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THE MODIFIED PROPORTIONAL FAIR SCHEDULER

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ABSTRACT

Until recently, wireless networks had been struggling against fading effects. However, 3G wireless networks have learned to profit from radio channel variations to augment their capacity while serving data traffic. Opportunistic schedulers take advantage from the delay-tolerance of these applications to ensure that transmission occurs when radio channel conditions are most favourable. A well-known opportunistic scheduler that strikes a good balance between fairness and efficiency, in an idealistic environment, is the "Proportional Fair" (PF) scheduler. Nevertheless, the hypotheses according to which its good performances are obtained are not valid in real environments. In this paper, we propose a modified version of PF that allows for a fair allocation of resources in realistic environments and introduces flexibility in sharing these resources between active users.

I. INTRODUCTION

Data Traffic is increasingly popular in 3G mobile networks. New technologies like HDR [1] and its equivalent in 3GPP, HSDPA [2], offer higher data rates than previous architectures notably through opportunistic scheduling. Opportunistic schedulers reap the benefits of multi-user diversity over short time-scales and determine how resources are allocated over longer time-scales. The classical PF scheduler is largely deployed in the aforementioned systems because it conjugates fairness and efficiency. However, PF is a rigid and non-adaptable scheduler as it falls short from enabling the system to define which trade-off between efficiency and fairness is targeted. Moreover, it only provides **temporal** fairness and can thus be considered as "unfair" as the throughput perceived by users (**utilitarian** fairness) decreases with distance. What is more, this restricted "fairness" is not fulfilled in real environments impacted by heterogeneous fading.

To cope with these drawbacks, we suggest in this paper an alternative scheduler, termed Modified PF (MPF), which is a hierarchical scheduler that allows, first, to obtain in a real environment the behaviour of PF in an idealistic environment, and second, to fully control the trade-off between fairness and global efficiency. To introduce the required control, we define different classes of users such that, on the one hand, users belonging to a given class have comparable SNRs (served according to PF, they will obtain comparable feasible rates) and, on the other hand, fading is homogeneous inside each class; as a consequence, applying PF to each class induces a strict temporal fairness for users belonging to the same class (we stress on the fact that it is not possible to reach this target when applying PF to the whole cell due to heterogeneous fading). At its first hierarchical level, MPF gives alternately one time slot to each class. At its second level, users inside each class are served by means of PF (the PF scheduler takes independent decisions inside each class). By doing so, we obtain in a real environment the behaviour that PF provides only in an idealistic environment. More importantly, we will use the proposed decoupling of the cell into various classes to control the resource allocation through a simple power control scheme.

The rest of the paper is organized as follows. In Section II., we present the cell partitioning into different classes. In Section III., we obtain analytical results (corroborated by simulation) for the mean rate per slot for a fixed number of users for PF and MPF in an environment with heterogeneous fading; in particular, we show how MPF, contrary to PF, provides exact temporal fairness to active flows. In Section IV., we give the system flexibility in the allotment of resources through a simple power control scheme. In Section V., we suggest to serve the nearest users in a CDMA fashion and justify how this solution improves performances. We give a brief conclusion in Section VI..

II. THE CELL PARTITIONING

To obtain different classes of users with comparable SNRs, we divide the cell into different zones; each Zone z corresponding to the set of users whose distance to the BS ranges from a minimal value r_z to a maximal value r_{z+1}. We decided to segment the cell in only three zones ($z = 0, 1, 2$) since increasing the number of zones limits the number of users per zone and therefore reduces the gain resulting from multi-user diversity. Next, we introduce the adopted radio model and then delimit the cell into the mentioned zones.

A. *The Radio Resource*

In this section, we describe the model for the radio resource and compute the feasible rate of each user accordingly.

1) *The Propagation Model*

The power received by a given user depends on the radio channel state and varies with time due to user mobility and fading effects. In our model, the mobility and the slow fading will not be included.

