An Efficient Subcarrier Assignment Algorithm for Downlink OFDMA

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Abstract— **In this paper, the Rate Adaptive optimization (RA) problem, which maximizes the sum of user data rates subject to total power constraint and individual guaranteed rates, is considered. Two tasks are commonly examined: bandwidth allocation and specific subcarrier assignment. A mechanism to provide a degree of fairness among users is coupled with the first task. Considering the second task, a novel algorithm, Rate Profit Optimization algorithm (RPO), is defined to assign specific subcarriers to different devices of a multiuser downlink OFDM system. In RPO, a new approach is proposed to assign a conflicting subcarrier (best subcarrier for several users). This algorithm is shown to exhibit good results regarding spectral efficiency and fairness with a complexity significantly lower than the Hungarian algorithm.**

Keywords— **OFDMA, Rate Adaptive optimization, subcarrier assignment, fairness.**

I. INTRODUCTION

OFDMA is a promising technique for broadband wireless networks and has been chosen for the IEEE 802.16 standard. OFDMA stands for Orthogonal Frequency Division Multiple Access and relies on the OFDM (Orthogonal Frequency Division Multiplexing) modulation technique. The bandwidth is divided into subsets which are assigned to distinct users during one OFDM symbol duration. Subcarrier assignment is based on channel state information (CSI). In OFDM-TDMA, a single user gets all the subcarriers during a symbol period and does not use bad subcarriers. Since a frequency in deep fade for a user may be a good frequency for another, OFDMA allows an efficient management of radio resources with a reduced number of wasted subcarriers. OFDMA also benefits from the immunity of OFDM against ISI (Inter Symbol Interference) caused by multiple paths ([1]).

Allocation resources strategies in OFDMA have been subject to active research last years. Two different problems have been investigated: Margin Adaptive optimization and Rate Adaptive optimization. The Margin Adaptive optimization (MA) tries to minimize the overall transmit power while maintaining minimum rate $r°_u$ for each user u. The MA problem, studied in [2]-[8], may be split in two parts ([2]): (i) subcarrier assignment assuming fixed modulation, (ii) bit loading over the assigned subcarriers to minimize the transmit power. The Rate Adaptive optimisation (RA) focuses on the maximisation of user rates subject to a power constraint. Such a problem has been investigated in [9]-[14]. Three subclasses of RA problems can be found: some papers maximize the minimum of user capacity ([10]); some papers maximize the global rate without user rate constraints (([13]), algorithm referred to as opportunist) and the rest of the papers maximize the global rate of the cell with user rate constraints (e.g. in [14] proportionality constraints are laid on subscribers' rates).

This paper deals with the RA problem and belongs to the third above-cited subclass. As in [5], [9] and [11]-[12], subcarrier assignment is divided into two tasks: (i) determine a number of subcarriers to be allocated to each user, (ii) assign each subcarrier to one specific user. In the sequel, these tasks are respectively referred to as task 1 and task 2. Regarding power allocation, equal power repartition on subcarriers is used ([10], [12]-[13]). In [13], equal power is shown to bring little rate degradation compared to the optimal waterfilling. This work has two objectives: (i) introduce a degree of fairness, during task 1, by adjusting a common guaranteed rate $r°_{floor}$ for users, (ii) define a heuristic algorithm for task 2 that exhibits attractive rate performance to compete with existing heuristics ([4]-[7]) and the Hungarian algorithm. The remainder of the paper is organized as follows. Section II presents the system model and the problem formulation. The subcarrier allocation scheme is investigated in section III: task 1 is revisited to provide fairness while an algorithm is proposed for task 2. In section IV, simulation results are presented: rate per subcarrier is evaluated; a fairness criterion is defined and plotted. The paper is concluded in section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system is formed by U users uniformly located in one cell with a single Base Station (BS). Resource assignment is studied in the downlink. The channel model consists of N independent parallel narrowband subcarriers over the bandwidth B. The channel gain $g(u,n)$, of user u on subcarrier n, is given by (similar to [15]):

