COMPARISON OF DIFFERENT SUBCHANNELIZATION MODES FOR OFDMA

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ABSTRACT

In this paper, the problem of subchannel composition and allocation for the downlink of an OFDMA system is investigated. The subchannelization mode FUSC adopted in 802.16 is described and compared with dynamic subcarrier allocation algorithms. Performances are evaluated regarding spectral efficiency and complexity. Both single cell and multi-cell scenarios are considered.

I. INTRODUCTION

Next broadband wireless access networks are likely to use OFDMA (Orthogonal Frequency Division Multiple Access) as multiple access technique. OFDMA combines Orthogonal Frequency Division Multiplexing (OFDM) modulation scheme and the Frequency Division Multiple Access technique (FDMA). OFDMA is a promising access scheme and has been chosen for IEEE 802.16-2004 ([1]) and its extension for mobility support IEEE 802.16e ([2]). This choice extends the list of standards where the modulation OFDM is used. Inter Symbol Interference (ISI) mitigation and high data rates support are the main advantages of OFDM.

OFDMA takes advantage of frequency diversity by dividing the bandwidth into several subsets which are allocated to distinct users. The different subsets are called subchannels. As different users may have the same best subcarriers, subcarrier allocation is a tough problem. The way to build subchannels and to allocate them is subject to active research. Academic research ([3]-[6]) have proposed dynamic subchannel composition according to channel conditions and user rate requirements. Such contributions consider a variable number of subcarriers per user. Three problems are commonly handled: (i) cell rate maximization subject to transmit power and minimum user rate constraints ([3]), (ii) transmit power minimization accounting for user rate requirements ([4], [6]), (iii) maximization of minimum user performance ([5]). In such research, allocation is done on a subcarrier basis. In practical standards such as [1]-[2], allocation is done on a subchannel basis; subchannels (i.e. fixed number of subcarriers) are built in advance.

In this paper, we compare predetermined subchannel composition and dynamic subchannel composition. As example of predetermined subchannel composition, we consider the FUSC mode (Fully Used SubChannels, adopted in [1]-[2] for the downlink) and adjacent subchannel composition, we consider proposals of [5]-[6]; we fix the number of subcarriers per user to allow comparisons with [1]-[2]. For simplicity, each user receives one subchannel.

The remainder of the paper is organized as follows. Section II describes the channel model and the system parameters.

Section III presents the pseudo-random subchannel composition of IEEE 802.16. Dynamic subchannel composition is investigated in section IV. In section V, simulation results are presented. Execution time of the algorithms is compared. Practical modulation and coding schemes (MCS) are used to evaluate spectral efficiency. In the multi-cell scenario, the frequency reuse factor is set to one and partial loading of FUSC subchannels is considered. Section VI concludes the paper.

II. SYSTEM MODEL

A. Channel modeling

The channel model consists of N parallel narrowband subcarriers over the bandwidth W. The path loss model is $K d(u)^{-\alpha}$ where d(u) is the distance between a given user u and his serving base station (BS), α is the pathloss exponent $(2 \le \alpha \le 4)$ and K is a constant for a given environment. The excess loss caused by different obstructions is modelled using a lognormal distribution variable a_{sh} (10 log(a_{sh}) is $\mathcal{N}(0, \sigma_{sh}^2)$) with 4 dB $\leq \sigma_{sh} \leq 12$ dB). All the subcarriers undergo the same shadowing effect for one user at one instant ([7]). Due to multipath propagation, the received signal on a subcarrier is the sum of several scattered waves. The amplitude of the received signal on each subcarrier has a Rayleigh distribution. We take into account the correlation between the signal envelopes of different subcarriers. Correlation of small scale fading is modelled by a first order Gauss-Markov process. The correlation coefficient decreases with subcarrier spacing. A common correlation threshold value used to determine the coherence bandwidth B_c is 0.5 ([8]). The relation between B_c and $\sigma_{\rm RMS}$ (root mean square delay spread) is then approximated by: $B_c = 1/(2 \pi \sigma_{\rm RMS})$.