Let P be the transmission power emitted by the BS, γ_k the free space path loss and x_k the fast fading (of unit mean) for a user k. The power received by user k situated at a distance r from the BS, at time t, is then given by:

$$P_k(r,t) = P \cdot \gamma_k(r) \cdot x_k(t) \quad (1)$$

The adopted model for the free space path loss is:

$\gamma_k = 1$ if $r \leq \epsilon$ and $\gamma_k = (\frac{\epsilon}{r})^\beta$ otherwise

where β is the path loss exponent (taking values between 2 and 5) and ϵ is the maximum distance at which the full power P is received.

2) The feasible rate

For user k, the signal-to-noise ratio and energy-per-bit to noise density ratio [4] are respectively equal to:

$$SNR_k = \frac{P_k}{(\eta + I_k)}, \quad \frac{E_b}{N_0} = \frac{W}{R_k} \cdot SNR_k \quad (2)$$

where R_k is the feasible rate of user k, W is the cell bandwidth, η is the background noise and I_k is the interference generated by other BSs.

For a given target error probability, $\frac{E_b}{N_0}$ must be greater than a given threshold σ_k. Assuming the equality, the feasible data rate of a user k is then $R_k = \frac{W}{\sigma_k} \cdot SNR_k$.
In the vast majority of references, σ_k is taken as a constant in order to preserve the linearity between R_k and SNR_k. However, this assumption is not valid when different types of modulation are used which is the case for HDR and HSDPA systems. Thus σ_k will vary with the feasible rate and hence with the distance from the BS. Therefore, we consider in our model different values of σ_k per zone (σ_k is then replaced by σ_z in R_k). In practice, the way we define the zones induces that inside Zone 0 and Zone 2, σ_k is indeed constant and equals 6.5dB and 2.5dB respectively [1]. For Zone 1, we define σ_1 as the mean value of σ_k in this zone. The feasible rate of user k in Zone z is then:

$$R_{k,\mathbf{z}} = \frac{W}{\sigma_z} \cdot SNR_k$$

We denote by C_0 the maximum peak rate offered by the used coder and by $r*$ the maximum distance at which this peak rate is achieved, i.e. $r \leq r* \Leftrightarrow R = C_0$.

We suppose that the interference is constant per zone (it increases with the zone index). Hence, I_k will be replaced by I_z. Using (1), (2) and knowing that C_0 is the maximum peak rate that can be attained, we have the following:

$$R_{k,\mathbf{z}}(r,t) = min\left[C_0, \frac{W}{\sigma_z} \cdot \frac{P \cdot \gamma_k(r) \cdot x_k(t)}{(\eta + I_z)}\right] \quad (3)$$

Assuming that C_0 can be achieved, i.e. $r* \geq \epsilon$, we get from the path loss model and (2) the following:

$$C_0 = \frac{W \cdot P}{\sigma_0 \cdot (\eta + I_0)} \cdot (\frac{\epsilon}{r*})^\beta$$

Using (3), the average feasible rate of a user k in Zone z, denoted by $C_{k,\mathbf{z}}$, is given by:

$$C_{k,\mathbf{z}}(r) = C_0 \cdot \mathbb{E}[min[(\frac{r*}{r})^\beta \cdot K_z \cdot x_k(t), 1]] \quad (4)$$

with $K_z = \frac{\sigma_0}{\sigma_z} \cdot (\frac{\eta + I_0}{\eta + I_z})$.

B. Delimiting the cell into zones

As mentioned earlier, PF falls short from realizing exact fairness in a realistic environment where users experience heterogeneous fading. To be more precise, in order for all users, served according to the PF algorithm, to get access to the channel the same asymptotical fraction of time and get the same average power independently of their distance to the BS, two main assumptions must be satisfied:

- The fading must be homogeneous and independent among users.
- The instantaneous rate must scale linearly with the instantaneous SNR.

Unfortunately, in practice, these assumptions are not valid: *first*, users do not experience the same type of fading which is a very complex phenomenon that varies widely across users. As a result, while the random variables representing the fading effects are independent among users, they are not identically distributed. The principal impact of this lack of homogeneity is an unfair distribution of slots amid active users: users with the most variable distributions (typically those who are the furthest away from the BS) receive the least amount of slots [3]. *Second*, the linearity between the feasible rate and SNR is too optimistic except for users with low SNR (again, typically for users far from the BS). Therefore, our target is to define the previously introduced zones in such a way that the two assumptions become more realistic inside each zone. For that reason, we will follow the approach taken in [5] as it serves well our purposes. We take a path loss exponent $\beta = 4$ (urban environment). The x_k are exponentially distributed (with unit mean) as we consider Rayleigh fading.

