DYNAMIC SUBCARRIER REUSE WITH RATE GUARANTY IN A DOWNLINK MULTICELL OFDMA SYSTEM

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ABSTRACT

In this paper, the downlink of a multi-cell OFDMA system is considered. Rate Adaptive Optimization is investigated with minimum rate requirement and in presence of co-channel interference. A low-complexity subcarrier allocation scheme is proposed. A particular procedure provides limitation of co-channel interference by dynamically adapting the subcarrier reuse factor. A rate requirement violation (RRV) threshold is introduced to decide whether or not the interference limitation procedure is to be used. The performance is evaluated for variant user's rate requirement. The evaluation includes comparison of the cell global rate and the outage probability with existing heuristics.

Index Terms— Co-Channel Interference, OFDMA, Rate Adaptive Optimization, Subcarrier Assignment, Frequency Reuse Factor.

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is a promising multiple access technique for next wireless broadband networks, for that matter it has been adopted as a mandatory access scheme in IEEE 802.16e. This technique benefits from the efficiency of OFDM (Orthogonal Frequency Division Multiplexing) to mitigate the ISI (Inter Symbol Interference) due to multipath fading and also benefits from the OFDM ability to support high data rates. Moreover, OFDMA takes advantage of the frequency diversity of the channel by dividing the bandwidth into several sets of subcarriers which are dynamically allocated to distinct users. The system throughput is then improved compared to fixed allocation schemes ([1]).

Because OFDMA provides flexible radio resource allocation, OFDMA has been subject to active research last years. A significant part of the contributions has been considering the downlink of single cell scenarios. For instance in [2], a suboptimal algorithm is proposed to maximize the minimum rate of a user while satisfying a power constraint; in [3], algorithms aim at maximizing the global rate with minimum rate requirement for each user and a total power constraint. This kind of optimization ([2]-[3]) is known as Rate Adaptive optimization (RA). The “dual” problem is known as Margin Adaptive optimization (MA) where the transmit power is minimized with rate requirement constraint for users ([4]). Those work ([2]-[4]) focus on a feasible subcarrier assignment and power allocation which are only based on the channel state information (CSI) in the cell. In a multi-cell environment, one also has to deal with co-channel interference resulting of bandwidth reuse. In the multi-cell context, problems which do not belong to the classical MA or RA categories can be defined. An example is given in [5] where the authors aim at minimizing the ratio of users whose rate requirement is not fulfilled; sectorization is introduced as well as distinct frequency reuse factors in order to handle different load distribution among the cells. They first allocate the resources to the different sectors which then behave independently during the rest of the assignment. In [6], the more classical MA problem is studied. Three steps are proposed: a user is attached to his best cell if there are enough subcarriers to satisfy his rate requirement. In this case, subcarriers are reallocated to all users of the cell, including the new one, according to channel gains and interference levels. Finally, the classical bit loading algorithm is performed to minimize the transmit power. Contributions to RA optimization in multi-cell systems can be found in [7]-[10]. Among these contributions, [7]-[8] do not consider user rate requirement. In [7], a threshold η is introduced so that a subcarrier which does not exhibit, for at least one user of the cell, a SINR (Signal To Interference and Noise Ratio) higher than η is set to be inactive in the cell. The active subcarriers are then allocated to the user with the best gain and the waterfilling algorithm (with SINR instead of SNR) is applied. In [8], a method is proposed to build the set of the so-called co-channel users for each subcarrier; assuming that co-channel users do not belong to the same cell, a user is added to the set if he maximizes the rate increase on the subcarrier. This implies that the rate of the new user is large enough and that the rates of the previous users do not decrease too much because of induced interference. The authors of [9] consider the general problem of subcarrier and power allocation with user rate requirement in the uplink. In [10], a downlink centralized scheme is proposed: the list of subchannels[1] reuse factors is predetermined; the number of subchannels using each reuse factor is computed depending on base stations needs. The allocation ends with independent subchannels allocation in the cells.

