# SIMILARITY MEASURES BETWEEN SAR AND OPTIC DATA

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Abstract—With the development of remotely-sensed multisensor satellites like Pleiades Cosmo-Skymed that have the particularity of providing both SAR and optic data, new techniques in image processing are needed. These techniques must take into account the complementarities and differences in nature of these data.

A preliminary operation for advanced techniques that use multisensor images such as fusion, classification, etc. is registration. In the case of SAR and optic data, we can do automatic registration if we exactly know the sensor parameters and have a digital terrain model (DTM) or a digital elevation model (DEM) at our disposal. If we do not have an exact knowledge of these parameters, the registration becomes difficult.

Another approach to achieve the automatic registration which does not need sensor parameters will rely on comparison measures between both data.

In this paper, we present a comparison of several similarity measures between multisensor SAR and optic images used in matching algorithms. An evaluation of these measures for synthetic data based on their distributions is given. Then results on real images are analyzed.

Index Terms—Similarity measures, multisensor image registration.

# I. INTRODUCTION

In the literature of multisensor remote sensing imagery, relatively few works address the problem of fine registration [1]. One of the reasons is that it must be generally based on a matching approach which presents difficulties in front of non similar data with different radiometric properties, such as SAR and optic images.

Usually, to rectify and adjust an automatic global registration, fine registration is needed and is often done manually by a human expert. However, in presence of data with a great number of local distortions (like, for instance, high resolution images with different elevations), we need to make this procedure automatic. The most adapted approach is the use of some matching criteria. In fact, selecting homologous primitives in both multisensor data and matching them is a good way to estimate a local deformation model. Applying this model on data, registration will be more accurate in all parts of the image.

Thus, the problem of registration becomes a problem of finding the nature of similarity existing between our multisensor data: SAR and optic.

Because the real scene imaged is common to these two data,

similarity measure must take into account the relationship existing between them. This relationship can be: common objects in images, primitives, contrast, distribution, noise, etc. In this work, we will study which kind of similarity between SAR and optic images can be found. In section II, we present some similarity measures that will be analyzed in our study. Then, we propose to use a statistical approach to evaluate performances of these measures and compare them. Since analytical expressions of probability density functions are not always available, we first study the robustness of simulation approaches to estimate them (section III). Then we study the similarity measures in section IV. First, we analyze the dependency of a similarity measure to parameters of SAR and optic scene data and we compute ROC (Receiver Operating Characteristics) curves expressing efficiency of a similarity measure for corner patterns. Finally, in section V, we validate results and conclusions on real images in a multisensor registration application.

# II. SIMILARITY MEASURES

Registration problem based on similarity measure can be expressed in the following analytical form:

$$Argmax_T(S_c(I, ToJ)) \tag{1}$$

Aligning an image J (slave) to an image I (master) is finding the optimal geometric transformation T that maximize the similarity  $S_c$  between images I and ToJ in the sense of criterion c. Here c is the similarity measure used to register two images. So, we note in this expression the importance of the similarity measure used in the optimization procedure.

In this work, we have studied the most known similarity measures in the literature. We present in this paper only the study done for the five measures that were considered as the most accurate measures in related works.

These measures can be classified into two different concepts:

- Similarities that express functional dependence:
  - Correlation coefficient (ρ): it measures linear dependence between two images [2],
  - Correlation ratio (η): this is a generalization of the correlation coefficient to a functional dependence [3],
  - 3) Woods criterion (*Woods*): this measure is quite similar to the previous one, but it takes into account the

variation coefficient (ratio of standard deviation and mean) [4].

- Similarities that express statistical dependence:
  - 1) Normalized mutual information (*IMN*): a variant of the mutual information that measures the amount of information that one image (seen as a random variable) contains about the other [5],
  - 2) Distance to independence  $(X^2)$ : it measures the degree of statistical dependence between data.

Performances of these measures will be compared based on an empirical study of their distributions (pdf : probability density function). So, we first validate the use of simulations to study similarity measures. Then, we evaluate them and compare their results on optic and SAR synthetic images: optic image with Gaussian noise and radar image with Gamma distribution of intensity.

# III. VALIDATION OF THE EMPIRICAL STUDY OF THE DISTRIBUTIONS CURVES

The criteria will be compared using their pdf depending on various parameters: analysis window size, contrast of the scene for the optic and SAR data, etc. Because it is not always possible to derive analytical expressions of the pdf, we propose to simulate samples to compute it.

To validate this approach, we first compare results given by a theoretical approach (analytical expression) and an empirical approach (computation of pdf using simulated data). To do this comparison, we study a very simple case of optic and SAR data where we can compute analytical expressions of a similarity measure. Then, results will be compared with those given by simulations.

