# SAR amplitude filtering using TV prior and its application to building delineation

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## Abstract

This paper investigates the use of a popular regularization model, the Total Variation minimization (TV), to filter SAR images and reduce speckle noise. This model is extensively used for its property of preserving edges. Due to the many local minima, TV minimization is difficult to achieve for non-convex likelihood terms such as that of SAR amplitude. Such a minimization can be performed efficiently by computing minimum cuts on weighted graphs. Exact minimization, although theoretically possible, can not be implemented due to memory constraints on large images required by remote sensing applications. The computational burden of the state-of-the-art algorithm for approximate minimization is also heavy. In this paper, we propose a new fast approximate discrete algorithm. The filtering is applied in the framework of building delineation for 3D reconstruction. Results on real images are presented.

## 1 Introduction

There are nowadays many SAR satellite sensors (EnviSat, Radarsat, Terra-SAR, ...) providing a huge amount of SAR images. The popularity of such sensors is linked to their all weather and all-time capabilities. However, SAR images are difficult to interpret, mainly because of the speckle phenomenon.

Speckle is due to the interferences of waves reflected by many elementary reflectors inside a resolution cell. Although speckle has been extensively studied and is well modeled in some particular cases [7] [9] [10], speckle reduction remains one of the major issues in SAR image processing. Many filters have been proposed in the last twenty years.

Our objective in this paper is to study the interest of Markovian modeling for SAR scene reflectivity restoration. Indeed, Markov Random Field (MRF) modelization provides a convenient way to express both data fidelity constraints and desirable properties of the filtered image. In this context, total variation minimization has been extensively used to constrain the oscillations in the regularized image while preserving its edges.

We first describe the chosen model (likelihood term and regularization term), and then study the optimization of such a functional. We first recall previous methods before describing a new algorithm which provides a *fast approximate* solution. The filtering process is then applied on real images for the delineation of buildings and their 3D reconstruction.

## 2 Markovian modeling

### 2.1 Principle

Let us denote by $v$ an image defined on a finite discrete lattice $S$ and taking its values in a discrete integer set $\mathcal{L} = \{0, \dots, L\}$. We denote by $v_s$ the value of the image $v$ at the site $s \in S$. We note by $s \sim t$ the neighboring relationship between sites $s$ and $t$, by $(s,t)$ the related clique of order two and by $N_s$ the local neighborhood of site $s$. Given an observed image $v$, a Bayesian analysis using the Maximum A Posteriori (MAP) criterion consists of finding a restored image $u$ which will represent in the following the “real” scene reflectivity. We will denote by $v_s$ the observed amplitude in the SAR image and $u_s$ is the square root of the scene reflectivity (to be homogeneous with amplitude).

We will use a restrictive modeling of the scene reflectivity. It is supposed to follow the cartoon model, which means homogeneous areas separated by well defined boundaries. The proposed approach uses a MAP criterion and supposes that the global field $u$ is markovian. Our aim is to introduce contextual relationship allowing a regularized solution $u$. The MAP problem is thus an energy minimization problem with :

$$E(u|v) = E(v|u)+E(u) = \sum_s U(v_s|u_s)+\beta \sum_{(s,t)} \psi(u_s,u_t)$$

where $U(v_s|u_s) = -\log(P(v_s|u_s))$ and $\psi$ is some regularization function.

## 2.2 Likelihood term

The likelihood terms $P(v_s|u_s)$ have been chosen as Nakagami distributions [7] for a $M$-look image:

$$P(v_s|u_s) = \frac{2M^M}{\Gamma(M)u_s^{2M}} v_s^{(2M-1)} e^{\left(-\frac{Mv_s^2}{u_s^2}\right)}$$

leading to the following energetic term :

$$U(v_s|u_s) = M[\frac{v_s^2}{u_s^2} + 2\log u_s]$$

This energy is not convex in $u_s$ due to the heavy tailed distribution of speckle images.