$$g(u,n)=K\, d(u)^{-\alpha}\, a_{sh}(u)\, a_f(u,n)$$

where $d(u)$ is the distance between user u and the BS, α is the pathloss exponent ($2\leq \alpha \leq 4$), K is a constant for a given environment, a_{sh} which represents the shadowing effect is a lognormal variable (i.e. $10\log(a_{sh})$ is $\mathcal{N}(0,\sigma^2_{sh})$ with 4 dB$\leq \sigma_{sh} \leq 12$ dB), and a_f is the small scale fading with Rayleigh distribution. All the subcarriers have the same shadowing effect for one user at one instant. In addition to flat fading, the subcarriers suffer from additive white Gaussian noise (AWGN), a random normal variable $\mathcal{N}(0,\sigma^2)$ with $\sigma^2=N_0\, B/N$. The corresponding channel gain to noise ratio (CgNR) is given by $CgNR(u,n)=g(u,n)/\sigma^2$. The received signal to noise ratio (SNR) is $\gamma_{u,n} = p_{u,n}\, CgNR(u,n)$ for subcarrier n assigned to user u. It is assumed that a subcarrier can not be shared simultaneously by different users and does not carry more than R_{max} bits per QAM symbol. Perfect CSI is assumed at the BS. The performances are evaluated over "captured" instantaneous channel states: results are averaged over M snapshots and analysed. The notations used in the sequel of the paper are summarized in table I.

TABLE I. NOTATIONS USED

N	Number of OFDM subcarriers
U	Number of users
B	Total bandwidth
$g(u,n)$	Channel gain of user u on subcarrier n
N_0	Noise power spectral density
$CgNR(u,n)$	Channel gain to noise ratio of user u on subcarrier n
$P_{T,Max}$	Total power constraint
$p_{u,n} = p$	Power allocated to user u on subcarrier n
$\gamma_{u,n}$	Received signal to noise ratio of user u on subcarrier n
Ω_u	Set of subcarrier allocated to user u
$r°_u$	Data rate constraint for user u
R_{max}	Maximum number of bits on a subcarrier per QAM symbol

The target data rate vector, $\underline{r}°=(r°_1, r°_2 \ldots r°_U)$, defines each user rate constraint. The RA problem can be formulated as follows:

$$Maximize \sum_{u=1}^{U} r_u \qquad subject\ to\ P_T \leq P_{T,max} \quad and \qquad \underline{r} \geq \underline{r}°$$

where r_u is computed over the set of subcarriers Ω_u, allocated to user u: $r_u = \sum_{n\in\Omega_u} r_{u,n}$. Let $h(.)$ be the "power-rate" function: $r_{u,n} = h(\gamma_{u,n})$. With a Shannon theoretic approach, on can set $h(\gamma_{u,n}) = \log_2(1+ \gamma_{u,n})$. Other functions $h(.)$ may be used to fit with practical modulation and coding schemes (MCS). To specify $h(.)$, we adopt the function $f(.)$ of [5] and we set $h(\gamma)=\min(f^{-1}(\gamma),R_{max})$ where R_{max} is the maximum number of bits per subcarrier in the more optimistic implemented MCS.

The problem formulated hereinbefore, may not have a solution for the given $\underline{r}°$, in this case another $\underline{r}°$ may be set. Next section will provide a proposal to adjust $\underline{r}°$.

III. SUBCARRIER ASSIGNMENT

Hereafter, subsection A describes our proposal to perform task 1 (i.e. determine the number of subcarriers for each user) and a mechanism to provide some degree of fairness among users. Algorithms performing task 2 are presented in subsections B-D.

