TerraSAR-X Data Feature Extraction: a Complex-Valued Data Analysis

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***Abstract*—The work present a complex-valued Gauss-Markov Random Field (GMRF) model for modeling and analyzing Synthetic Aperture Radar (SAR) data.**
The model is based on the extension of the classical GMRF model [1] to complex domain. It is developed under the point of view that radar signals are complex-valued signals. The model parameters are estimated in a Bayesian frame by *Maximum A Posteriori* estimate and are characterizing the image content on the pixel vicinity. The model order allows to characterize different scale structure by the neighborhood system of cliques. The conventional method are not using complex-valued data for texture characterization but they are considering the data transformation from Cartesian to polar coordinates. The results are presented on TerraSAR-X data.

I. Introduction

TerraSAR-X satellite is providing a huge amount of High Resolution (HR) Synthetic Aperture Radar (SAR) data, which have to be processed and interpreted. To work in the direction of automatic interpretation is a difficult task involving many different scientific fields: information and communication theory, statistics, machine learning, etc. The automatic interpretation spans through a signal-based analysis in order to characterize the signal and, further, the image content. This part of the work is pointed to an analysis of the TerraSAR-X data and the validation of the model for feature extraction [2], [3]. The method is in the field of spectral parameter analysis. We want to find a robust parameter representation of the signal by estimating the autocorrelation sequence. The extracted feature, i.e. primitive features, are representative of the signal characteristics and, thus, descriptors of the scene content. By the primitive feature or, incidentally, by their combinations we want to characterize the texture and more extensively the objects and the structures contained in the image for further indexing. Including this functionalities in image information mining systems will help the automatic interpretation of SAR image which is a difficult task. We propose a novel GMRF for complex valued SAR image analysis and parameter estimation. The goal is to find robust parameters to model the data. The model is bidimensional and fits the data directly in the complex domain, where classical approaches [4] use to non-linarly transform the data from cartesian to polar coordinates, as show in Fig. 1, because of the statistics of the speckle [5].
In the following, we describe briefly the concepts and the methodologies and the adopted Gauss-Markov Random Field (GMRF) model. Thus, we present the results and we end with some observations on the possible applications and perspectives.

II. Complex-Valued Random Variables

We consider a n-dimensional vector of complex-valued random variable $\mathbf{z} = \mathbf{x} + j\mathbf{y}$ of elements $\mathrm{z}_i = \mathrm{x}_i + j\mathrm{y}_i$ with $i = 1, 2, \ldots, n$ and covariance matrix

$$\mathbf{\Sigma_z} = \mathrm{E}\Big[(\mathbf{z} - \boldsymbol{\mu}_{\mathrm{z}})(\mathbf{z} - \boldsymbol{\mu}_{\mathrm{z}})^H\Big] \tag{1}$$

where $\mathrm{E}\big[\,\cdot\,\big]$ is the expectation, $\cdot^H$ is the Hermitian operator (i.e. transpose conjugate operator) and $\boldsymbol{\mu}_{\mathrm{z}} = \boldsymbol{\mu}_{\mathrm{x}} + j\boldsymbol{\mu}_{\mathrm{y}}$ is the vector of the mean values.
In general, $\mathbf{z}$ can be represented as a $2n$-dimensional vector of real-valued random variables based on the isomorphism between $\mathbb{C}^n$ and $\mathbb{R}^{2n}$. On the othar hand, the $2n \times 2n$ real-valued covariance matrix representation has not the simmetric form necessary for the isomorphism with the complex-valued covariance matrix. A block $\mathbf{M}_{ik}$ of the real-valued covariance matrix has the following form

$$\mathbf{M}_{ik} = \begin{bmatrix} \mathrm{E}\Big[(\mathrm{x}_i - \mu_{\mathrm{x}_i})(\mathrm{x}_k - \mu_{\mathrm{x}_k})\Big] & \mathrm{E}\Big[(\mathrm{x}_i - \mu_{\mathrm{x}_i})(\mathrm{y}_k - \mu_{\mathrm{y}_k})\Big] \\ -\mathrm{E}\Big[(\mathrm{y}_i - \mu_{\mathrm{y}_i})(\mathrm{x}_k - \mu_{\mathrm{x}_k})\Big] & \mathrm{E}\Big[(\mathrm{y}_i - \mu_{\mathrm{y}_i})(\mathrm{y}_k - \mu_{\mathrm{y}_k})\Big] \end{bmatrix} \tag{2}$$

where the indices i and j denote two elements of the vector $\mathbf{z}$. The global statistical information of a n-dimensional zero-mean complex-valued random variable vector $\mathbf{z} - \boldsymbol{\mu}_{\mathrm{z}}$ is contained in the $2n \times 2n$ real-valued covariance matrix, because it is taking in consideration the set of $2n$ random variables.
Under certain conditions, that we are going to describe forward all the statistical information is contained in $\mathbf{\Sigma_z}$.
The SAR signal is the complex envelope of a zero-mean band-limited Gaussian process. In case of target the mean value can be always subtracted in order to comply with this hypothesis without loose of generality.