Zone 0: Users in this zone are located between $r_0 = 0$ and r_1 and get the maximum peak rate C_0 with probability ≥ 0.95, i.e. from (3) and (4):

$$\mathbb{P}\big(R_{k,0}(r,t) = C_0\big) \geq 0.95 \Rightarrow \mathbb{P}\big((\frac{r*}{r})^4 K_0 x_k \geq 1\big) \geq 0.95$$

$$\Rightarrow e^{-(\frac{r}{r*})^4} \geq 0.95 \Rightarrow r \leq r_1 = (-\ln(0.95))^{1/4} r*$$

The mean rate of user k in Zone 0 is then $C_{k,0}(r) \approx C_0$.

Zone 2: Users in this zone are located between r_2 and $r_3 = \Re$ ($\Re$ is the cell ray) and do not get the maximum peak rate C_0 with probability ≥ 0.95, i.e. from (3) and (4):

$$\mathbb{P}\big(R_{k,2}(r,t) \neq C_0\big) \geq 0.95 \Rightarrow \mathbb{P}\big((\frac{r*}{r})^4 K_2 x_k < 1\big) \geq 0.95$$

$$\Rightarrow e^{-(\frac{r}{r*})^4 \cdot \frac{1}{K_2}} \leq 0.05 \Rightarrow r \geq r_2 = (-\ln(0.05) K_2)^{1/4} r*$$

The mean rate of user k in Zone 2 is then $C_{k,2}(r) \approx C_0 \cdot (\frac{r*}{r})^4 \cdot K_2$ and the distribution of the feasible rates is approximately that of $C_{k,2}(r) \cdot x_k(t)$.

Zone 1: Users in this zone are consequently those who are situated at a distance ranging from r_1 to r_2 and who get the maximum peak rate C_0 with non-negligible probability. We assume that, for this intermediate zone, the distribution of the feasible rate is the same for all users:

$$x'_k(t) \cdot C_{k,1} = \int_{r_1}^{r_2} C_0 \cdot min[(\frac{r*}{r})^4 \cdot x_k(t) \cdot K_1, 1] \cdot \frac{2rdr}{r_2^2 - r_1^2} \quad (5)$$

with x'_k being the unit mean random variable representing the variations around the mean rate $C_{k,1}$ in Zone 1. This approximation is made in order to obtain homogeneous fading in this zone.

Despite the heterogeneity of fading all over the cell, by dividing it into the three previous zones, we can fairly assume the homogeneity of fading within each zone, required to obtain a "fair" PF (applied among users of the same zone). In summary, for user k in Zone z, we can write the feasible rate as $R_{k,\mathbf{z}} = C_{k,\mathbf{z}} \cdot X_{k,\mathbf{z}}$ by defining $X_{k,\mathbf{z}}$ as being the variations due to fading around the mean rate $C_{k,\mathbf{z}}$. More explicitly, we have for Zone 0 $X_{k,\mathbf{0}} = 1$, for Zone 1 $X_{k,\mathbf{1}} = x'_k$ and for Zone 2 $X_{k,\mathbf{2}} = x_k$.

III. Analytical study of PF Vs. MPF

The PF algorithm is thoroughly studied in [6] in the case of homogeneous fading. We start by presenting some results from the cited paper that we use in our analysis.
At time slot t, PF schedules the user with the highest feasible rate relative to its current average throughput, i.e.,

$$\text{user } k^* = argmax_k[\frac{R_k(t)}{T_k(t)}]$$

where $T_k(t)$ is the exponentially smoothed throughput:

$$T_k(t+1) = (1 - \frac{1}{\tau}) \cdot T_k(t) + \frac{1}{\tau} \cdot R_k(t) \cdot \mathbb{1}_{user(t)=k} \quad (6)$$

with $\mathbb{1}_{user(t)=k}$ being the indicator function which equals 1 if user k was chosen at time slot t and 0 otherwise. τ is a time constant that captures the time-scales of the PF scheduler.