Minimum bit rate constraints are considered in both MA ([2]-[9]), and RA problems ([11]-[12], [14]). In [4]-[5] and [11], the required rates are used to make an estimate of the number of subcarriers to assign to users. Indeed, for user u, the number of subcarriers N_u is determined with the average CgNR $\overline{g_u}$ (on all subcarriers) and the required bit rate $r°_u$; N_u is initialised to $\lceil r°_u/R_{max} \rceil$. The power needed to transmit $r°_u$ bits on m subcarriers is set to: $P'_u(m) = (m/\overline{g_u}) \times f(r°_u/m)$. The algorithm increments N_u for user u which exhibits the biggest power reduction $\Delta_u = P'_u(N_u) - P'_u(N_u+1)$. In [4]-[5], the algorithm ends when all the subcarriers are allocated. In [11], when the algorithm ends, $\Sigma_u P'_u(N_u) < P_{T,max}$ but sometimes all subcarriers are not used. Hereafter, a RA specific algorithm to compute $\{N_u\}_{1\leq u\leq U}$ is proposed (increasing N_u when user u minimizes the power is rather suitable for MA problems). Our algorithm uses all subcarriers and the available power $P_{T,max}$. For fairness concerns, $\{N_u\}_{1\leq u\leq U}$ will be computed with $\underline{r}°=r°\times\underline{1}$; however the algorithm performs as well with any vector $\underline{r}°$ (in the general case, $r°_u \neq r°_v$ when $u \neq v$).

A. Task 1: bandwidth allocation with adjustable degree of fairness

To determine for each user u, the number of subcarriers N_u, a common target minimum rate $r°$ is fixed. If too optimistic, $r°$ will be decreased (by the fairness adjustment mechanism). All users share the same rate guaranty $r°$, so N_u is initialised to $\lfloor N/U \rfloor$. Equal power on subcarriers is considered: $p=P_{T,max}/N$.

We define gap_u the difference between the estimated rate of user u (over m subcarriers) and the minimum rate $r°$: $gap_u(m) = m\times h(p\ \overline{g_u}) - r°$. While $\Sigma_u N_u < N$, the user with the minimum gap receives one extra subcarrier. When $\Sigma_u N_u = N$ and at least one predicted rate is below $r°$, the basic idea is to "take from the rich and give to the poor". More precisely: if $\min_u(gap_u(N_u))<0$ and provided that $\max_u(gap_u(N_u\text{-}1))>0$, then the algorithm performs $N_{u'}=N_{u'}-1$ for "rich" user $u' = \arg\max_u(gap_u(N_u-1))$ and $N_{u''}=N_{u''}+1$ for "poor" user $u''=\arg\min_u(gap_u(N_u))$. This algorithm, called BARE (bandwidth allocation on rate estimation), ends successfully when $gap_u(N_u) \geq 0$ for each u. In other cases, the power constraint is too low to meet the common user rate guaranty $r°$. In such cases, we introduce a fairness mechanism which decreases $r°$ of λ bits ($\lambda\in\Re^*$) before restarting BARE. The process ends when $r°$ is satisfied ($\min_u(gap_u(N_u) \geq 0$).

Fairness degree adjustment mechanism

While $\min_u(gap_u(N_u)<0)$ *&* $r°>\lambda$
 $r°=r°- \lambda$
 BARE
end

The output of the while loop, $r°_{floor}$, is a target common minimum rate, for which the power constraint is satisfied. This fairness mechanism contrasts with the opportunist algorithm which does not provide any rate guaranties to users; it assigns each subcarrier to the user with the highest CgNR on it ([13]). In [11], there are minimum rate constraints but no $\underline{r}°$ adjustment.

B. Hungarian algorithm for task 2

Once the number of subcarriers needed for each user is determined, the authors of [11] apply the Hungarian algorithm. The input of the Hungarian algorithm must be a square cost matrix. Each user is then duplicated in N_u fictive users to obtain a total of N fictive users. Each of N fictive users can only get one subcarrier. A square cost matrix (size $N\times N$ instead of $U\times N$) can then be formulated. Dealing with RA optimization, the entry $c(u,n)$ in the cost matrix is the rate $r_{u,n}$ that user u would have on subcarrier n: $r_{u,n} = h(p\ CgNR(u,n))$. The complexity of this algorithm is critical regarding real time implementation (see IV.E). To reduce complexity several heuristics have been proposed for subcarrier assignment.