In this paper, we propose a subcarrier assignment scheme where a procedure of interference limitation is provided in case of violation of rate requirements. The set of active subcarriers of a cell is not predetermined. Equal power repartition (as in [10]) is chosen on active subcarriers of each cell. The power for

[1] In [10], group of subcarriers randomly chosen in the frequency band.

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users with bad channel conditions is not increased to avoid increasing interference. Instead, subcarriers with low frequency reuse factor (FRF) are allocated to such users. Like in [8]-[9], a set of co-channel users (Π_n) is formed for each subcarrier *n*, taking into account user rate requirements ([9]). The remainder of the paper is organized as follows. Section II presents the system model and the problem formulation. The subcarrier allocation schemes are investigated in section III. In section IV, the cell rate and the outage probability of the proposed scheme is compared with proposals of [8]-[9] and [2] (adapted to multi-cell with FRF=1). Section V concludes the paper.

II. System Model and Problem Formulation

A. System Model

We consider the downlink transmissions of a multi-cell OFDMA system. The notations used in the paper are listed in table I. A system with a limited number *B* of cells is considered (a wrap around technique is used). A square cell shape is chosen. Each cell has one base station (BS) which is cell-centered. Unlike in [6], we assume that users are already attached to their serving BS. We consider *U* users per cell. In a given cell *b*, $1 \le b \le B$, a local index *u* is used, $1 \le u \le U$; the global index ω of a user is specified by the pair (u,b). Users are uniformly located in a cell. A user is either a "near user" or a "far user", according to a threshold parameter D_{max} which is the maximal distance between a "near user" and his serving BS.

It is assumed that a subcarrier *n* can be assigned to at most one user per cell. Each subcarrier carries at most R_{max} bits per QAM symbol according to available modulation and coding schemes (MCS). The path loss model is $K\,d^{-\alpha}$ where *d* is the distance between a given user and a given BS, α is the pathloss exponent ($2 \le \alpha \le 4$) and *K* is a constant for a given environment. The large scale shadowing effect is characterized by a log-normal distribution ($\mathcal{N}(0,\sigma^2_{sh})$ in dB units with 4 dB≤ σ_{sh} ≤12 dB). Small scale fading is modelled with a Rayleigh distribution. Channel gain of subcarrier *n* between user ω and base station *k* is denoted as $g(\omega,k,n)$, $1 \le k \le B$, $1 \le n \le N$. We consider additive white Gaussian noise (AWGN) which is characterized on each subcarrier by a Gaussian random variable $\mathcal{N}(0,\sigma^2)$ where $\sigma^2 = N_0\, W/N$. Each user $\omega = (u,b)$ receives the useful signal part from the serving BS *b* and the interference part from neighboring BS $k \neq b$ using the same subcarriers. We assume an equal power allocation *p* on subcarriers. The level of interference on subcarrier *n*, suffered by user ω with index *u* in cell *b*, is thus given by: $I_n(u,b) = p \sum_{k \neq b} g(\omega,k,n)$. Knowing the number of BSs which use the subcarrier *n*, and assuming perfect channel state information, the level of interference is known by the RRM (Radio Resource Management). The channel gain to interference and noise ratio (CgINR) on subcarrier *n* regarding user ω with index *u* in cell *b*, is given by $CgINR(u,b,n) = g(\omega,b,n) / (I_n(u,b)+\sigma^2)$. The signal to interference and noise ratio (SINR)is $\gamma(u,b,n) = p\; CgINR(u,b,n)$. Performance evaluations are snapshot based; the results from the "captured" system states are averaged and analyzed.

Table I: System parameters

W	Total bandwidth
N	Number of OFDM subcarriers
B	Number of cells
U	Number of users in one cell
D_{max}	Maximum distance between a near user and his serving BS
$g(\omega,k,n)$	Gain on subcarrier *n* between user $\omega=(u,b)$ and cell *k*
N_0	Noise power spectral density
$CgINR(u,b,n)$	Channel gain to interference and noise ratio on subcarrier *n* for user with index *u* in cell *b*
$P_{T,Max}$	Total power constraint in each cell
P^b_T	Total power transmitted in cell *b*
p	Power allocated on each subcarrier
$\gamma\,(u,b,n)$	Received signal to noise ratio on subcarrier *n* for user with index *u* in cell *b*
$\Omega_{u,b}$	Set of subcarrier allocated to user with index *u* in cell *b*
Π_n	Set of co-channel users of subcarrier *n*
$r°_{u,b}$	Data rate constraint for user *u* in cell *b*
R_{max}	Maximum number of bits on a subcarrier per QAM symbol