Thus, considering $\boldsymbol{\mu}_{\mathrm{z}} = 0$, for any couple $\mathrm{z}_i = \mathrm{x}_i + j\mathrm{y}_i$ and $\mathrm{z}_j = \mathrm{x}_j + j\mathrm{y}_j$, the following hypothesis are true:

$$\mathrm{E}\big[\mathrm{x}_i^2\big] = \mathrm{E}\big[\mathrm{y}_i^2\big] = \sigma_i^2 \tag{3}$$

$$\mathrm{E}\big[\mathrm{x}_i\mathrm{y}_i\big] = 0 \tag{4}$$

$$\mathrm{E}\big[\mathrm{x}_i\mathrm{x}_k\big] = \mathrm{E}\big[\mathrm{y}_i\mathrm{y}_k\big] = \mathrm{r}_{ik}\sigma_i\sigma_k \qquad i \neq k \tag{5}$$

$$\mathrm{E}\big[\mathrm{x}_i\mathrm{y}_k\big] = -\mathrm{E}\big[\mathrm{y}_i\mathrm{x}_k\big] = \mathrm{s}_{ik}\sigma_i\sigma_k \qquad i \neq k. \tag{6}$$

The receiver extracts the components in phase and quadrature from the received signal, thus, the complex amplitude for the two channel is the same (3). The two components in quadrature and phase are orthogonal and, thus, uncorrelated (4). The reciprocity and the properties of symmetry of the Fourier transform ensure (5) and (6) for the band-limited signal: the Wiener-Kintchine theorem states that the autocorrelation function is the inverse Fourier transform of the power spectral density, the Fourier transform of a real signal gives a complex signal with a even real part (5) and a odd imaginary part (6). The above conditions, (3), (4), (5) and (6), ensure the isomorphism between the complex-valued covariance matrix and the real-valued representation, thus the blocks of the complex-valued covariance matrix become

$$\mathbf{M}_{ii} = \sigma_i^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

$$\mathbf{M}_{ik} = \sigma_i\sigma_k \begin{bmatrix} \mathrm{r}_{ik} & \mathrm{s}_{ik} \\ -\mathrm{s}_{ik} & \mathrm{r}_{ik} \end{bmatrix} \qquad i \neq k \tag{8}$$

The elements of the complex covariance matrix are $m_{ii} = 2\sigma^2$ and $m_{ik} = 2\sigma_i\sigma_k(\mathrm{r}_{ik} - j\mathrm{s}_{ik})$ where $i \neq k$.
Thus, the expression of the n-dimensional complex-valued multivariate Gaussian can be derived from the $2n$-dimensional real-valued multivariate Gaussian and take the form

$$p(\mathbf{z}) = \frac{1}{\pi^n \det \mathbf{\Sigma}} \exp\left\{-\mathbf{z} \quad \mathbf{M}^{-1}\mathbf{z}^H\right\} \tag{9}$$

where $\det(\cdot)$ is the determinant operator. In the next section we are going briefly to present the classical Bayesian frame, and, thus, our proposed complex-valued GMRF model.

III. Gauss-Markov Random Field Model

Considering the consideration of the previous section and [6],the Gauss-Markov Random Field (GMRF) model for complex valued pixels has the following form:

$$p(\mathrm{z}_s|\mathrm{z}_{s+r}, r \in \mathcal{N}) = \frac{1}{2\pi\sigma^2} \exp\left\{ - \frac{\left(\begin{bmatrix} \mathrm{x}_s \\ \mathrm{y}_s \end{bmatrix} - \mu \right)^2}{2\sigma^2} \right\} \tag{10}$$

$$\mu = \sum_{r\in\mathcal{N}} \begin{bmatrix} \xi_r & -\tau_r \\ \tau_r & \xi_r \end{bmatrix} \begin{bmatrix} \mathrm{x}_{r+s} \\ \mathrm{y}_{r+s} \end{bmatrix} \tag{11}$$

where $\mathrm{z} = \mathrm{x} + j\mathrm{y}$ is the complex valued pixel, $\theta = \xi + j\tau$ is the complex-valued parameter vector, σ^2 is the model variance and the sum is over all the pixel r belonging to the vicinity neighbor $\mathcal{N}$. In the analysis we follow the Bayesian inference to respect the considered model H_i:

$$p(\boldsymbol{\theta}|\mathrm{z}_s, H_i) = \frac{p(\mathrm{z}_s|\boldsymbol{\theta}, H_i)p(\boldsymbol{\theta}|H_i)}{p(\mathrm{z}_s|H_i)} \tag{12}$$

where $p(\mathrm{z}_s|\boldsymbol{\theta})$ is the GMRF model, $p(\boldsymbol{\theta})$ is the parameter prior which is uniform and $p(\mathrm{z}_s)$ is the model evidence and it is constant at this level of inference. We want to find the values of the parameter vector which maximize the previous expression, thus it takes the form of the Maximum a Posteriori (MAP) estimate:

$$\hat{\boldsymbol{\theta}}_{MAP} = \arg\max_{\boldsymbol{\theta}}\{\log p(\mathrm{z}_s|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})\} \tag{13}$$

As further measure we compute the evidence of the model which is a quantitative measure of how well the model is fitting the data. It is computed as follow:

$$p(\mathrm{z}_s|H_i) = \int_\Theta p(\mathrm{z}_s|\boldsymbol{\theta}, H_i)p(\boldsymbol{\theta}|H_i)\mathrm{d}\boldsymbol{\theta} \tag{14}$$

where the integration is all over the parameter space Ω and the integral can be computed analytically. The variance of the model:

$$\sigma^2 = \mathrm{E}\left\{(\mathbf{z} - \mathbf{G}\hat{\boldsymbol{\theta}})^2\right\} \tag{15}$$

is the expectation of the difference between the data and the best fit of the model over the data, $\mathbf{G}$ is the matrix of cliques.

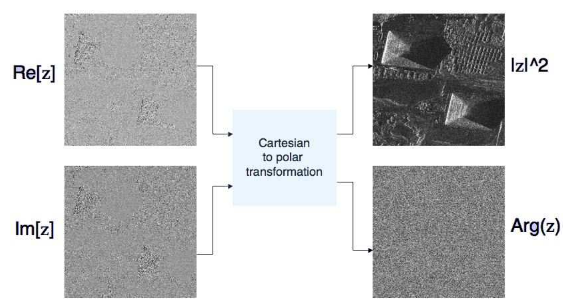
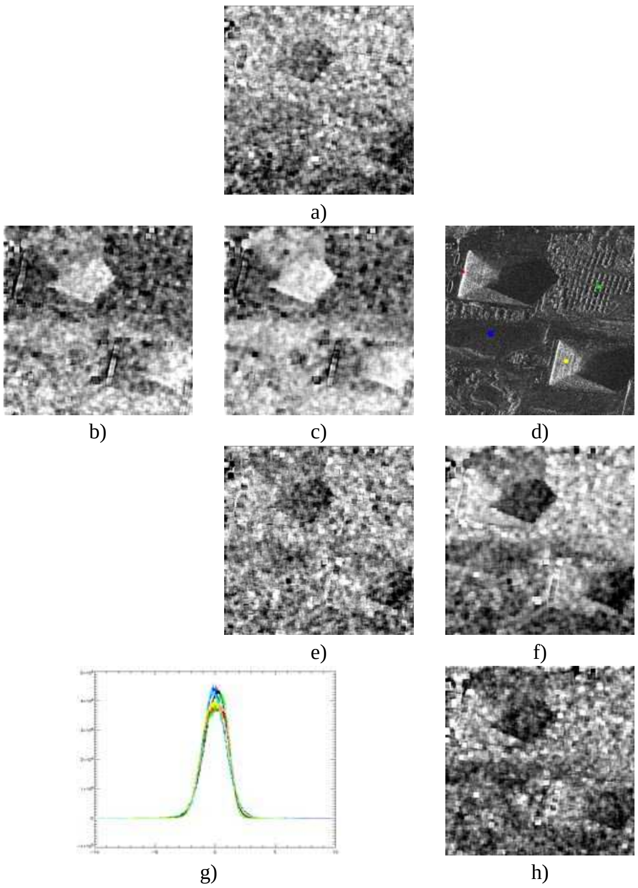


Fig. 1. Example of SAR image and transformation from Cartesian to polar coordinates.

Fig. 2. Spot Light HR SLC image of El Geeza, Egypt. Polarization HH and incident angle ~ 53. Azimuth resolution ~ 1.10 m and slant range resolution ~ 0.59 m