C. Existing heuristics for task 2

The authors of [4] define the Amplitude Craving Greedy Algorithm (ACG). The basic idea is the following one: without required bit rates per user, maximizing the overall data rate would be achieved by assigning each subcarrier n to the best user $\tilde{u} = \arg\max_{u} (CgNR(u,n))$. To satisfy required bit rates per user, it is enough to consider $(N_u)_{1\le u\le U}$ computed during task 1. Then, the ACG algorithm performs as follows: processed per increasing index order (or in a randomized order ([5])), each subcarrier n is assigned to the best user u satisfying $|\Omega_u|<N_u$. In [6]-[7], the drawback of ACG is exposed. Investigating the MA problem, it is found that the subcarrier processing order used in ACG leads to high bit error rate ([6]) or high transmit power ([7]). The subcarrier processing order can thus be improved. The authors of [6] propose the "modified ACG" algorithm where subcarriers are ordered in increasing order regarding $\min_u$ (CgNR(u,n)). Except the order, assignment is done as in ACG. The authors of [7] propose the "improved ACG" algorithm. At each step of this algorithm, subcarrier $\tilde{n}$ is found and assigned to user $\tilde{u}$ where $(\tilde{u},\tilde{n}) = \arg\max_{n} \max_{u,|\Omega_u|<N_u} (CgNR(u,n))$ (1). Algorithms ACG, "modified ACG" and "improved ACG" have been defined in MA, but are fully usable in RA when an algorithm such as BARE is used to perform task 1.

D. Proposed heuristic for task2: RPO algorithm

Here an algorithm is proposed which still allows users to get their best subcarrier, $n'(u)$=arg $\max_n$ (CgNR(u,n)), when there is no conflict. Indeed, at a given stage, several users may have the same best subcarrier n; this event is referred to as conflict. In [4]-[7], a subcarrier n is allocated to user $u°(n)$ =arg $\max_{u\in\Pi}$(CgNR(u,n)). In our algorithm, the user selection may be different for a conflicting subcarrier. To resolve the conflict on subcarrier n, we consider the rate that a competing user u would have on his second best subcarrier $n''(u)$ = arg $\max_{n\neq n'(u)}$ (CgNR(u,n)).

The Rate Profit Optimization (RPO) algorithm proceeds as follows. The best subcarrier of each unsatisfied user is determined. Without conflict the candidate subcarriers are assigned to their best user. Let C be the set of conflicting subcarriers. For subcarrier $n \in C$, let Λ_n be the set of users whose best subcarrier is n. We define the profit of user $u\in\Lambda_n$ under the hypothesis that u gets subcarrier n instead of user $u°(n)$ (index $u°(n)$ is shortened to $u°$ when n is fixed):

profit (u)=*rateGap*(u)-*rateGap*($u°$)

where *rateGap*(u) = *rate*(u,n) - *rate*(u,$n''(u)$) is the rate difference between user u best and second-best subcarrier, with *rate*(u,n)=$h(\gamma_{u,n})$. As a matter of fact, the global rate is increased by *rateGap*(u) under the hypothesis that u gets subcarrier n; while the global rate is decreased by *rateGap*($u°$) since $u°$ doesn't get subcarrier n under this hypothesis. To optimize the global rate, the subcarrier n is allocated to user u'=arg $\max_{u\in\Lambda_n}$ (*profit*(u)). A user with N_u subcarriers will not compete against other users during the rest of the assignment process. The algorithm ends when all subcarriers are assigned.

TABLE II. ALGORITHM NOTATIONS

$n'(u)$	arg $\max_n$ (CgNR(u,n)) i.e. best subcarrier of user u
$n''(u)$	arg $\max_{n\neq n'(u)}$ (CgNR(u,n)) i.e. second best subcarrier of user u
A	Set of unallocated subcarriers
C	Set of subcarriers choosed by more than one user
Ω_u	Set of sucarriers allocated to user u
Π	Set of unsatisfied users ($\|\Omega_u\|<N_u$)
$u°(n)$	arg $\max_{u\in\Pi}$ (CgNR(u,n)) i.e. best user of subcarrier n
Λ_n	Set of users such as n'(u)=n ; ($n\in$C)