The channel gain g(u,n), of user u on subcarrier n, is summarized as: $g(u,n)=K d(u)^{-\alpha} a_{sh}(u) a_f(u,n)$ where a_f has a Rayleigh distribution and represents the small scale fading. Additive white Gaussian noise (AWGN) is characterized on each subcarrier by a Gaussian random variable $\mathcal{N}(0, \sigma^2)$ with $\sigma^2 = N_0 W/N$. The channel gain to noise ratio (CgNR) is then: $c_e nr(u,n) = g(u,n)/\sigma^2$.

If *B* interfering cells are considered, the channel gain to interference and noise ratio (CgINR) is $c_ginr(u,n)=g(u,n)/(\sigma^2+I(u,n))$. The level of interference suffered by user *u* on subcarrier *n* is expressed as $I(u,n) = \sum_{b=1...B} K d_b(u)^{-\alpha} a_{sh}^{\ b}(u) a_f^{\ b}(u,n) \delta_{b,n}$. In this expression, $d_b(u)$ and $a_{sh}^{\ b}(u)$ are the distance and the shadowing effect between user *u* and the interfering BS *b*, $a_f^{\ b}(u,n)$ is the small scale fading on subcarrier *n*; $\delta_{b,n}$ is one if subcarrier *n* is used in interfering cell *b*, otherwise $\delta_{b,n}$ is zero. In a reuse one deployment with full loading, $\delta_{b,n}=1$ for $1 \le b \le B$ and $1 \le n \le N$.

B. System parameters

We consider the downlink of one cell with one BS and U users. First, we ignore inter-cells interference. Secondly, we consider B interfering cells (cf. V). For both scenarios, spectral efficiency is computed for subchannelization modes presented in III-IV. Equal power p is allocated to subcarriers (in [9], the equal power strategy provides good performance compared to waterfilling). The signal to interference and noise ratio is SINR(u,n)= $p c_g inr(u,n)$. We assume the same MCS for the N_{sc} = 48 subcarriers of a subchannel. The MCS is determined by the subchannel effective SINR (cf. V) according to Table I ([1]).

Among the *N* subcarriers (Fast Fourier Transform size), the number of data subcarriers is N_{data} . In the scalable version of the physical layer ([2]), the subcarrier spacing Δf and the useful symbol duration $T_u=1/\Delta f$ are independent of the bandwidth *W*; $T_u=91.43 \ \mu$ s. The guard time represents 1/8 of T_u : $T_g=11.67 \ \mu$ s. The total symbol duration is thus $T_s=102.86 \ \mu$ s.

III. SUBCHANNELIZATION MODES IN 802.16

Several modes of subchannelization are described in [1-2] among which can be found FUSC (Full Usage of SubChannels), PUSC (Partial Usage of SubChannels), and AMC (Adaptive Modulation and Coding). Section III describes these modes in the downlink.

A. FUSC

In FUSC (not defined in uplink), subchannels are composed of N_{sc} subcarriers during one OFDM symbol duration. Taking advantage of channel diversity, subchannels are made of subcarriers spread over the frequency band. The bandwidth is divided into N_{sc} groups of $N_{scg}=N_{data}/N_{sc}$ consecutive subcarriers, after excluding the initially assigned pilots. A subchannel is made of one subcarrier from each group.

The formula which governs subchannel composition can be summarized as follows: $k_s = GS(s, k) + SS(s, k, DL_PermBase)$. In this formula, k_s designates the $(k+1)^{th}$ subcarrier of subchannel *s*, *GS* stands for group selection and *SS* stands for subcarrier selection.

The function GS, depends on indexes s and k. It indicates the group wherein the $(k+1)^{th}$ subcarrier of subchannel s will be picked out. GS (s,k) is a multiple¹ of N_{scg} where N_{scg} is the number of subcarriers in a group.

