0.2
PE	120	3	100	0.09912	-0.05112	-3.1

As shown in Table 1, the measured value of the Mandel parameter only slightly deviates from its predicted value for WCP. The remaining deviation can be attributed to statistical fluctuations due to the finite number of detected photons. We checked that a larger set of detected events gives smaller deviation between them. Moreover, the measured photon statistics is insensitive to the method used for pulse generation, either by chopping the laser beam with an AOM or by pulsing the laser diode driving current. In the later case, although little attention was paid to the realization of a low-noise current generator, the added electronic noise is sufficiently small and does not affect the Poissonian photon number distribution within our measurement precision.

5 Conclusion

Many commercial QKD systems [28] are based on information encoding on faint coherent pulses of light which can for instance be obtained by strongly attenuating the beam emitted by a pulsed laser diode. Although these semiconductor emitters usually lead to an excess of intensity noise of a few dB compared to the shotnoise level, we have confirmed experimentally the well known property that after strong attenuation, the photon number distribution follows a Poisson law.

Furthermore we have implemented, as a calibration protocol for attenuated pulsed laser-diode intensity noise measurement, a method first introduced to analyze the intensity noise of a non-classical pulsed light source. This analysis, directly done in the time domain, can be straighforwardly applied to characterize any photon number statistics in the prospect of application to a QKD system.

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