RPO Algorithm

```
A={1..N}   # all subcarriers
Π={1..U}   # all users
while |A|>1
        for each u ∈ Π          # selection among unallocated subcarriers
                n'(u)= arg max_{n∈A} (CgNR(u,n));
                n''(u)= arg max_{n∈A, n≠n'(u)} (CgNR(u,n));
        end for
        Let C be the set of subcarrier such that n'(u)== n'(v) and u≠v;
        for each u∈ Π                          # Simple allocation cases
                if n'(u)∉C,  Ω_u=Ω_u ∪{n'(u)};  end if
                A=A- {n'(u)} ;
                if |Ω_u| = =N_u ,  Π=Π-{u};  end if
        end for
        while |C|≠0, pick n in C               # Conflict cases resolution
                u°=argmax_{u∈Π}(CgNR(u,n)) ;
                r_{u°,1}=h(γ_{u°,n}) ;
                r_{u°,2}=h(γ_{u°,n''(u)}) ;
                Λ_n is formed;
                for each u∈Λ_n
                        r_{u,1}=h(γ_{u,n}) ;
                        r_{u,2}=h(γ_{u,n''(u)}) ;
                        profit(u)= (r_{u,1}- r_{u,2})- (r_{u°,1}- r_{u°,2}) ;
                end for
                u'=arg max_{u∈Λn} (profit(u))
                Ω_{u'}=Ω_{u'} ∪{n};
                if |Ω_{u'}|= =N_{u'},  Π=Π-{u'};  end if
                C=C- {n} ;
                A=A- {n} ;
        end while
end while
The last subcarrier (if any), is assigned to the last user of Π.
```

IV. SIMULATION RESULTS

A. Simulation Parameters

Simulation parameters are shown in table II. The maximum number of bits per subcarrier R_{max}=4.5 in reference to the highest MCS in IEEE 802.16: 64-QAM with a coding rate of 3/4. Equal power $p=P_{T,max}/N$ ([10], [12]) varies from 1mW to 35mW (0-15dBm) and σ_{sh} is fixed to 8 dB. The maximum distance of a user is 5 km. We are interested in comparing the rate per subcarrier, a factor of fairness (defined in section C), the outage probability and the execution time of different algorithms. The Hungarian algorithm is plotted because it gives the optimal subcarrier assignment. The opportunist algorithm ([13]) is plotted as the upper bound of the rate per subcarrier. In [10], the maximization of minimum user capacity ($\max(\min_u r_u)$) is investigated, the corresponding "fair heuristic" is also considered. The proposed RPO algorithm is compared to "improved ACG", "modified ACG", ACG and bDA (basic Dynamic Assignment) algorithm proposed in [12]. In bDA algorithm, users are ordered from high to low priority and user u gets the totality of N_u subcarriers at a time. The results are averaged over M=20000 channel states snapshots for each power value.

TABLE III. SIMULATION PARAMETERS

B (MHz)	1
U	8
N	128
λ(bit)	1
R_{max} (bits)	4.5
K : Path loss constant	10^{-4}
α : Path loss exponent	2.8
$Dmax$ (km)	5
σ_{sh} (dB)	8
Mean of exponential variable : a_f^2	1
Standard deviation of a_f^2	1
Power p per subcarrier (mW)	1…35
N_0 Thermal noise density (dBm/Hz)	-174

B. Spectral efficiency

Figure 1 shows the average data rate per subcarrier. When the power per subcarrier p varies from 1 to 15mW, the average SNR varies between 10 and 30 dB (SNR∈[-10 dB, 55 dB]). Providing minimum rate or equal rates to users reduce the global rate performance. When comparing opportunist and Hungarian algorithms, it can be inferred that minimal rate guaranties reduce the global rate from 30.2% to 4.3% as p increases from 1 to 15mW. In the other hand, comparison between opportunist and "fair heuristic" show that equal rate guaranty reduces the global rate from 46.7% to 7% as p increases from 1 to 15mW.