Since the random variables representing the fading are i.i.d, we have that $T_k(t) = C_k \cdot U_k(t)$, where C_k is the mean rate of user k and U_k are identically distributed (but not independent) random variables. If $\frac{1}{\tau} \to 0$, then:

$$T_k \to C_k \cdot \frac{g(n)}{n} \quad (7)$$

where n is the total number of active users and $g(n) = \mathbb{E}[max(X_1, .., X_n)]$ is the PF scheduling gain with homogeneous fading, defined as the ratio of what the user receives as compared to a simple RR (Round Robin) scheduling. In practice, τ has large values as this offers the opportunity of waiting a long time before scheduling a user when its channel quality is maximal: the scheduler is then expected to better exploit multi-user diversity. Hence, we adopt formula (7) in our analysis.

Next, in subsection A., we analyse the MPF scheduler. Since there is not an exhaustive study of PF with heterogeneous fading in the literature, we propose, in subsection B., an approximate analysis for PF. In subsection C., we confirm the validity of our results through simulation.

A. The MPF Scheduler

In our model, an independent PF scheduler is applied among users belonging to the same zone and thus experiencing homogeneous fading. Therefore, we can adopt the result in (7) for each zone. Namely, in Zone z, the exponentially smoothed throughput of user k is given by:

$$T_{k,\mathbf{z}} \to C_{k,\mathbf{z}} \cdot \frac{g_z(n_z)}{n_z} \quad (8)$$

where n_z is the total number of active users in Zone z and $g_z(n_z) = \mathbb{E}[max(X_{1,\mathbf{z}}, .., X_{n_z,\mathbf{z}})]$.

In MPF, the average rate of a user k in Zone z is then: [2]

$$\chi_{k,\mathbf{z},MPF} = \frac{C_{k,\mathbf{z}}}{n_z} \cdot g_z(n_z) \cdot \mathbb{P}(\alpha_z) \quad (9)$$

with event α_z={Zone z is served}.

B. The PF Scheduler

We analyse a model where PF selects a user among all users present in the cell while adopting for the exponentially smoothed throughput the value taken by formula (8). We make this approximation in order to have a tractable model by supposing that formula (8) remains valid for a user in a given Zone z, when all users are served according to plain PF. We prove the validity of this assumption through simulation in subsection C..

In PF, the average rate of a user k in Zone z is then:[2]

$$\chi_{k,\mathbf{z},PF} = \frac{C_{k,\mathbf{z}}}{n_z} \cdot \mathbb{E}[Z_z \cdot \mathbb{1}\{\frac{Z_z \cdot n_z}{g_z(n_z)} > \frac{Z_j \cdot n_j}{g_j(n_j)}, \forall j \neq z\}] \quad (10)$$

with $Z_j = max\{X_{1,\mathbf{j}}, ..., X_{n_j,\mathbf{j}}\}$.

Access Probability: To evaluate the impact of heterogeneous fading on the access probability when PF is applied among all users in the cell, we define the following probabilities: $a = \mathbb{P}(Z_2 \cdot \frac{C_2}{T_2} > Z_1 \cdot \frac{C_1}{T_1})$, $b = \mathbb{P}(Z_2 \cdot \frac{C_2}{T_2} > \frac{C_0}{T_0})$ and $c = \mathbb{P}(Z_1 \cdot \frac{C_1}{T_1} > \frac{C_0}{T_0})$.

Thus, the probability to serve a user in Zone 2 is $P_2 = \frac{a \cdot b}{n_2}$, to serve a user in Zone 1 is $P_1 = \frac{(1-a) \cdot c}{n_1}$ and to serve a user in Zone 0 is $P_0 = \frac{(1-b) \cdot (1-c)}{n_0}$. If we had homogeneous Rayleigh fading all over the cell, the probability of a user k to be selected would be the same for all users in the cell and is given by $P = \mathbb{P}(X_k = max(X_1, .., X_n)) = \frac{1}{n}$.