## 2.3 Regularization term

As said in the introduction, the TV regularization prior is well adapted when dealing with strong discontinuities. Besides this prior has good properties for minimization since it is a convex function. The energetic term corresponding to the discretization of TV can be written as follows:

$$E(u) = \beta \sum_{(s,t)} w_{st}|u_s - u_t|$$

with $w_{st} = 1$ for the 4-nearest neighbors and $w_{st} = 1/\sqrt{2}$ for the 4 diagonal ones. We will not explicitly write the weights $w_{st}$ in the following equations.
$\beta$ is the so-called hyperparameter used to tune the relative importance of prior knowledge with respect to likelihood.

## 2.4 Energy minimization

For the regularization of amplitude images we have the following energy to minimize:

$$E(u|v) = \sum_s M[\frac{v_s^2}{u_s^2} + 2\log u_s] + \beta \sum_{(s,t)} |u_s - u_t| \quad (1)$$

Since the likelihood term is not convex, the global energy is in general not convex. The optimization of such an energy is discussed in the following section.

# 3 Optimization

## 3.1 Previous works

For many years, optimization of MRF energies has been done using simulated annealing [6] or ICM [1]. Both have important drawbacks: the first one is very slow, whereas the second one is very sensitive to initialization and converges far from the global minimum in pratice. More recently, efficient discrete optimization schemes have been proposed based on graph-cut search, i.e. the computation of a s-t minimum cut or, by duality, a maximum flow in a graph.

Two exact algorithms could be applied to solve exactly the problem. The first one has been proposed in [8] and is able to exactly optimize any likelihood term combined with a convex regularization term. Nevertheless, the constructed graph is quite huge since the number of nodes is the number of pixels multiplied by the number of grey-levels. This memory size is prohibitive for any application to remote sensing images. Another exact solution is provided in [5]. The algorithm works for levelable energies, which means that the energy can be written as a sum on the level sets of $u$. Though the graph construction leads to a reduced size graph for convex likelihood and prior energies, the graph is of comparable size as that of [8] in the general case. Since the convexity of the posterior energy is not guaranteed in our model (due to the non-convex log-likelihood of the amplitude), a fast algorithm based on a scaling search can not be applied [5].

Concerning approximate optimization, $\alpha$-expansion algorithm proposed in [3] can be applied. Starting from a current solution, this algorithm proposes to each pixel either to keep its current gray-level, or to take a value $\alpha$ as new gray-level. The energy associated to this class of changes is minimized using a graph-cut. The succession of $\alpha$-expansions over all possible values in $\mathcal{L}$ until convergence leads to a solution which is shown to be close to the global minimum. The set of all possible values can be large, therefore leading to heavy computational burden. We suggest in the next section a faster algorithm which is more suitable when large images or joint regularization are considered.

## 3.2 Proposed algorithm

Graph-cut approach provides a way to explore a combinatorial set of changes involving simultaneously all pixels. Following [3], we denote such changes *large moves*. Instead of allowing a pixel to either keep its previous value or change it to a given one ($\alpha$-expansion), we suggest that a pixel could either remain unchanged or its value be increased (or decreased) by a fixed step. Such an approach has first been described independently in [2, 4, 11] and applied recently with unitary steps in [2]. We however use these large moves in a case of non-convex data term. The trial steps are chosen to perform a scaling sampling of the set of possible pixel values.

We describe in the following subsections the set of large moves considered, the associated graph construction, and the approximate optimization scheme.

### 3.2.1 Local minimization

First, let us introduce the set of images that lie within a single move in our algorithm. Then, $S_d(u^{(n)}) = \{u \,/\, \forall s \in S, \exists k_s \in \{0,1\}, u_s = u_s^{(n)} + k_s d\}$ is the set of images whose pixel value $u_s$ is either unchanged or increased by step $d$. We define the "best" move has the one that minimizes the restriction of the energy to the set $S_d(u^{(n)})$:

$$u^{(n+1)} = \underset{u^{(n+1)} \in S_d(u^{(n)})}{\arg\min} E(u^{(n+1)}|v).$$

The restriction of the energy to $S_d(u^{(n)})$ corresponds to an energy involving only the binary variables $(k_s)_{s \in S}$. According to [12], an energy of binary variables arising from a first-order Markov model can be minimized by computing a minimum cut on a related graph provided it satisfies the following submodular property:

$$\psi(0,1) + \psi(1,0) \geq \psi(0,0) + \psi(1,1).$$

To compute the "best" move using a s-t minimum-cut algorithm, the following must therefore hold:

$$\psi(u_s, u_t+d) + \psi(u_s+d, u_t) \geq \psi(u_s, u_t) + \psi(u_s+d, u_t+d). \quad (2)$$