The function SS designates a specific subcarrier into the group pointed by GS(s,k): $0 \le SS(s,k, DL_PermBase) \le N_{scg}$ -1. The function SS is governed by a permutation list of N_{scg} integers (between 0 and N_{scg} -1) which is proper to subchannel s (this list is denoted as p_s , see Table II)². The parameter $DL_PermBase$ serves as an offset; it is given by the DL-MAP³ and differs following the zone⁴ of the DL subframe.

Fig.1 illustrates FUSC subchannel composition. The horizontal lines demarcate the different groups of N_{scg} consecutive subcarriers. Each subcarrier of a subchannel belongs to a different group (there is only one point or cross between two horizontal lines). The subcarriers of a subchannel are regularly spaced over the whole bandwidth.

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	Modulation	Coding Rate	Required SINR (dB)					
	QPSK	1/2	6					
	QPSK	3/4	9					
	16-QAM	1/2	12					
	16-QAM	3/4	15					
	64-QAM	2/3	18					
	64-QAM	3/4	21					

TABLE II: FUSC PARAMETERS					
W Total bandwidth		10 MHz			
N	Total number of subcarriers	1024			
N _{data}	Number of data subcarriers	768			
N_{sc}	Number of data subcarriers per	48			
	subchannel				
N _{scg}	Number of data subcarriers per	16			
	group				
p_0	Permutation list: p_s is p_0 cyclically	[6 14 2 3 10 8 11 15			
	shifted to the left s times	9 1 13 12 5 7 4 0]			
S	Number of subchannels	16			
U	Number of users	16			

B. PUSC

The (downlink) PUSC mode divides the bandwidth into 6 parts called major groups. It enables another frequency reuse factor than one (in FUSC, only reuse one is possible). By default, segments are composed of 2 consecutive major groups which can be assigned to distinct cells (or distinct sectors of the same cell if sectorization is assumed). Segmentation is possible because subchannels are made of subcarriers over one major group (instead of the whole band in FUSC). Inside one major group, first pilots are assigned; then the remaining subcarriers are partitioned into groups ([1 p.564], ([2 p.530]). A subchannel consists of one subcarrier per group. The formula governing subchannel composition is common to FUSC. Unlike uplink PUSC, pilots are assigned (in each major group) before subchannels composition. It allows thinking that pilots of a major group can be shared by the subchannels of this major group. It is different in uplink PUSC where each subchannel has its own pilots.

C. AMC

The AMC mode allocates 6 bins (defined as 9 adjacent subcarriers including one pilot) to users. Several subchannel types are defined. For instance, the subchannel type $i \times j$ means that *i* consecutive bins are allocated over *j* OFDM symbols ([1]). The main difference of this mode is that subcarriers of a subchannel are adjacent instead of being distributed over the bandwidth. In this paper, we are interested in performance of adjacent subcarriers subchannelization (ASS, similar to subchannel type 6×1 of AMC mode).

 $^{^{1}}GS(s,k) = m_{k,s} N_{scg}$ and $m_{k,s} = (k+13s)mod N_{sc}$.

² SS (s,k,IDcell) = { $p_s(m_{k,s} \mod N_{scg}) + DL_{PermBase}$ } mod N_{scg}

³ It is a broadcast MAC management message.

⁴ Contiguous OFDMA symbols using the same subchannelization mode.



IV. DYNAMIC SUBCHANNEL COMPOSITION

In this paper, we consider that a subchannel is made of N_{sc} subcarriers over one single OFDM symbol. We see that FUSC subchannels are built in advance without channel state consideration. In section A-B, subchannels are optimized for a specific user. In section C, subchannels are built in advance but may be allocated based on channel considerations.

A. Doufexi & Armor algorithm

The authors of [5] provide a DSA (Dynamic Subcarrier Allocation) algorithm as an alternative to pseudo-random subchannel composition. The algorithm proposed in [5] builds U subchannels in parallel. Each subchannel is exclusively formed for a single user. As a result, allocating a subcarrier to a subchannel is synonym of allocating the subcarrier to a specific user.