The simple model is Constant/Gamma data: constant areas on optic image and generalized Gamma distribution on SAR image. The underlying scene is supposed to be divided into two areas  $W^1$  and  $W^2$  where we can locate a corner (see Fig.1.). Let I represents the SAR image and J the optic image. For the SAR image,  $\mu_I^1$  (resp.  $\mu_I^2$ ) is the mean of  $W_I^1$  (resp.  $W_I^2$ ),  $n_1$  (resp.  $n_2$ ) is the pixel number of  $W_1$  (resp.  $W_2$ ), S(i,j) the sample of a generalized Gamma distribution noise with mean equal to 1 and L is the number of looks. For the optic image,  $m_J^1$  (resp.  $m_J^2$ ) is the gray level intensity of  $W_J^1$ (resp.  $W_J^2$ ). So, the studied model is:

$$\begin{split} I(i,j) &= \left\{ \begin{array}{ll} \mu_{I}^{1}S(i,j) \ , & if \ (i,j) \in W_{I}^{1} \\ \mu_{I}^{2}S(i,j) \ , & if \ (i,j) \in W_{I}^{2} \end{array} \right. \\ J(i,j) &= \left\{ \begin{array}{ll} m_{J}^{1} \ , & if \ (i,j) \in W_{J}^{1} \\ m_{J}^{2} \ , & if \ (i,j) \in W_{J}^{2} \end{array} \right. \end{split}$$

Based on this simplified model, we can clearly express response of similarity measures on the corresponding corners in the SAR and the optic images. For example, computing theoretical expression of the correlation coefficient on extracted corners, we obtain:

$$\rho_{I,J}^2 = \begin{cases} \frac{1}{1 + \frac{(n_1 + n_2)(n_1 c_I^2 + n_2)}{Ln_1 n_2 (c_I - 1)^2}} & ; if \quad m_1 \neq m_2 \\ 0 & ; else \end{cases}$$
(2)

Where  $c_I = \frac{\mu_I^1}{\mu_I^2}$  is the contrast between the two regions  $W_I^1$  and  $W_I^2$ . Plot of this function depending on  $c_I$  and L is given in Fig.2.

Then, we simulate this model into images and we draw plots of the correlation coefficient responses. Examples of the two plots are presented in the same figure (Fig.2.) to compare them.



Fig. 1. Simplified model of a corner: (left) image without noise corresponding to the optic data (J); (right) image with gamma distribution corresponding to the radar data (I).



Fig. 2. Theoretical and empirical plots of the correlation coefficient response depending on contrast and number of looks in SAR image. The empirical plots are drawn using 20 samples for each contrast value and number of looks.

We conclude that the empirical study of a similarity measure can be a good approach to evaluate it since the results are similar to the theoretical ones. For the rest of this paper, all plots to evaluate a similarity criterion are done based on simulations.

# IV. EVALUATION OF SIMILARITY MEASURES

In this part, we consider a model closer to the reality to evaluate accuracy of measures. For the optic image, we generate a Gaussian noise and for the SAR image, we generate a Gamma distribution. Position of the regions on simulated images is the same as in the last simple model.

Note that in this part, only the number of samples of the two regions in the window is taken into account since the criterion is computed for the true match position.

## A. Evaluation of criteria

Criteria used to evaluate performance of measures are based on the pdf computed empirically using simulations. For each fixed distribution parameters, we generate in practice 100 simulations of our model to estimate pdfs. Then, efficiency of a similarity measure is studied according to two ways:

- 1) Understanding measure behavior by studying their dependence on distribution parameters: contrast  $(c_I = \frac{\mu_I^1}{\mu_I^2})$  and number of looks  $(L_I)$  on SAR image and contrast  $(c_J = \frac{\mu_J^1}{\mu_J^2})$  and standard deviation  $(\sigma_J)$  on optic image.
- 2) Evaluating the measure by drawing ROC plots: probability  $P_d$  of detecting a corner versus the probability  $P_F$  of false alarm. Expressions of these probabilities are given by:

$$P_d(s,\Theta) = \int_s^1 f(t/\Theta)dt \tag{3}$$

$$P_F(s) = P_d(s, c_J = 1, c_I \neq 1)$$
 (4)

 $\Theta$  is the parameter vector of the distribution model. f(t) is the density probability function of a measure. The set of parameters ( $c_J = 1, c_I \neq 1$ ) means that no corner exists in the optic image whereas it was found in the SAR image.

All plots corresponding to the five similarity measures are drawn and used to compare the behavior of each measure.

Images simulated in this part, that correspond to the real scene, are closer to Fig.3. Only distribution parameters  $(c_I, L_I, c_J, \sigma_J)$  are changing during simulations.



Fig. 3. Left : image with Gaussian noise (contrast  $c_J$ , standard deviation of the noise  $\sigma_J$ ). Right : Image with Gamma distribution (contrast  $c_I$ , number of looks  $L_I$ ).

#### B. Pdf of the similarity measure

We first trace pdf plots with fixed distribution parameters in these cases :

- 1) For SAR (resp. optic) contrast impact on a similarity measure, we generate three series of pdfs:  $c_I = 1$  (resp.  $c_J = 1$ ) (no contrast in image),  $c_I = 1.5$  (resp.  $c_J = 1.5$ ) (good contrast) and  $c_I = 2$  (resp.  $c_J = 2$ ) (high contrast).
- 2) For the number of looks impact, we generate two series of pdfs:  $L_I = 1$  and  $L_I = 3$ .
- 3) For optic standard deviation influence, we generate two series of pdfs:  $\sigma_J = 3$  and  $\sigma_J = 10$ .