Note that in most cases, the prior model $\psi$ depends only on the difference $u_s - u_t$. This is the case in the model described in previous section. For such prior models, condition (2) becomes:

$$\psi(u_s - u_t - d) + \psi(u_s - u_t + d) \geq 2\psi(u_s - u_t)$$

which is the definition of the convexity of $\psi$.
In conclusion, the *local* problem of finding the vectorial field $u^{(n+1)}$ located within a single move (i.e. $u^{(n+1)} \in S_d(u^{(n)})$) that minimizes the posterior energy $E(u^{(n+1)}|v)$ can be *exactly* solved by computing a minimum cut on a graph (described in next paragraph) provided that the regularization potential is convex and depends only on the difference $u_s - u_t$.
The model we described in previous section consists of the sum of a non-convex likelihood term and a convex prior term. The above property therefore holds for this model and we give in the next paragraphs an algorithm for approximate global minimization based on exact local minimizations performed using graph-cuts.

### 3.2.2 Graph construction

We build a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, following the method of [12], to minimize the restriction of the energy to allowed moves of step $d$. The graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is directed, with nonnegative edge weights and two terminal vertices: the source $S$ and the sink $P$. The graph structure and the edge weights are chosen such that any cut[1] has a cost (i.e. sum of the cut edges capacities) corresponding to the energy to minimize. We create a vertice for each site $s$, all connected respectively to the source and the sink through two edges with capacity $c_{s,1}$ (resp. $c_{s,0}$). Finally, each clique $(s,t)$ gives rise to an edge with capacity $c_{s,t}$.
The capacities are set according to the additive method described in [12]. The first term in equation (1) is represented by the weights:

$$\begin{cases} c_{s,1} = \max(0, U(v_s|u_s^{(n)} + d)) - U(v_s|u_s^{(n)})) \\ c_{s,0} = \max(0, U(v_s|u_s^{(n)}) - U(v_s|u_s^{(n)} + d))). \end{cases}$$

To this weights are added the weights representing each clique (second term of equation (1)):

$$\begin{cases} c'_{s,1} &= \beta \cdot \max\left(0, \psi(u_s^{(n)} + d, u_t^{(n)}) - \psi(u_s^{(n)}, u_t^{(n)})\right) \\ c'_{s,0} &= \beta \cdot \max\left(0, \psi(u_s^{(n)}, u_t^{(n)}) - \psi(u_s^{(n)} + d, u_t^{(n)})\right) \\ c'_{t,1} &= \beta \cdot \max(0, \psi(u_s^{(n)} + d, u_t^{(n)} + d) \\ & \quad -\psi(u_s^{(n)} + d, u_t^{(n)})) \\ c'_{t,0} &= \beta \cdot \max(0, \psi(u_s^{(n)} + d, u_t^{(n)}) \\ & \quad -\psi(u_s^{(n)} + d, u_t^{(n)} + d)) \\ c_{s,t} &= \beta \cdot (\psi(u_s^{(n)}, u_t^{(n)} + d) + \psi(u_s^{(n)} + d, u_t^{(n)}) \\ & \quad -\psi(u_s^{(n)}, u_t^{(n)}) - \psi(u_s^{(n)} + d, u_t^{(n)} + d)) \end{cases}$$

### 3.2.3 Approximate global minimization

When non-convex data terms such as Nakagami law described are considered, the global minimization problem can not be exactly solved without considering each possible configuration (i.e. building a huge graph). On the other hand, when all terms are convex, it has been proven in [4] that a succession of local minimizations leads to the global minimum. An exploration based on different scalings of the step is then suggested to speed up convergence.

