JOINT FILTERING OF SAR INTERFEROMETRIC AND AMPLITUDE DATA IN URBAN AREAS BY TV MINIMIZATION

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ABSTRACT

This paper investigates the use of a popular regularization model, the Total Variation minimization (TV), to filter SAR interferometric images (amplitude and phase data). This model is extensively used for its property of preserving edges and is therefore well adapted for urban areas. Using a TV model adapted to multi-dimensionnal data, we propose to do a joint filtering of phase and amplitude images. Due to the many local minima, the minimization of such a model is hard to perform. A new fast approximate discrete algorithm is presented. The filtering is applied in the framework of 3D reconstruction. Results on real images are presented.

Index Terms— SAR imagery, graph-cut, Markov Random Field, interferometry

1. INTRODUCTION

There are nowadays many SAR satellite sensors (EnviSat, Radarsat, ALOS ...) providing a huge amount of SAR images. The popularity of such sensors is linked to their all-weather and all-time capabilities, combined with their polarimetric and interferometric potential. The interferometric data, which are phase difference images, give either elevation or movement information. The launch of new sensors with improved resolution in 2007 (TerraSAR-X and CosmoSkyMed) opens new fields of applications. Particularly, the computation of Digital Elevation Models (DEM) becomes feasible with metric interferometric images, specially when tandem configurations will be available. These new data will contribute to urban monitoring which is an important issue for governmental agencies (risk analysis, disaster management, environmental protection, urban development planning,...). In this paper we are interested in filtering SAR images for the purpose of building delineation to perform 3D reconstruction.

Although many works have already been dedicated to SAR filtering, we explore in this paper the *joint* filtering of amplitude and phase data. We will formulate the problem as the Maximum A Posteriori estimation and considers the minimization of an energy that combines two type of information: a data driven term and a regularization term. As we will see in the following, due to the physical mechanisms of radar processing, this energy is not convex (whatever the regularization function) and optimization is a hard task. In this paper, a dedicated algorithm is proposed which is based on graph-cut approaches [1]. It provides an approximate solution but it is fast and requires a limited amount of memory.

Thanks to the French Spatial Agency CNES for their financial support (project R-S06/OT04-010).

2. PROPOSED MODEL

It is assumed that an image u is defined on a finite discrete lattice S and takes values in a discrete integer set $\mathcal{L} = \{0, \ldots, L\}$. We denote by u_s the value of the image u at the site $s \in S$. We note by (s,t) a clique of order two related to a chosen neighborhood system and by N_s the local neighborhood of site s. A solution $\hat{u}$ regularizing u is searched for. It can be shown that under the assumption of Markovianity of $\hat{u}$ and with some independence assumption on u conditionally to $\hat{u}$, the MAP problem is an energy minimization problem:

$$\begin{aligned} \hat{u}^{(MAP)} &= \arg\min_{\hat{u}} E(\hat{u}|u) \\ \text{with} \quad E(\hat{u}|u) &= \sum_s U(u_s|\hat{u}_s) + \beta \sum_{(s,t)} \psi(\hat{u}_s, \hat{u}_t) \end{aligned}$$

$U(u_s|\hat{u}_s) = -\log p(u_s|\hat{u}_s)$ and ψ is a function modeling the prior chosen for the solution. In the case of the minimization of the Total Variation, $\psi(\hat{u}_s, \hat{u}_t) = w_{st}|\hat{u}_s - \hat{u}_t|$ ($w_{st} = 1$ for the 4-nearest neighbors and $w_{st} = 1/\sqrt{2}$ for the 4 diagonal ones; we will not explicitly write the weights w_{st} in the following equations).