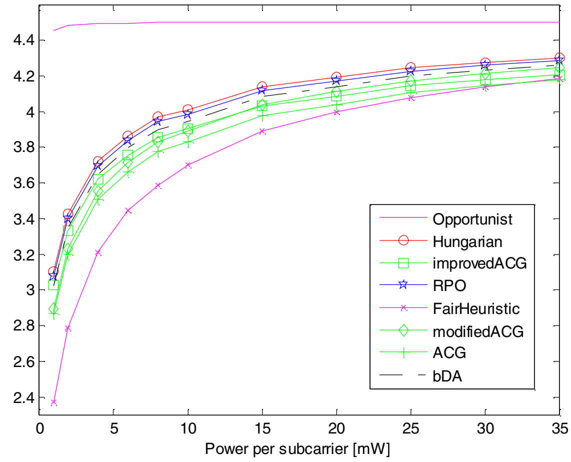

Figure 1. Rate per subcarrier versus transmitted power $p=P_{T,max}/N$

The RPO algorithm exhibits the closest performance to that of the Hungarian algorithm among the sub-optimal heuristics. Whatever the power per subcarrier, the rate performance reduction (compared to Hungarian) remains between 0.5 and 1%. The RPO algorithm is followed by the bDA algorithm (with a rate reduction around 2% compared to Hungarian). At high power values (p >15 mW), "modified ACG" outperforms "improved ACG" rate performance and comes close to bDA (however when α>3, "improved ACG" stays above "modifiedACG"). As the power per subcarrier increases, the rate difference between ACG and the fair heuristic decreases.

C. Fairness

The different user rates in the cell are compared. The ratio between the worst user rate and the best user rate is called fairness factor (Fig.2) and denoted as F. Strict fairness is characterised by a factor F equal to one. It can be inferred from Fig.2 that F increases as p increases. The "fair heuristic" has the highest fairness factor F>0.9. After the fair heuristic, the Hungarian algorithm presents the best fairness factor. Because bad subcarriers are treated first in "modifiedACG", its fairness factor closely follows the Hungarian one. RPO ensures to users either their best subcarrier or a good second best subcarrier; for this reason RPO fairness factor also stays close to that of Hungarian. RPO outperforms bDA, ACG and "improvedACG". The factor F of the opportunist algorithm is almost null.

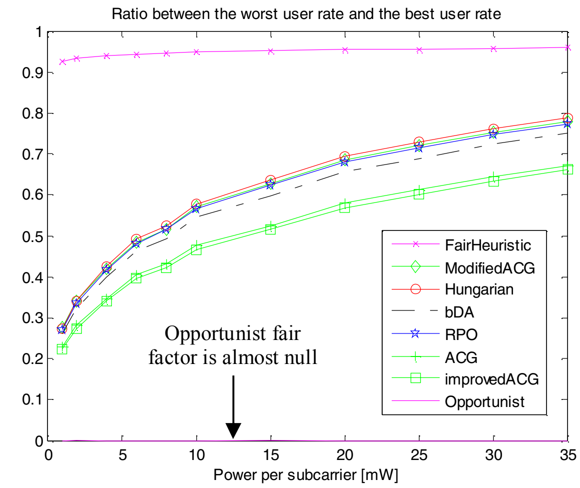

Figure 2. Fairness factor F versus transmitted power $p=P_{T,max}/N$.

D. Floor rate $r°_{floor}$ and outage probability

The common rate guaranty $r°_{floor}$ (obtained from the fairness mechanism in task 1) increases from 10 to 30 bits/symbol as the transmit power increases. The outage probability, P_{out}, is hereafter the probability that a user transmit less than $r°_{floor}$ bits per (OFDM) symbol duration. The parameter P_{out} measures the failure rate of task 2 in fulfilling objectives fixed by task 1 in terms of $r°_{floor}$. It provides a quality indicator for the algorithms proposed to compute task 2. Concerning the Hungarian algorithm, over M=20000 snapshots for each power value, the number outage is null. The order of magnitude of the Hungarian outage probability is likely inferior or equal to 10^{-6}. The order of magnitude of the RPO fairness factor is 10^{-5} since there is at most one outage over M=20000 snapshots. It can be seen from Fig. 3 that RPO outperforms all the existing heuristics regarding $r°_{floor}$ fulfilment.