[2]The detailed result is found in [8]

Zone	0		1		2	
	PF	MPF	PF	MPF	PF	MPF
NUM	0.034	0.034	0.033	0.0297	0.013	0.010
SIM	0.033	0.033	0.033	0.0315	0.012	0.011

Table 1: Average Rate per user

C. *Numerical results*

We present in this section our numerical experiments performed to illustrate the previous results. The number of users in each zone is fixed and equal to 10 ($n_0 = n_1 = n_2 = 10$ and thus $n = 30$). We take $C_0 = 1$, $r* = 1$, $\Re = 2 \cdot r*$ (larger values of $\Re$ induce very small rates at the border of the cell [5]), $K_2 = 1.7$ and $K_1 = 1$ (realistic parameters). As a result, we get $r_1 \approx 0.5$ and $r_2 \approx 1.5$. Users are served according to PF and according to MPF. As MPF serves Zones alternately, we have $\mathbb{P}(\alpha_0) = \mathbb{P}(\alpha_1) = \mathbb{P}(\alpha_2) = 1/3$.

In all experiments, we determine the average rate per user and display the results for users belonging to the same zone for PF and MPF. We compare the results obtained by simulation to those obtained numerically from $\chi_{z,PF}$ and $\chi_{z,MPF}$. Results are shown in Table 1 and indicate that the analytical formulae provide highly accurate estimates of simulation results and thus our proposed approximation for PF is valid.
As for performance, we see, in MPF, that there is a conservation in the mean rate in Zone 0 and Zone 1, contrary to Zone 2 where a slight degradation is witnessed. This loss is compensated for by the increase in the probability to access the channel for users in Zone 2. Indeed, P_0 and P_1 are roughly equal to 1/30 (as in the homogeneous case from P) while P_2 is approximately equal to 0.017 (whereas $P = 1/30$). This means that with heterogeneous fading, the PF algorithm favours close users who are scheduled about twice as often as far users. Yet, with MPF, the probability to choose a user is $\mathbb{P}(\alpha_z) \cdot 1/n_z = 1/3 \cdot 1/10$ (recall that users in the same Zone z have the same probability $\frac{1}{n_z}$ to be selected) exactly as in the ideal homogeneous system. Far users will then have the same chance to be scheduled as close users and hence the lost equity in the distribution of slots among users will be restored.

IV. Power Control

The proposed segmentation of the cell in three zones offers us the possibility to choose between utilitarian fairness and efficiency through a simple power control scheme. We know that, in HDR/HSDPA systems, the BS transmits to only one user at a time at full power in order to get rid of intra-cell interference. However, we can tune the transmitted power depending on the served zone in order to reduce inter-cell interference for some users. For this purpose, we suggest in this section to serve users in Zone z at power $\varphi_z \cdot P$ with $0 < \varphi_z \leq 1$. We show how this scheme enable us, by alleviating inter-cell interference, to choose between favouring far users who suffer from inherently small rates and favouring close users to augment overall throughput.

We consider hexagonal networks where the interference suffered by users in a cell is almost utterly generated by the 6 neighbouring BSs. This assumption is valid when $\beta \geq 4$. Due to the decoupling of the system we performed, a BS serves alternately Zones 0, 1 and 2. Hence, we can easily achieve the following: for each BS serving Zone z (mod 3), 3 of the BSs surrounding it serve Zone $(z+1)$ (mod 3) while the 3 others serve Zone $(z+2)$ (mod 3) according to the plan in Figure 1. Thus, users in Zone 2, for instance, always endure inter-cell interference resulting from serving Zones 0 and 1 at power $\varphi_0 \cdot P$ and $\varphi_1 \cdot P$ respectively.

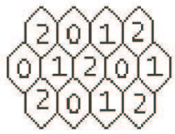

Figure 1:

The rate of user k in Zone z in this model is then:

$$R'_{k,\mathbf{z}} = \frac{W}{\sigma_z} \cdot \frac{\varphi_z \cdot P_k}{\eta + I(z+1) + I(z+2)}$$

where $I(z+1)$ is the inter-cell interference resulting from the surrounding BSs serving Zone $(z+1)$ (mod 3) (I' in the original model) and $I(z+2)$ is the inter-cell interference resulting from the surrounding BSs serving Zone $(z+2)$ (mod 3) (I'' in the original model).