2.1. Distributions of interferometric phase and amplitude

The synthesized radar image z is complex-valued. Its amplitude $|z|$ is very noisy due the interferences that occur inside a resolution cell. Under the classical model of Goodman, the amplitude a_s of a pixel s follows a Nakagami distribution depending on the square root of the reflectivity $\hat{a}_s$. This likelihood leads to the following energetic term:

$$U(a_s|\hat{a}_s) = M\left[\frac{a_s^2}{\hat{a}_s^2} + 2\log\hat{a}_s\right]$$

In the case of SAR interferometric data, the interferometric product is obtained by complex averaging of the hermitian product γ of the two SAR images. A good approximation of the phase ϕ_s distribution is a Gaussian which leads to a quadratic energy:

$$U(\phi_s|\hat{\phi}_s) = \frac{(\phi_s - \hat{\phi}_s)^2}{\hat{\sigma}^2_{\phi_s}}$$

The standard deviation $\hat{\sigma}^2_{\phi_s}$ at site s is approximated by the Cramer-Rao bound $\hat{\sigma}^2_{\phi_s} = \frac{1-\rho_s^2}{2L\rho_s^2}$ (with L the number of average samples and ρ_s the coherence of site s). For low coherence areas (shadows or smooth surfaces, denoted $Shadows$ in the following), this Gaussian approximation is less relevant and a uniform distribution model is better $p(\phi_s|\hat{\phi}_s) = \frac{1}{2\pi}$. In this paper, we are interested in high resolution interferometric data. In many cases, the elevation range is contained within one fringe so we do not have to handle the problem of phase unwrapping.

2.2. Regularization term

The proposed method aims at preserving simultaneously phase and amplitude discontinuities. Indeed, the phase and amplitude information are hopefully linked since they reflect the same scene. Amplitude discontinuities are thus usually located at the same place as phase discontinuities and conversely. We propose in this paper to perform the joint regularization of phase and amplitude. To combine the discontinuities a disjunctive $\max$ operator is chosen. The joint prior model is defined by:

$$E(\hat{a},\hat{\phi}) = \sum_{(s,t)} \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|), \qquad (1)$$

with γ a parameter that can be set to 1, and otherwise accounts for the relative importance given to the discontinuities of the phase ($\gamma > 1$) or of the amplitude ($\gamma < 1$). The global joint energy term is then (with some weighting of the likelihood terms):

$$E(\hat{a},\hat{\phi}|a,\phi) = \frac{1}{\beta_a}\sum_s M[\frac{a_s^2}{\hat{a}_s^2} + 2\log\hat{a}_s]$$
$$+\frac{\gamma}{\beta_\phi}\sum_s \frac{(\phi_s - \hat{\phi}_s)^2}{\hat{\sigma}^2_{\phi_s}} + \sum_{(s,t)} \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|)$$

Shadow areas Due to the specific properties of shadow areas (random phase implying no likelihood term), they are separately detected and an adapted regularization term is defined. The regularized fields $\hat{a}$ and $\hat{\phi}$ at sites s located inside the detected shadow areas $Shadows$ are governed only by the regularisation term. With the prior term defined in equation (1), the phase $\hat{\phi}_s$ for $s \in Shadows$ that minimizes the energy corresponds to an interpolation of the phase value at the surrounding sites. Shadow areas however are most of the time at ground level and not at an intermediate height between the top of the structure that creates the shadow and the ground at the shadow end. A modified regularization term that better describes this prior knowledge is therefore used for cliques involving one or both site(s) inside the shadow regions: $E(\hat{a},\hat{\phi}) = \sum_{(s,t)} E(\hat{a},\hat{\phi})_{(s,t)}$ with $E(\hat{a},\hat{\phi})_{(s,t)}$ defined as:

(i) if $s \notin Shadows$ and $t \notin Shadows$,
$E(\hat{a},\hat{\phi})_{(s,t)} = \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|)$,

(ii) if $s \in Shadows$ and $t \notin Shadows$ and $\hat{\phi}_s \leq \hat{\phi}_t$
$E(\hat{a},\hat{\phi})_{(s,t)} = |\hat{a}_s - \hat{a}_t| + \gamma|\hat{\phi}_s - \hat{\phi}_t|$,

(iii) if $s \in Shadows$ and $t \notin Shadows$ and $\hat{\phi}_s > \hat{\phi}_t$
$E(\hat{a},\hat{\phi})_{(s,t)} = |\hat{a}_s - \hat{a}_t| + 2\gamma|\hat{\phi}_s - \hat{\phi}_t|$,

(iv) if $s \in Shadows$ and $t \in Shadows$
$E(\hat{a},\hat{\phi})_{(s,t)} = |\hat{a}_s - \hat{a}_t| + \gamma\left(\hat{\phi}_s - \hat{\phi}_t\right)^2$.