E. Evaluation of complexity

The computational complexity of the Hungarian method is said to be as high as $O(N^4)$ ([3],[8]). As alternative, bDA is $O(N\times U \log N)$ while "improved ACG" is $O(N^2)$. Indeed, (1) is $O(N+U)$. Since (1) is executed N times, "improved ACG" is $O(U\times N+N^2)$ simplifying to $O(N^2)$ when $U<<N$. Dealing with RPO, the main while loop executes N times in the worst case, search of best and second subcarrier is $O(N)$; during the conflict resolution, computation of maximum user profit is at worst $O(U^2)$. The worst case complexity is then $(N^2+U^2\times N)$

simplifying to $O(N^2)$ when $U<<N$. Figure 4 shows task 2 CPU time versus the number of subcarriers. The RPO execution time is similar to bDA and "improved ACG" with better rate performance. The Hungarian algorithm runs at least three order of magnitude lower than the heuristics.

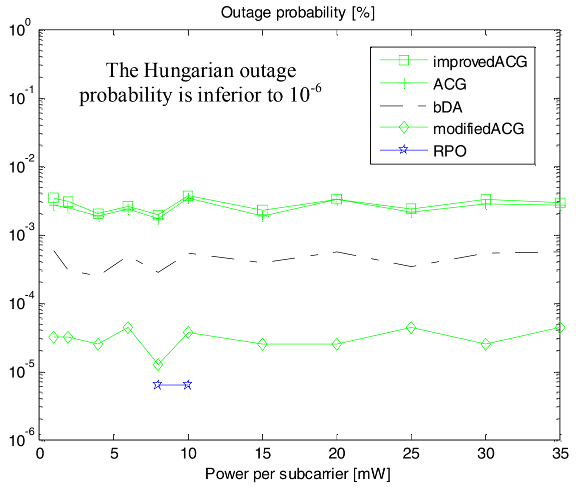

Figure 3. Outage probability versus transmitted power $p=P_{T,max}/N$

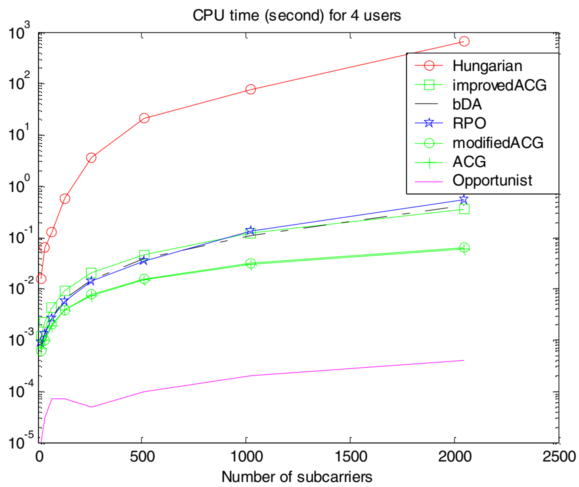

Figure 4. Comparison of execution time of different proposal for task 2.

V. CONCLUSION

In this work, we reviewed previous investigations on resource allocation in OFDM networks. The RA problem can be split into two tasks: determination of the number of subcarriers for each user and specific subcarrier assignment. A RA-oriented approach to compute the number of subcarrier per user with a degree of fairness is described. Furthermore, we proposed an algorithm for subcarrier assignment that can replace the Hungarian algorithm and several known heuristics. The complexity of this algorithm, named RPO, is significantly lower than the Hungarian algorithm and its execution time is similar to "improved ACG" and bDA algorithms. The RPO fairness factor is higher than bDA, ACG and "improved ACG" and stays close to that of the Hungarian algorithm. Regarding the rate per subcarrier, the RPO algorithm outperforms all the described heuristics. Besides, the RPO rate per subcarrier is at most 1% below the Hungarian algorithm performance. Next investigations will consider resource allocation in a multicell system with co-channel interference.

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