To compare $R'_{k,\mathbf{z}} = \frac{W}{\sigma_z} \cdot \frac{\varphi_z \cdot \gamma_k \cdot P}{(\eta + \varphi_{z+1} I' + \varphi_{z+2} I'')}$ with the rate obtained without power control $R_{k,\mathbf{z}} = \frac{W}{\sigma_z} \cdot \frac{P \cdot \gamma_k}{(\eta + I' + I'')}$, we compute the following ratio:

$$\psi_z = \frac{R'_{k,\mathbf{z}}}{R_{k,\mathbf{z}}} = \frac{\varphi_z \cdot (\eta + I' + I'')}{\eta + \varphi_{z+1} I' + \varphi_{z+2} I''}$$

A. *Numerical Results*

We assume that $I' \approx I''$ and η is negligible in comparison with inter-cell interference.

If we wish to improve the performance of far users, we can set $\varphi_2 = 1$ (thus the BS will serve users in Zone 2 at full power) and $\varphi_0 < \varphi_1 < 1$ (so that Zone 1 receives more power than Zone 0). Consequently, we obtain $\psi_2 = \frac{2}{\varphi_0 + \varphi_1} > 1$ and the less power we give close users, the more we increase the rate perceived by far users. While $\psi_0 = \frac{2\varphi_0}{1+\varphi_1} < \psi_1 = \frac{2\varphi_1}{1+\varphi_0} \leq 1$. If φ_0 and φ_1 satisfy the subsequent additional relation $1 + \varphi_0 = 2 \cdot \varphi_1$, we can preserve the performances in Zone 1 ($\psi_1 = 1$) and only reduce the rate in Zone 0, the latter will not be that disadvantaged because often users in Zone 0 receive excess power that goes to waste due to their proximity to the BS. For instance, for $\varphi_0 = 1/2$ and $\varphi_1 = 3/4$, we get $\psi_2 = 1.6$, $\psi_1 = 1$ and $\psi_0 \approx 0.6$.

As for increasing the global throughput, we can set $\varphi_1 = 1$ (as already mentioned, serving users in Zone 0 at full power could lead to resource wastage) and $\varphi_2 < \varphi_0 < 1$. Consequently, we obtain $\psi_1 = \frac{2}{\varphi_0+\varphi_2} > 1$ and $\psi_2 = \frac{2\varphi_2}{1+\varphi_0} < \psi_0 = \frac{2\varphi_0}{1+\varphi_2} \leq 1$. Once again, if φ_0 and φ_2 verifies the subsequent additional relation $1 + \varphi_2 = 2 \cdot \varphi_0$, we can preserve the performance in Zone 0 with $\psi_0 = 1$.

V. MPF+CDMA

The authors in [7] showed that because the SNR does not scale linearly with the feasible rate for all users in the cell, scheduling one user at a time may not always result in maximum channel utilization. In particular, they showed that for almost orthogonal channels and for high SNRs, regular CDMA outperforms PF scheduling in terms of average. We deduce then that CDMA scheduling is more beneficial for users in Zone 0 as they enjoy very good channel conditions. Additionally, users in Zone 0 cannot take advantage from opportunistic scheduling because of their almost constant rates. Since we operated this segmentation of the cell, we have the freedom to treat each zone differently. Therefore, we will serve users in Zone 0 in a CDMA fashion while keeping on serving users in Zones 1 and 2 according to the PF algorithm.

A. *Average rate per slot*

We consider the case where we serve simultaneously n_0 users in Zone 0, each user receiving a power of $P_k = \frac{P}{n_0}$. The average rate R_k of user k can be assumed to follow a logarithmic relation (which is a better approximation than the linear assumption obtained in (2)):

$$R_k(\zeta_k(t), n_0) = W \log_2(1 + \frac{\zeta_k(t)}{\theta \cdot \zeta_k(t) \cdot (n_0 - 1) + n_0})$$

where $\zeta_k(t) = \frac{P \cdot (\frac{\epsilon}{r_0})^\beta \cdot g_k(t)}{\eta + I_0}$ is the SNR of user k in Zone 0, $g_k(t)$ the instantaneous channel gain (which is constant and equal to 1 in Zone 0) and θ the orthogonality factor that represents the fraction of power transmitted by the BS that appears as interference to the user.