We follow here an heuristic method that combines the *exact* determination of the best moves, with no guarantee on how close to the global minimum we get. Next sections will illustrate on some simulated and real data that the obtained results are satisfying in practice with a speed adequate for application use.

The joint-regularization algorithm is summarized here:

1: **for all** $s \in S$ **do**
2: $\quad u_s^{(0)} \leftarrow \{L/2, \ldots, L/2\}$
3: **end for**
4: $n \leftarrow 0$
5: **for** $i = 1$ to $precision$ **do**
6: $\quad d_i \leftarrow L/2^i$
7: $\quad$ **for all** $d_i \in S(d_i)$ **do**
8: $\quad\quad u^{(n+1)} \leftarrow \arg\min_{u^{(n+1)} \in S_d(u^{(n)})} E(u^{(n+1)}|v)$
9: $\quad\quad n \leftarrow n + 1$
10: $\quad$ **end for**
11: **end for**

Line 8 represents the exact binary energy minimization obtained by computing a minimum cut on a graph build according to previous section. Note that if we perform unitary steps $d_i \in S(1)$ until convergence at the termination of our algorithm, exact minimization is then guaranteed for convex energies [4].

[1] a cut is a partition of the vertices into two disjoint sets $\mathcal{S}$ and $\mathcal{P}$ such that $S \in \mathcal{S}$ and $P \in \mathcal{P}$

# 4 Results and application to building delineation

## 4.1 Simulated images

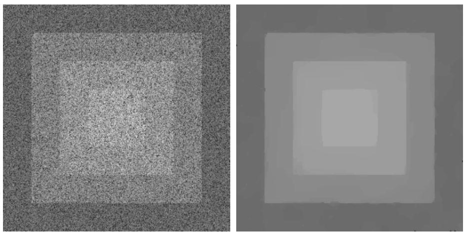

**Figure 1:** Simulated SAR image with Nakagami distributions (cartoon image) and the filtered result with TV regularization.

An example of result using a simulated image corresponding perfectly to the chosen model (cartoon + Nakagami) is presented figure 1. Speckle is strongly reduced while edges are well preserved.

## 4.2 Real images and building delineation

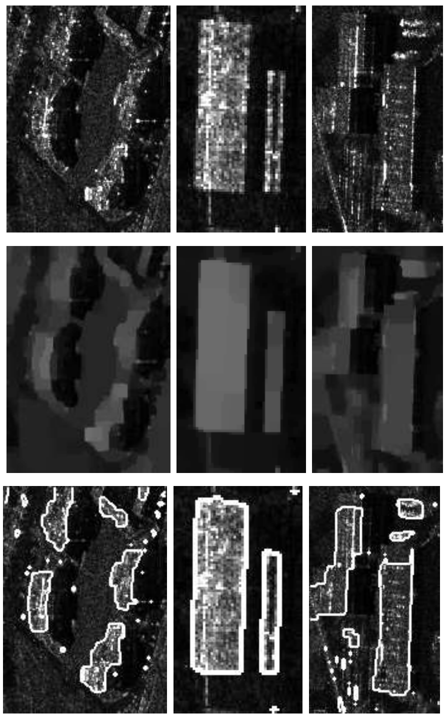

**Figure 2:** Three extracts of original 1-look SAR image ©ONERA, their associated filtered results (below), and the automatic building delineation (bottom).

In this part, real SAR images of urban areas have been filtered. Then automatic thresholding is applied to detect potential buildings. The building footprints are then used for 3D reconstruction using interferometric information or lay-over/shadow analysis.

# 5 Conclusion

In this paper we have described a fast algorithm for SAR amplitude filtering with TV regularization. This leads to efficient speckle reduction while preserving edges and is therefore of interest for building delineation.
Further work should include the joint regularization of interferometric phase and amplitude images using a coupled edge process. In this framework, the efficiency of the proposed discrete approximate algorithm will be crucial.

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