B. Problem Formulation

Let $\underline{r}°_b=(r°_{1,b}\;\; r°_{2,b}\ldots\;\; r°_{U,b})$ and P^b_T be the user rate requirements and the total transmitted power in cell *b*. A multi-cell version of the RA problem can be formulated as follows:

$$\text{Maximize } \sum_{b=1..B} \sum_{u=1..U} \sum_{n\in\Omega ub} \log_2(1+\gamma\,(u,b,n))$$
$$\text{subject to } P^b_T < P_{T,Max},\; 1\le b\le B \;\; \text{and} \;\; \underline{r}_b \ge \underline{r}_b°,\; 1\le b\le B.$$

The formulated problem is a constrained nonlinear programming problem. Under the equal power hypothesis, *p* is given by $P_{T,Max}/N$. The solution can be given by an $N\times B$ array: CoChannelUser(n,b) gives the local index *u* ($1\le u\le U$) of the user in cell *b*, who receives subcarrier *n*; CoChannelUser(n,b)=0 if cell *b* does not use subcarrier *n*. The solution space is finite so an optimal solution exists and an exhaustive search would find one. Since this method is intractable, a low complexity suboptimal algorithm is proposed in section III.A.

III. Subcarrier allocation schemes

A. Opportunist Subcarrier Allocation with interference limitation (OSA-IL)

Our algorithm (cf. Fig.1), proceeds one subcarrier at a time ([8]) without preliminary evaluation of the number of subcarrier per cell ([5]) or per user ([3]). The set of co-channel users contains at most one user per cell ($|\Pi_n| \le B$). A first phase, called opportunist subcarrier allocation, tries to fulfill the rate requirements with FRF=1 (for all subcarriers). Sharing the concern of [5] about satisfaction of rate requirements, we introduce a rate requirement violation ratio (RRV). If, at the end of the first phase, the RRV exceeds a threshold η, an interference limitation phase is launched to reduce the number of unsatisfied users; otherwise the allocation is terminated. The RRV is given by $(\sum_{b=1..B} V_b)/(B\times U)$ where V_b is the number of users of cell *b* who do not satisfy their rate requirement.

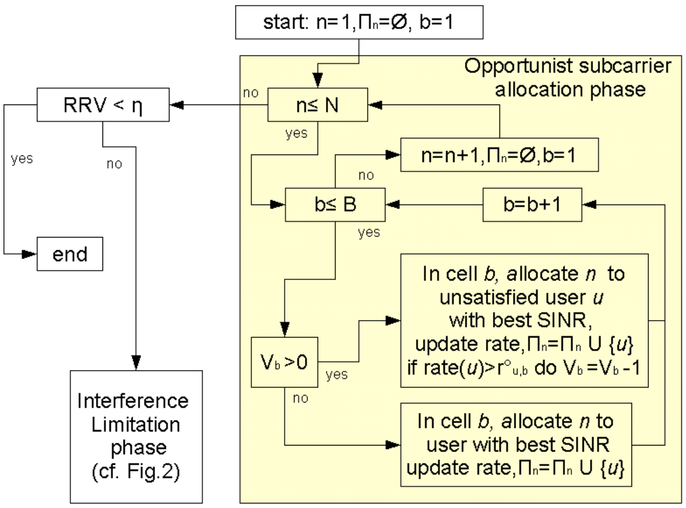

Figure 1: OSA-IL flow chart

1) Opportunist Subcarrier Allocation phase

The principles of the first phase (cf. colored box in Fig.1) here follow. Each subcarrier is used in every BS (FRF=1 for all subcarriers). A fixed subcarrier n is allocated to user u in cell b who has the highest SINR value $\gamma(u,b,n)$ and has not yet fulfilled his rate requirement. If each user of cell b already satisfies $r(u,b) > r^\circ_{u,b}$ (i.e. V_b=0), the subcarrier is allocated to the user with the highest SINR. The user rate is updated as well as the number of unsatisfied users of each cell. After the processing of the N subcarriers, the RRV is computed.