We show in the next figure an example of a these plots.



Fig. 4. Pdf of the correlation coefficient when varying contrast on SAR image ( $c_I^1 = 1, c_I^2 = 1.5, c_I^3 = 2$ ). Contrast on optic image is fixed to 1.5 and standard deviation is fixed to 3. The number of looks on SAR image is equal to 1.

We will only mention in this article conclusions extracted from these plots. In fact, varying distribution parameters we note that, as expected, all measures give best response if  $c_I$ and  $L_I$  are high and  $\sigma_J$  is low and they do not depend or depend slightly on  $c_J$  (which is less expected).

#### C. Evaluation of the similarity measures

We present, to evaluate and compare similarity measures, their ROC curves; first, with low noise ( $\sigma_J = 3$ ) on optic image then with a high noise ( $\sigma_J = 10$ ). Parameters chosen for estimation of  $P_d$  (resp.  $P_F$ ) are  $c_I = 1.5$ ,  $c_J = 1.5$  and  $L_I = 1$  (resp.  $c_I = 1.5$ ,  $c_J = 1$  and  $L_I = 1$ ).



Fig. 5. ROC plots for the five similarity measures (Gaussian noise with  $\sigma_J=3$ )



Fig. 6. ROC plots for the five similarity measures (Gaussian noise with  $\sigma_J=10$ )

Based on these two figures, we can extract practical conclusions. The first one is that statistical similarity measures are more adapted to SAR and optic data than functional dependence measures. The second one is that only the correlation coefficient, as a functional dependency measure, gives a good response with low noise.

Thus, the choice and the use of a similarity measure to match corners in both optic and SAR images depend on the quality of optic images. With low noisy optic images, correlation coefficient is an efficient criterion to use for registration. And to guarantee a good matching in high noisy optic images, normalized mutual information is considered as the best and the most adapted similarity measure for our multisensor data.

# V. VALIDATION AND APPLICATION TO MULTISENSOR IMAGE REGISTRATION

#### A. Validation

To validate conclusions drawn in the precedent section, we test the five similarity measures on real data. The strategy consists on selecting corners on optic images and manually finding their homologous in the SAR corresponding images. Then, we repeat this procedure automatically by introducing similarity measure and a searching window (size: 21x21). For each measure, we compute the error of matching which corresponds to the Euclidean distance between the homologous corner found with automatic procedure and the one fixed manually (which is obviously the right correspondent corner). Tests are carried out on a pair of SAR and optic data (2048x2048) with a high resolution and globally registered based on a global registration approach (see the next paragraph). On this image, we extracted 7 regions where we found local alignment errors (generally caused by the presence of relief). In each region, we extract corners and we apply our strategy to compute error matching by the five similarity measures.



Fig. 7. Example of region (128x128) where two corners are extracted and automatically matched by IMN measure (left: SAR image, right: optic image). The size of window used for the matching process is (21x21) and the exploration area is  $\pm$  12 pixels around each pixel.

In the next table we present a summary of results obtained when computing mean errors for all corners extracted in regions :

	$\rho^2$	$\eta$	woods	IMN	$X^2$
Pixel Error	4,04	8,37	6,2	3,83	5,8

TABLE I

MATCHING ERROR : MEAN OF ERRORS, FOR EACH SIMILARITY MEASURE, RECORDED WHILE MATCHING THE 7 REGIONS.

It is clear that results obtained with real images are closer to those predicted empirically on simulated model with high noise on optic image. So, we can confirm the interest of the normalized mutual information compared to other similarity measures. Nevertheless, the correlation coefficient has also a good behavior

# B. Application

Finding the best similarity measure between SAR and optic data gives a solution to do a fine registration between these multisensor data. In our work, we started by making a global registration based on Fourier-Mellin approach [6]. Then, to rectify locally registration errors, we estimate a non rigid local transformation based on matching technique which uses the best similarity measure adapted to SAR and optic images.

# VI. CONCLUSION

In this work, we have studied performances of the most known similarity measures to do a fine registration between SAR and optic data. The study is based on a statistical and empirical approach. First, we have analyzed the pdf of each similarity measure and studied the influence of some parameters (contrast, level of noise, etc.). Then, the efficiency of these measures is evaluated through ROC plots. Conclusions drawn show the most adapted measure to a pair of SAR and optic images depending on distribution parameters.

The comparison presented here is of course not exhaustive, since it would be interesting to include more scene types (point target, cross-roads, etc.). However, this paper opens many prospects to future works. In fact, we can add to our empirical approach a theoretical study by computing analytical expressions of similarity measures to our scene model (Gaussian/Gamma), like expression computed in section III for correlation coefficient.

We can also extend this work to other types of primitives than corners: targets, linear structures, regions, etc. and thus we draw new conclusions related to the relationship existing between one similarity measure and one primitive used for the matching problem.

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