TABLE I

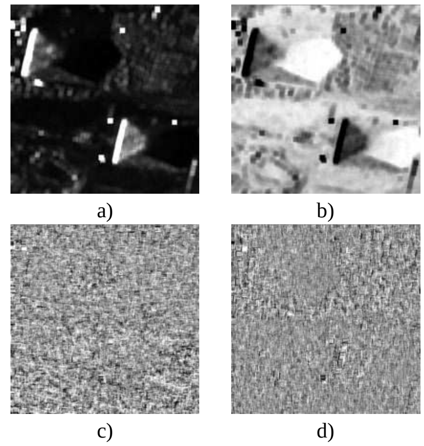
ABSOLUTE VALUES OF THE COMPLEX-VALUED ESTIMATED PARAMETERS AND ORIGINAL IMAGE IN FIG. D). THE POSITIONS ARE REPRESENTATIVE OF THE CLIQUE IN THE NEIGHBORHOOD, I.E. THE DIRECTION TO WHICH THE PARAMETER REFERS. IN FIG. G) THE PLOT OF THE REAL PART OF THE PARAMETERS IS SHOWN TO BE GAUSSIAN.



a) b) c) d) e) f) g) h)

TABLE II

FROM TOP LEFT CLOCKWISE: VARIANCE OF THE MODEL, EVIDENCE OF THE MODEL, PHASE OF THE VERTICAL CLIQUE AND PHASE OF THE HORIZONTAL CLIQUE. THE MODEL VARIANCE IS PROPORTIONAL TO THE RADAR BACKSCATTER. THE EVIDENCE IS HIGHER WHERE THE MODEL FITS BETTER THE DATA. THE PHASE OF THE VERTICAL AND HORIZONTAL ARE SHOWING THE CHARACTERISTICS OF A GRADIENT.



a) b) c) d)

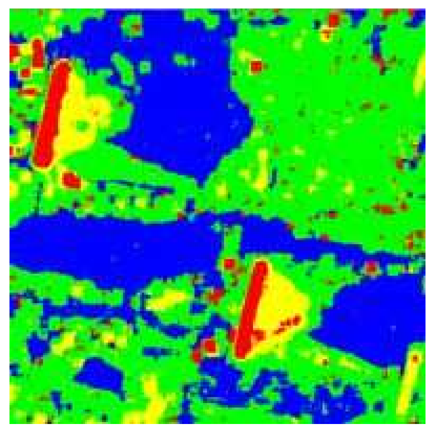


Fig. 3. Maximum Likelihood (ML) feature based classification. The training set is shown in I: Fig. d). Red: bright; yellow: medium bright; green: texture; blue: dark.

IV. RESULTS

We processed an image from a scene acquired over El Geeza, Fig. 2. The image is Single Look Complex (SLC) acquired in High Resolution (HR) Spot Light (SL) mode, polarization HH and incident angle ~ 53. Azimuth resolution ~ 1.10 m and slant range resolution ~ 0.59 m.
The absolute values of the complex-valued estimated parameters are shown in Tab. I, together with the original image Tab. I: Fig. d). The positions are representative of the clique in the neighborhood, i.e. the direction to which the parameter refers. The histograms of the absolute value of the zero-mean parameter are shown on the top right: they have all the same Gaussian shape.
The original image, Tab. I: Fig. d), shows the selected training set for the Maximum Likelihood (ML) feature based classification, shown in Fig. 3. The classes are choosen in order to corrispond to the visual appearence of the image: red: bright; yellow: medium bright; green: texture; blue: dark areas.
The textured areas are not well separated and there are many false alarms.
In Tab. II, from top left clockwise, the variance of the model, the evidence of the model, the phase of the vertical clique and the phase of the horizontal clique are shown. The model variance, Tab. II: Fig. a), is proportional to the radar backscatter. The evidence, Tab. II: Fig. b) is higher where the model fits better the data. The phase of the vertical and horizontal in Tab. II Figs. c) and d) are showing the characteristics of a gradient. Different structures and image characteristics can be captured by different model orders, i.e. a different number of parameters. On the other hand a greater model order increases the computation time.

V. CONCLUSIONS

A complex GMRF model for texture and structure modeling can be used for automatic detection and classification of textured and structured areas, man made or natural target. It can help analysts in the interpretation of complex SAR images.

The model shows the limit given by an Analyzing Window (AW) based estimation and analysis: to have a sufficient number of pixels a large AW is needed, on the other hand a small AW would adapt better the data heterogeneity. The analysis and the model has to be refined by introducing edges and strong scatterers detection. Further, the effectiveness of data transformations or data pre-processing, e.g. to reduce the dynamic of the signal or to reduce the noise, has to be investigate in order to find the optimal data space. Study and analysis of alternative data space or data transformation, e.g. complex multilooking for noise reduction. Study and analysis of the parameter feature space. Further analysis in the feature space, e.g. singular value decomposition to reduce space dimensions. Distortion-based validation of the estimated parameters. Mutual information based analysis to measure the information hidden in the phase. Extension of the model on multi-band polarimetric data. On the other hand, the next research step is going to be the inclusion of the model in a full Bayesian approach as data prior and considering the SAR image formation model [7]. Further the comparison of the model with non-linear Huber-Markov model, applied in [8] for edge preserving.

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