2) Allocation with interference limitation

When the RRV is higher than η, the first allocation is ignored. The interference limitation phase (Fig.2) still proceeds per subcarriers, the FRF (frequency reuse factor) is dynamically adapted to reduce the RRV. Apart from the number of cells using a given subcarrier, the allocation principle inside a cell does not change (compared to III.A.1); users with better channel conditions (easier to satisfy) still receive subcarriers before users with poorer channel conditions.

A subcarrier is allocated with the current FRF $frf(i)$ (frf is an array containing all the authorized FRF sorted in decreasing order; $frf(i) = x_i / B$ where x_i is the number of cells using the subcarrier). Updating the current FRF depends on the level of interference suffered by the previous allocated subcarriers. Whatever the subcarrier, the average SINR of co-channel users ($\bar{\gamma}_n = (\sum_{u\in \Pi n} \gamma\,(u,b,n)) \,/\, |\Pi_n|$) should be above a threshold δ to guarantee an acceptable level of interference on the subcarrier. A counter C keeps the number of subcarriers whose average SINR $\bar{\gamma}_n$ is below δ. When the counter reaches the limit C_{max}, the current FRF is decreased (if it is not already the lowest). Indeed, if the co-channel users average SINR is weak, it will be worst for the remaining unsatisfied users because they have poorer channel conditions. Decreasing the current FRF provides remaining unsatisfied users with better subcarriers (suffering from less interference than the current subcarrier). However, on a specific subcarrier n, even the lowest FRF may not be enough for some users ($\gamma(u,b,n) < \varepsilon$). In that case, this specific subcarrier can be assigned in each BS. If all users are satisfied, FRF=1 for all remaining unallocated subcarriers. In Fig.2, let Θ be the set of cells where $V_b = 0$ (Θ^C is the complementary set); the set of unsatisfied (respectively satisfied) BS which use the current subcarrier is denoted as Φ_1 (respectively Φ_2); BSs are selected regarding the highest CgNR among the unsatisfied (respectively satisfied) users to build Φ_1 (respectively Φ_2).