The cases where $s \notin Shadows$ and $t \in Shadows$ are treated in a symmetrical manner. Outside shadow areas (case i), the regularization term is the same as previously. To limit the effect of a given shadow area on the regularization of the amplitude, we independently regularize phase and amplitude in and at the limit of the shadows (cases ii to iv). To force the regularized phase inside a shadow to follow ground level, we penalize more heavily over-estimation (case iii) than under-estimation (case ii). Finally, a quadratic constraint (case iv) enforces a flat/smooth ground inside a shadow area. Note that in each case (i to iv) the prior term $E(\hat{a},\hat{\phi})_{(s,t)}$ is convex and so is the prior energy $E(\hat{a},\hat{\phi})$. The convexity of the prior energy is essential to apply the minimization algorithm described in the following section.

3. ENERGY MINIMIZATION

Minimizing a non-convex energy is a difficult task as the algorithm may fall in a local minimum. Algorithms such as the Iterated Conditional Modes require a "good" initialization and then performs local changes to reduce the energy. Graph-cut approach provides a way to explore a combinatorial set of changes involving simultaneously all pixels. Following [1], we denote such changes *large moves*. Instead of allowing a pixel to either keep its previous value or change it to a given one (α-expansion), we suggest that a pixel could either remain unchanged or its value be increased (or decreased) by a fixed step. Such an approach has first been described independently in [2–4] and applied recently with unitary steps in [2]. We however use these large moves in a case of non-convex data term. The trial steps are chosen to perform a scaling sampling of the set of possible pixel values. We express the algorithm in the general case of joint regularization.

3.1. Local minimization

First, let us introduce the set of images that lie within a single move in our algorithm. For the sake of generality, we denote by $\hat{\mathbf{u}}$ the vectorial field arising by associating to each component one of the images to jointly regularize. Then,

$$\mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)}) = \{\hat{\mathbf{u}} \,/\, \forall s \in S, \exists k_s \in \{0,1\}, \hat{\mathbf{u}}_s = \hat{\mathbf{u}}_s^{(n)} + k_s\mathbf{d}\}$$

is the set of images whose pixel value $\hat{\mathbf{u}}_s$ is either unchanged or increased by step $\mathbf{d}$. We define the "best" move $\hat{\mathbf{u}}^{(n)} \mapsto \hat{\mathbf{u}}^{(n+1)}$ has the one that minimizes the restriction of the energy to the set $\mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)})$:

$$\hat{\mathbf{u}}^{(n+1)} = \underset{\hat{\mathbf{u}}^{(n+1)} \in \mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)})}{\arg\min} E(\hat{\mathbf{u}}^{(n+1)}|\mathbf{u}).$$

The restriction of the energy to $\mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)})$ corresponds to an energy involving only the binary variables $(k_s)_{s\in S}$. According to [5], an energy of binary variables arising from a first-order Markov model can be minimized by computing a minimum cut on a related graph provided it satisfies the following submodular property:

$$\psi(0,1) + \psi(1,0) \geq \psi(0,0) + \psi(1,1).$$

To compute the "best" move using a s-t minimum-cut algorithm, the following must therefore hold:

$$\psi(\hat{\mathbf{u}}_s, \hat{\mathbf{u}}_t+\mathbf{d})+\psi(\hat{\mathbf{u}}_s+\mathbf{d}, \hat{\mathbf{u}}_t) \geq \psi(\hat{\mathbf{u}}_s, \hat{\mathbf{u}}_t)+\psi(\hat{\mathbf{u}}_s+\mathbf{d}, \hat{\mathbf{u}}_t+\mathbf{d}). \tag{2}$$