The authors of [5] try to maximise the average channel gain received by a user without minimising the average channel gain of the others. Each user receives one subcarrier at a time until all users get N_{sc} subcarriers. Each user u (abiding by a given order) can choose the subcarrier n^* which satisfy $n^* = \arg \max_n (cgnr(u,n))$. A parameter Q_u gives information on quality of already allocated subcarriers to user u; Q_u is updated following $Q_u = Q_u + g(u,n^*)$. When all users have received their k^{th} subcarrier, the order in which each user chooses his $(k+1)^{th}$ subcarrier is obtained by sorting users in increasing order regarding Q_u . In this algorithm (referred to as "Doufexi&Armor"), the channel gain is thus used as the metric to allocate subcarriers. In a multi-cell context, the algorithm can be applied with $c_ginr(u,n)$ instead of g(u,n).

B. modifiedACG algorithm

The amplitude craving greedy (ACG, [4]) algorithm allocates subcarriers in a random order. A user u expects N_u subcarriers. Let Ω be the set of users which are still expecting subcarriers i.e. the set of users who received less than N_u subcarriers. Each subcarrier is allocated to the best user $u'=\arg \max_{u \in \Omega} (c_g nr(u,n))$. The basis of the algorithm here follows: without constraints on the number of subcarriers to allocate to each user, the optimal solution (to maximize the rate) would allocate each subcarrier to its best user ([9]).

The authors of [6] propose a differential feedback reduction scheme which supports relative CSI instead of perfect CSI. They show that ACG subcarrier processing order may be improved to avoid worst case allocation. In the modifiedACG algorithm, the subcarriers are sorted once in decreasing order regarding *worstCgNR_n* = min_u($c_gnr(u,n)$); so that subcarriers are ranged from smaller minimum user gain to larger minimum user gain. Subcarrier allocation principle is similar to ACG. In this paper, the modifiedACG is applied with a constant number N_{sc} of subcarriers per user. In a multi-cell context, the algorithm uses $c_ginr(u,n)$ instead of $c_gnr(u,n)$.

C. Adjacent Subcarrier Subchannelization (ASS)

In this part, suchannels are composed of N_{sc} adjacent subcarriers. If such subchannels are allocated to users regardless of their average channel gain, it is denoted as "random ASS". To improve performance an algorithm may govern subchannel selection for users. Let $\overline{g}_{u,s}$ be the mean CgNR of user *u* over subchannel s:

$$\overline{g}_{u,s} = \frac{1}{N_{sc}} \sum_{n=((s-1)N_{sc})+1}^{sN_{sc}} c_g nr(u,n)$$

An intuitive and simple algorithm considers users for 1 to U; a user receives the available subchannel which exhibits the best average CgNR (resp. CgINR in a multi-cell context). It is referred to as "intuitive ASS", it is similar to subchannel type 6×1 of AMC mode ([1]-[2]). If the number of subcarriers in a coherence bandwidth $(\lfloor B_c / \Delta f \rfloor)$ is nearly N_{sc} , the different subcarrier channel gains in a subchannel may be regarded as similar. The signalling overhead is then reduced in ASS compared to individual subcarrier allocation.

D. Complexity

We evaluate complexity of algorithms presented in previous sections. The number of iterations of the "Doufexi&Armor" algorithm is N/U. At each step, U users are sorted which takes: $O(\log U)$; U users choose their best subcarrier and this takes $O(U \times N)$. Finally, the complexity is $O(N^2+N/U\log U)$ which simplifies to $O(N^2)$. Dealing with the modified ACG algorithm, sorting the CgNR (channel gains) is the dominant operation. Sorting N elements takes O(U) (for one subcarrier, the maximum CgNR is picked among U). The complexity is thus $O(Nlog N+U \times N)$ simplifying to $O(N \log N)$ when $U \le N$.

Table III gives exemples of CPU times of "Doufexi&Armor" algorithm ([5]) and modifiedACG algorithm ([6]) for N=1024, and U=16. The modifiedACG algorithm is two times faster than "Doufexi&Armor" algorithm whereas intuitive ASS is two hundred times faster than the latter.