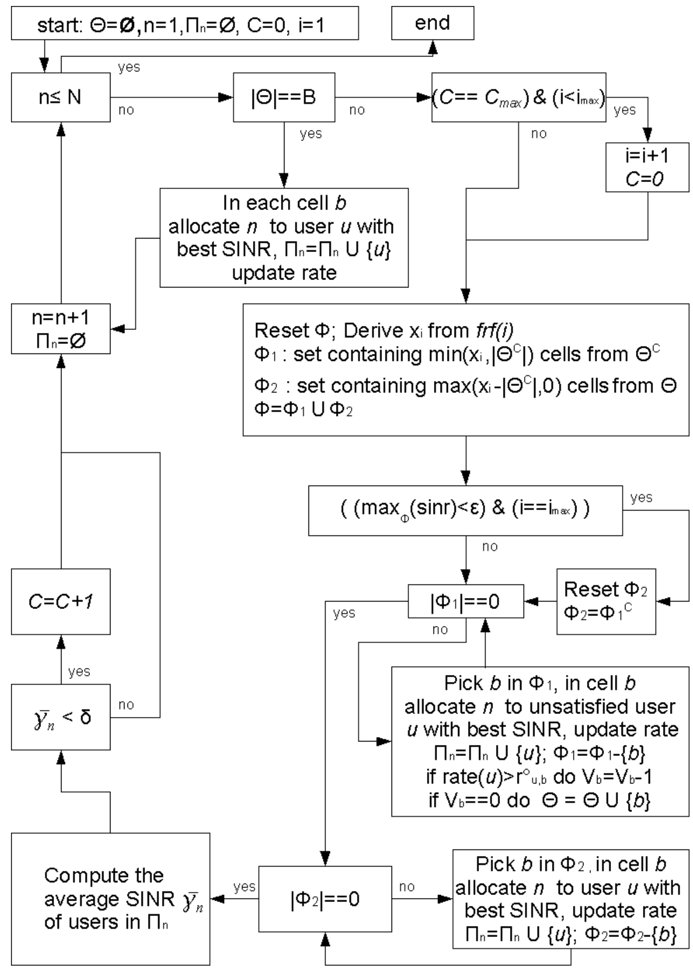

Figure 2: Interference Limitation Phase flow chart

OSA-IL is a low complexity algorithm; for subcarrier and BS the dominant operation is a search of the maximum over U terms. A first evaluation of the complexity gives $O(N\times B\times U)$.

B. Short allocation (or OSA)

A special case of the algorithm is obtained when η =1. In that case, the interference limitation phase is never launched. This algorithm is thus called Opportunist Subcarrier Allocation (OSA).

C. Existing heuristics

1) Heuristic A with constant power

In [8], the author maximizes the global rate without user rate requirement (when rate requirements are considered, the required number of subcarriers is minimized). We consider Heuristic A (chapter 2 in [8]) in our equal power context. Each subcarrier is allocated to one user of a new cell if the subcarrier rate is increased. The rate brought by the new user must exceed the rate decrease of existing co-channel users.

2) Heuristic B with constant power

In [9], an algorithm is proposed for the uplink where each user has a rate and a power constraint. The author proposes a two steps algorithm with initialization part followed by an iterative capacity refinement part. This last part will not be considered since it is stated in [9] that it may increase the outage. We are interested in the initialization algorithm B adapted to the downlink context with constant power on subcarriers. As in [8], the algorithm is based on the maximal packing principle: a new user is accepted on a subcarrier if there is a modulation reassignment for the existing users such that the global rate of the subcarrier is increased. The subcarrier n to be treated and its first co-channel user u realize the best SNR $\gamma(u,b,n)$ (among unallocated subcarriers and unsatisfied users). To add users in Π_n, the new subcarrier rate is computed for all the unsatisfied users. The new user maximizes the rate increase on the subcarrier; if the rate increase is negative, no user is inserted and a new subcarrier is chosen. When all users are satisfied, all users compete for the unallocated subcarriers. This algorithm shows good results however the CPU time may be high; a first evaluation of the complexity gives $O(N^2 \times B \times U)$.

3) "Fair" heuristic with maximum reuse factor

The problem investigated in [2] is the maximization of minimum user capacity (max ($\min_u r_u$)). "Equal" rate is guaranteed to users so the algorithm is denoted hereafter as *Fair Heuristic*. For comparison matters, we apply the algorithm proposed in [2] to each cell separately. In [2], constant power is also assumed. In each cell, the algorithm is applied with all the subcarriers: the frequency reuse factor is thus one. In each cell, the user with the minimum current allocated rate is allowed to choose his best subcarrier. The process goes on until all subcarriers are assigned in the cell.

IV. Simulation Results

Simulation parameters are listed in table II. We are interested in comparing the average cell rate and the outage probability between OSA-IL (opportunist subcarrier allocation with interference limitation,), OSA, Heuristic A and B and finally the Fair Heuristic. All users have a common rate requirement $r°$ varying from 16 kbps to 2 Mbps. The results are averaged over 1000 rounds (i.e. 1000 snapshots of the channel states).

A. Average cell throughput

We evaluate, for each cell b, the cell throughput (aggregated rate over the U users). Fig.3 shows cell throughput averaged over the B cells. Heuristic A and Fair Heuristic show a constant rate since there is no user rate requirement. In Fair Heuristic, users with poor channel conditions (far users) receive enough bandwidth to reach the same rate than near users (which explains that Fair Heuristic has the lowest cell throughput) whereas far users do not receive bandwidth in Heuristic A (which explains that Heuristic A exhibits the highest cell throughput). In presence of rate requirement, the global rate decreases until a value $r_l°$ ($r_l°$=600 kbps for OSA, $r_l°$=800 kbps for OSA-IL and $r_l°$=1 Mbps for Heuristic B), then the global rate increases as r° increases beyond $r_l°$.