Note that in most cases, the prior model ψ depends only on the difference $\hat{\mathbf{u}}_s - \hat{\mathbf{u}}_t$. This is the case in the model described in the previous section. For such prior models, condition 2 becomes:

$$\psi(\hat{\mathbf{u}}_s - \hat{\mathbf{u}}_t - \mathbf{d}) + \psi(\hat{\mathbf{u}}_s - \hat{\mathbf{u}}_t + \mathbf{d}) \geq 2\psi(\hat{\mathbf{u}}_s - \hat{\mathbf{u}}_t)$$

which is the definition of the convexity of ψ.

In conclusion, the *local* problem of finding the vectorial field $\hat{\mathbf{u}}^{(n+1)}$ located within a single move (i.e. $\hat{\mathbf{u}}^{(n+1)} \in \mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)})$) that minimizes the posterior energy $E(\hat{\mathbf{u}}^{(n+1)}|\mathbf{u})$ can be *exactly* solved by computing a minimum cut on a graph (described in next paragraph) provided that the regularization potential is convex and depends only on the difference $\hat{\mathbf{u}}_s - \hat{\mathbf{u}}_t$.

The model we described in previous section consists of the sum of a non-convex likelihood term and a convex prior term. The above property therefore holds for this model and we give in the next paragraphs an algorithm for approximate global minimization based on exact local minimizations performed using graph-cuts.

3.2. Graph construction

We build a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, following the method of [5], to minimize the restriction of the energy to allowed moves of step $\mathbf{d}$:

$$\underset{(k_s)_{s\in S}}{\arg\min} \sum_s U(\mathbf{u}_s|\hat{\mathbf{u}}_s^{(n)}+k_s\mathbf{d})+\beta\sum_{(s,t)}\psi(\hat{\mathbf{u}}_s^{(n)}+k_s\mathbf{d}, \hat{\mathbf{u}}_t^{(n)}+k_t\mathbf{d}) \tag{3}$$

The graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is directed, with nonnegative edge weights and two terminal vertices: the source $\mathscr{S}$ and the sink $\mathscr{P}$. The graph structure and the edge weights are chosen such that any cut[1] has a cost (i.e. sum of the cut edges capacities) corresponding to the energy to minimize. We create a vertice for each site s, all connected respectively to the source and the sink through two edges with capacity $c_{s,1}$ (resp. $c_{s,0}$). Finally, each clique (s,t) gives rise to an edge with capacity $c_{s,t}$. The capacities are set according to the additive method described in [5].

3.3. Approximate global minimization

When non-convex data terms such as Nakagami law described in section 2.1 are considered, the global minimization problem can not be exactly solved without considering each possible configuration (i.e. building a huge graph). On the other hand, when all terms are convex, it has been proven in [3] that a succession of local minimizations leads to the global minimum. An exploration based on different scalings of the step is then suggested to speed up convergence.

We follow here an heuristic method that combines the *exact* determination of the best moves, with no guarantee on how close to the global minimum we get. In the following section we will illustrate on some real data that the obtained results are satisfying in practice with a speed adequate for application use.

In one dimension, a scaling search is performed by looking for the best move with steps $d_i^+ = L/2^i$ and $d_i^- = L/2^i$ for i from 1 to the desired precision (i.e. quantization level). In N dimensions, there are $3^N - 1$ vectorial steps $\mathbf{d}_i$ to consider for a given step size d_i:

$$\mathbf{d}_i \in \mathscr{S}(d_i) \overset{\text{def}}{=} \{0, -d_i, +d_i\}^N / \{0, \ldots, 0\}.$$

The joint-regularization algorithm is summarized here:

1: **for all** $s \in S$ **do**
2: $\quad\hat{\mathbf{u}}_\mathbf{s}^{(0)} \leftarrow \{L/2, \ldots, L/2\}$