TABLE III: EXECUTION TIMES IN SECONDS

Doufexi&Armor	modifiedACG	intuitive ASS
0.0565	0.0238	0.0003

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V. SIMULATION RESULTS

Simulation parameters are described in Table IV. We focus on one reference (or target) hexagonal cell. In this cell, users are located at the same range of the serving BS (in all scenarios described hereafter). For simplicity, a user receives one subchannel. Subchannel allocation may last several time slots but we only consider snapshots. Spectral efficiency provided by the different algorithms are compared. Results are averaged over 1000 channel state snapshots. We assume equal power allocation. In the sequel, let P_{BS} denote the total BS power and let *p* denote the power per subcarrier.

In scenario 1, the target cell is isolated, interferences are ignored. The power P_{BS} varies from 28 to 49 dBm (p varies from -7 dBm to 20 dBm). For a subchannel, the MCS is chosen according to the effective SNR and Table I. The effective SNR of a subchannel s is given by $2^{\text{MIC}(s)}$ -1 ([10]) where MIC stands for mean instantaneous capacity. The subchannel MIC is the average capacity computed accross N_{sc} subcarriers of the subchannel; the capacity of a subcarrier (allocated to u) is expressed as: $c(u,n)=\log_2(1+\text{SNR}(u,n))$.

In scenario 2, we consider *B* interfering cells distributed on two rings around the target cell. We consider one sector per cell with omni directional antennas. The reuse factor is one i.e. all frequencies can be reused in the interfering cells (the frequency reuse pattern is 1x1x1 according to notations in [10]). All BS transmit at the same power P_{BS} . Cells are fully loaded unless specified otherwise. The subchannel MCS depends on the subchannel effective SINR. First, the spectral efficiency is plotted as P_{BS} varies with a fixed inter BS distance. Regarding FUSC subchannelization, partial loading is investigated. In a second time, spectral efficiency is plotted for a variable inter BS distance varies and a fixed BS power.

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Path loss constant: K	1.4 10 ⁻⁴			
Path loss exponent: α	3.5			
Log-normal standard deviation: σ_{sh}	8.9 dB			
Root mean square (RMS) delay spread: σ_{RMS}	295 ns			
Thermal noise: N_0	-174 dBm/Hz			
Noise Figure at BS receiver	6 dB			
Number of data subcarriers: N _{data}	768			
Subcarrier spacing: Δf	10.93 kHz			
Number of subchannels in one cell: S	16			
Number of users in the target cell	16			
Number of interfering cells: B	18			
Distance of users from serving BS in the target cell	500 m			

TABLE IV: SIMULATION PARAMETERS

A. Scenario 1: single cell context

The spectral efficiency of the different algorithms is plotted in Fig.3. Doufexi&Armor algorithm shows the best spectral efficiency and outperforms FUSC up to 0.58 bit/s/Hz. The modifiedACG improves the spectral efficiency up to 0.35 bit/s/Hz compared to FUSC. With a 295 ns RMS delay spread, the coherence bandwidth is about 539 kHz which is close to subchannel size $N_{sc}\Delta f$ =524 kHz. For that coherence bandwidth, intuitive ASS and modifiedACG (with individual subcarriers) have similar performance. This makes intuitive ASS preferable: it is faster and requires less signalling overhead. Besides, compared to Doufexi&Armor algorithm,

the intuitive ASS performance reduction is only 5% when P_{BS} =43 dBm (p=14 dBm). Performance of random ASS and FUSC are similar. Multi-user diversity gain can not be achieved if CSI is not considered or shows to be outdated. With one subchannel and P_{BS} =43 dBm (p=14 dBm), a user achieves a rate of 1.9 Mbit/s (resp. 1.81 and 1.66) with Doufexi&Armor (resp. intuitive ASS and FUSC).