TABLE II: SIMULATION PARAMETERS

U (number of users per cell)		10
B (number of cells)		9
Size of a cell (km)		2
D_{max} (maximum distance of a "near user",km)		1
η (threshold of RRV in OSA-IL)		0.001
δ (threshold of average SINR of co-channel users, dB)		3
ε (thresh. of the maximum SINR of co-channel users, dB)		-3
C_{max} (limit number of subcarrier with $\bar{\gamma}_n < \delta$)		5
Central frequency (GHz)		3.3
W (total bandwidth, MHz)		5
N (number of subcarriers)		512
frf		[1 8/9 7/9 6/9 5/9 4/9 3/9]
$r°$ (user rate requirement, kbps)		16…2000
R_{max} (bits)		4.5
Resulting path loss exponent: α		4
Shadowing	Mean	0
	Standard deviaton (dB)	6
Small scale fading	Mean	1
	Standard deviaton (dB)	$\sqrt{(2/\pi)}$
Power per subcarrier (dBm)		15
N_0 (dBm/Hz)		-174

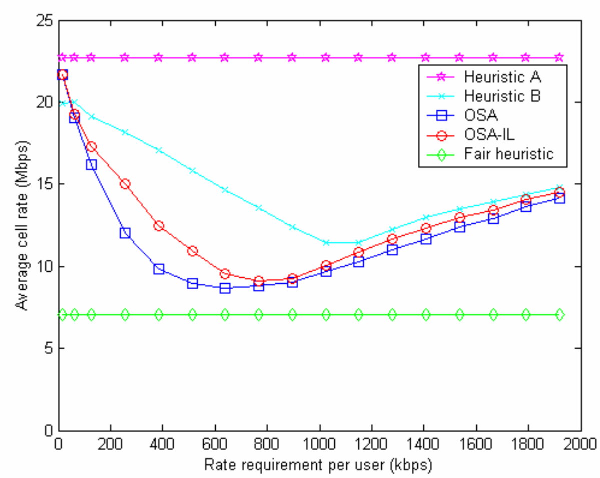

Figure 3: Cell throughput versus $r°$

When $r° < r_l°$, bandwidth grant to far users reduces the global rate. Above $r_l°$, $r°$ is almost unachievable: the set of unsatisfied users is so large that subcarriers are exclusively allocated to the best users, which increases the global rate. Heuristic B outperforms both OSA and OSA-IL regarding the global rate. The interference limitation phase (OSA-IL) improves, to some extent, the global rate compared to OSA. Enhancement in the subcarrier processing order can improve the OSA-IL global rate. However, there is clearly a tradeoff between achievable rate and complexity.

B. Outage Probability

The outage probability, P_{out}, is the probability that a user transmits less than the rate requirement $r°$. This parameter is critical to appreciate the algorithms regarding the problem formulated in II.B. The "outage probability" of fair heuristic and Heuristic A are just plotted for comparison purpose. Fig. 4 shows the outage probability estimated on all the $B \times U$ users. The interference limitation phase has been introduced to reduce the RRV of the OSA algorithm. The goal is achieved especially for far users (Fig. 5) whose distance from the base station is above D_{max} (OSA-IL performance are much closer to that of Heuristic B than OSA).