[1] a cut is a partition of the vertices into two disjoint sets S and P such that $\mathscr{S} \in \mathcal{S}$ and $\mathscr{P} \in \mathcal{P}$

3: **end for**
4: $n \leftarrow 0$
5: **for** $i = 1$ to $precision$ **do**
6: $\quad d_i \leftarrow L/2^i$
7: $\quad$ **for all** $\mathbf{d}_i \in \mathscr{S}(d_i)$ **do**
8: $\quad\quad \hat{\mathbf{u}}^{(n+1)} \leftarrow \arg\min_{\hat{\mathbf{u}}^{(n+1)} \in \mathscr{S}_{\mathbf{d}}(\hat{\mathbf{u}}^{(n)})} E(\hat{\mathbf{u}}^{(n+1)}|\mathbf{u})$
9: $\quad\quad n \leftarrow n + 1$
10: $\quad$ **end for**
11: **end for**

Line 8 represents the exact binary energy minimization obtained by computing a minimum cut on a graph build according to section 3.2. Note that if we perform unitary steps $\mathbf{d}_i \in \mathscr{S}(1)$ until convergence at the termination of our algorithm, exact minimization is then guaranteed for convex energies [3].

4. JOINT REGULARIZATION OF INSAR IMAGES IN URBAN AREA

We now consider joint regularization on high-resolution data acquired over the city of Toulouse, France. The image shown in figure 1(a) is 1200×1200 pixels extracts from single-pass interferometric SAR images acquired by RAMSES (ONERA SAR sensor) in X-band at sub-metric resolution. The amplitude image is a 2-look image obtained after averaging the intensity of the two images of the interferometric pair. The interferogram has been computed on a 3×3 window and the coherence over detected shadow-areas set to 0.

From the regularization results of figure 1 it can be noticed that the noise has been efficiently reduced both in amplitude and phase images. The sharp transitions in the phase image that correspond to man-made structures are well preserved. Joint regularization gives more precise contours than independent regularization as they are co-located from the phase and amplitude images (minimum cost images have transitions that occur between the same neighboring pixels). Small objects also tend to be better preserved by joint-regularization as illustrated in figure 1.

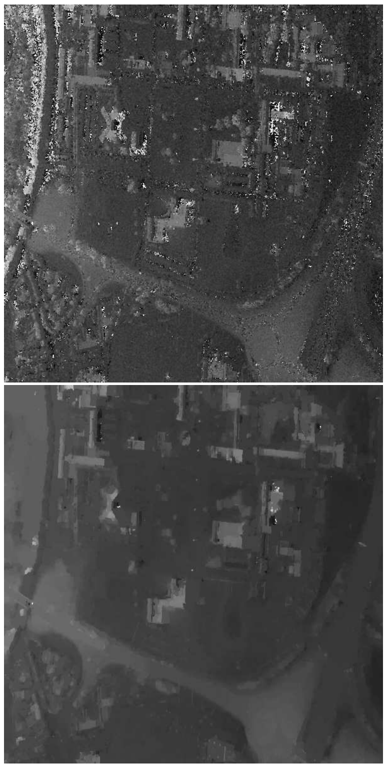

Fig. 1. Joint regularization of InSAR images (1200×1200 pixels): above noisy phase, below jointly regularized phase.

5. CONCLUSION

Speckle noise can be effectively reduced in SAR images with a Markov Random Field approach. TV minimization results in smoothed homogeneous regions while preserving sharp transitions. The Markovian formulation provides a convenient way to incorporate priors and to perform joint regularization. We have shown on real data that this can help to prevent over-regularization effects of objects that are visible in different images (such as amplitude and interferometric phase). Moreover, the contours of the jointly regularized images are more precise as all information is merged.

The quality of the results could be improved for 3D urban modeling by introducing more evolved prior knowledge in combination with contextual interpretation of the urban scene. The MRF model is flexible enough to incorporate higher level prior models. Including radar geometric deformations compensation in the regularization process could be an interesting step toward successful use of the regularized images.

6. REFERENCES

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