It is sensible to assume that $\zeta_k(t)$ is a stationary and ergodic stochastic process for every user. Thus, the asymptotic data rate, where $\zeta = \frac{P \cdot (\frac{\epsilon}{r_0})^\beta}{\eta + I_0}$, is given by:

$$\begin{aligned}\overline{R_k} &= lim_{T\to\infty} \frac{1}{T} \int_0^T R_k(\zeta_k(t), n_0) dt \\ &= \int_0^\infty R_k(\zeta_k, n_0) \cdot \delta(\zeta_k - \frac{P \cdot (\frac{\epsilon}{r_0})^\beta}{\eta + I_0}) d\zeta_k = R_k(\zeta, n_0)\end{aligned} \tag{11}$$

The average rate obtained for user k while using the PF algorithm is then:

$$R_k(\zeta, n_0 = 1) \cdot \mathbb{P}(serving\, user\, k) = \frac{W}{n_0} \cdot \log_2(1 + \zeta) \tag{12}$$

To compare (11) and (12), we compute the ratio ρ:

$$\rho(\zeta, \theta, n_0) = \frac{n_0 \cdot \log_2(+\frac{\zeta}{\theta \cdot \zeta \cdot (n_0-1) + n_0})}{\log_2(1 + \zeta)} \tag{13}$$

If the base station uses orthogonal codes to transmit to distinct users, the intra-cell interference is virtually eliminated, which corresponds to $\theta = 0$. However, when there is multi-path fading, this form of interference is only partially reduced which implies $\theta \in [0, 1]$. Using orthogonal codes in our case requires synchronisation of all users which is easily done for the downlink channel. In addition, the channel for users in Zone 0 is hardly experiencing any fading at all and the risk of receiving delayed copies which are not orthogonal any more is significantly minimized. Thus, considering that $\theta \to 0$ is a quite reasonable hypothesis. Knowing that the mean SNR for users in Zone 0 is at least equal to 9.5dB [1], we deduce that $\rho(\zeta \geq$9.5dB, $\theta = 0$, $n_0 = 10) > 2.77$. Hence, we conclude that we profit considerably from the MPF+CDMA scheme.

VI. Conclusion

This work proposed a modified version of the well-known PF scheduler that overcame its flaws, such as its lack of flexibility in controlling resources and its biased temporal fairness. Furthermore, our MPF, through the segmentation of the cell to different categories, enabled the system to serve each category according to the most adapted scheme leading to an increase in overall performances.

References

[1] Bender P., Black P., Grob M., Padovani R., Viterbi A., CDMA/HDR: A Bandwidth-Efficient High-Speed Wireless Data Service for Nomadic users, IEEE Communications Magazine, pp. 70-77, 2000.

[2] Love R., Gosh A., Nikides R., Jalloul L., Cudac M., High-Speed Downlink Packet Access Performance, Proceedings of IEEE VTC Spring, 2001.

[3] Holtzman J., Asymptotic analysis of Proportional Fair Algorithm, Proceedings of IEEE, 2001.

[4] Viterbi A.J., CDMA: Principles of Spread Spectrum Communication, Addison-Wesley, 1995.

[5] Bonald T., Proutière A., Wireless Downlink Data Channels: User Performance and Cell Dimensioning, MobiCom'03, 2003.

[6] Kushner H.J., Convergence of Proportional-Fair Sharing Algorithms Under General Conditions, IEEE Transactions on Wireless Communications, Vol.3, No.4, 2004.

[7] Berggren F., Jantti R., Multiuser Scheduling over Rayleigh Fading Channels, Proceedings of IEEE Globecom 2003.

[8] Khawam K., Kofman D., Altman E., The Weighted Proportional Fair Scheduler, QShine 2006.