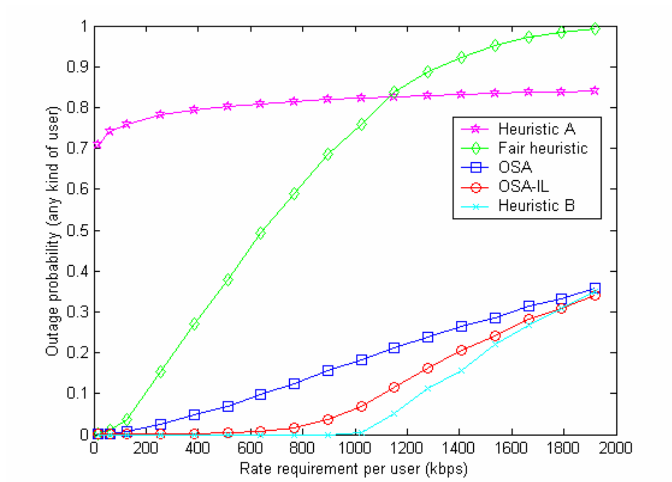

Figure 4: Probability that a user transmits less than $r°$ versus $r°$

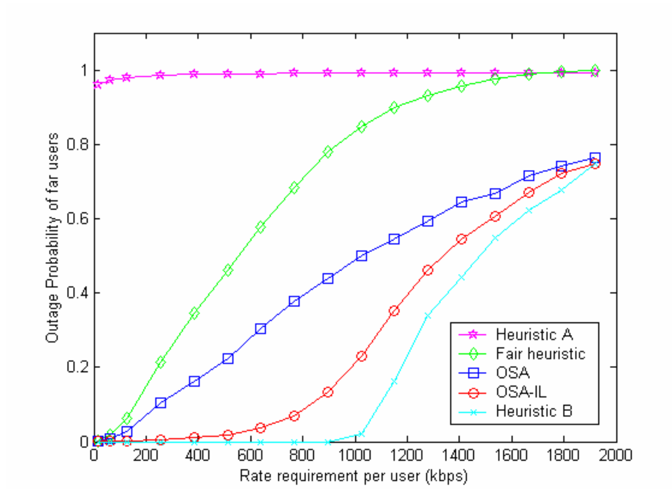

Figure 5: Probability that a far user transmits less than $r°$ versus $r°$

C. *Execution time*

Execution time of Heuristic B significantly increases with N: from N=128 to N=2048 (respectively N=8192), Heuristic B execution time is multiplied by 50 (respectively 800) instead of 7 (respectively 12) for OSA-IL. The relative complexity of Heuristic B is defined as the ratio between execution time of Heuristic B and that of OSA-IL. The upper part of Fig.6 shows the Heuristic B relative complexity when U varies (N fixed) while the lower part plots Heuristic B relative complexity when N varies (U fixed). It is shown that OSA-IL outperforms Heuristic B regarding execution time.

V. CONCLUSION

We have proposed a subcarrier allocation scheme OSA-IL (Opportunist Subcarrier Allocation with Interference Limitation) with a special case OSA which differs in the way to handle the co-channel interference. OSA-IL outperforms OSA regarding the global rate and the outage probability. If the maximal outage probability of a far user is fixed to 5%, the minimum rate ensured by OSA-IL is 640 kbps instead of 128 kbps with OSA and 64kbps with the Fair Heuristic ([2]). The proposed OSA-IL is effective in maximizing the global rate while maintaining a low outage probability even to far users. An algorithm, Heuristic B ([9]) shows better results than OSA-IL ($r°$ =1Mbps for far users with an outage of 5%) at the cost of higher complexity. For simplicity matter, results have been plotted with a constant number of users per cell and a common rate requirement. However, the algorithm is also adapted to different load distribution among the cells.

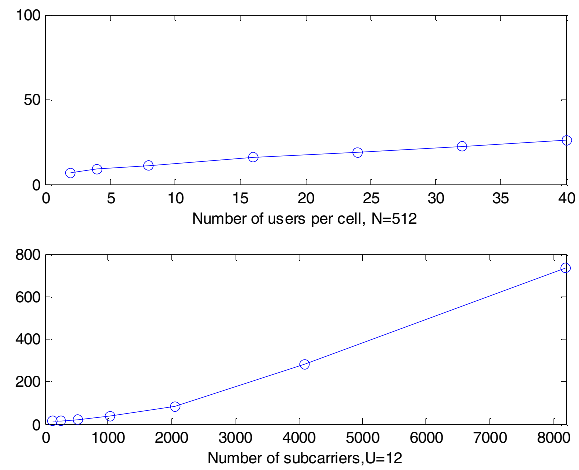

Figure 6: Relative complexity of Heuristic B versus the number of users per cell and next versus the number of subcarriers

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