Figure 3: Spectral efficiency (bit/s/Hz) vs power per subcarrier

B. Scenario 2: multicell context

In a first time, the distance between two nearby BS is fixed to 2 km. The spectral efficiency of the different algorithms is plotted in Fig.4. The transmit power P_{BS} increases from 28 to 49 dBm. It can be seen that the transmit power increase does not increase the spectral efficiency as much as in the single cell context (Fig.3). Indeed, the increase of the transmit power also increases the level of interference.

To improve spectral efficiency, we consider partial loading of FUSC subchannels. Each cell uses only S_{used} subchannels over the *S* available subchannels. In Fig.4, the FUSC spectral efficiency is plotted for a partial loading of 40% and 60%. In case of 60% loading, FUSC subchannelization has the best spectral efficiency. For P_{BS} =43 dBm (p=14 dBm), a user achieves a rate of 0.98 Mbit/s with 40% loading, 0.75 Mbit/s with 60% loading and finally 0.52 Mbit/s with full loading. Although partial loading increases the subchannel rate, it does not increase the global rate since fewer subchannels can be used (cf. Table V).



Figure 4: Reuse one, spectral efficiency (bit/s/Hz) vs power

Subchannelization	Doufexi	modified	FUSC	FUSC	FUSC
scheme	&Armor	ACG	(100%)	60%	40%
	(100%)	(100%)			
Rate (Mbit/s),	13.32	11.05	8.67	7.06	6.01
p = 14dBm					

In Fig.5, The distance between two BS (denoted as $d_{BS/BS}$) varies from 1 to 5 km. The power of all BS is fixed to P_{BS} =43 dBm (p =14 dBm). As in single cell scenario, the path loss exponent α is 3.5. The spectral efficiency improves as the inter BS distance increases. For $d_{BS/BS} = 5$ km, all subchannelization modes have nearly the same spectral efficiency than in a single cell context (at p = 14 dBm). It can be inferred that, with $\alpha=3.5$, cells are isolated if $d_{BS/BS} \ge 5$ km. In Fig. 6, we plot the spectral efficiency reduction coefficient η_R . Let η_S be the spectral efficiency obtained in single cell configuration and for a given set of parameters $(p, \alpha, d_{BS/BS})$; let η_M be the spectral efficiency obtained in multi cell configuration and for the same set of parameters. The performance reduction η_R is $(\eta_S - \eta_M)/\eta_S$. Coefficient η_R decreases when $d_{BS/BS}$ increases. The speed of η_R diminution depends on α : the higher α , the more the decrease according to $d_{BS/BS}$ is fast. Indeed, the higher α , the lower the interference between cells. Performance reduction is higher in FUSC than in Doufexi&Armor scheme.



Figure 5: Reuse one, spectral efficiency (bit/s/Hz) vs inter BS distance



Figure 6: Performance reduction coefficient (%) compared to single cell vs inter BS distance

VI. CONCLUSION

In this paper, we have studied different modes of subchannelization for downlink OFDMA. The main modes of IEEE 802.16 have been presented and the performances of FUSC in term of spectral efficiency have been evaluated. FUSC exhibits the lowest performance compared to other schemes. However, FUSC is independent of CSI knowledge so the rate provided is solid guarantee for users. Two algorithms ([5], [6]) are examined which build dynamically subchannels as an alternative to deterministic subchannel composition of FUSC. Such algorithms exhibit high spectral efficiency but a good CSI is needed per subcarrier. This requires a signalling overhead which may not be acceptable. Adjacent subcarrier subchannelization (or ASS i.e. consecutive subcarriers grouped together to form a subchannel) is examined to reduce computational time and signalling overhead of DSA algorithms. A random ASS allocation is similar to FUSC. However, when ASS allocation is based on average SNR, it exhibits better performance than FUSC and only slight performance degradation is observed compared to DSA algorithms. In a second time, a multi-cell context has been considered with reuse one. Partial loading improves FUSC spectral efficiency. A compromise must be reached since global rate decreases as partial loading increases.

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