A Semi-analytical Method to Model Effective SINR Spatial Distribution in WiMAX Networks

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Abstract—The stationary probabilities of different modulation and coding schemes (MCS) are required for dimensioning an OFDMA based network. In this paper, we introduce a semi-analytical approach to find out these stationary probabilities for a WiMAX network in downlink (DL) with users served by the best base station (BS). Using Monte Carlo simulations, we find the spatial distributions of effective SINR for different values of cell range. With the help of distribution fit, we show that GEV distributions can be used as a good fit for different frequency reuse schemes. Furthermore, by applying curve fitting, we demonstrate that the parameters of GEV distributions, as a function of $\sigma_{SH}$ values, can be expressed using polynomials. These polynomials can then be used off-line (in place of time consuming simulations) to find out effective SINR cumulative distribution function (CDF), and hence the stationary probabilities of MCS, for any desired value of $\sigma_{SH}$. We further show that these polynomials can be used for other cell configurations with acceptable deviation and significant time saving.

Keywords: OFDMA, PUSC, IEEE 802.16e, WiMAX, SINR_{eff}, MIC, best base station.

I. INTRODUCTION

WiMAX, a broadband wireless access technology, is based on IEEE standard 802.16-2005 [1]. Physical layer of WiMAX is characterized by orthogonal frequency division multiple access (OFDMA). With OFDM, available spectrum is split into a number of parallel orthogonal narrowband subcarriers. These subcarriers are grouped together to form subchannels. The distribution of subcarriers to subchannels is done using three major permutation methods called: partial usage of subchannels (PUSC), full usage of subchannels (FUSC) and adaptive modulation and coding (AMC). The subcarriers in a subchannel for first two methods are distributed throughout the available spectrum while these are contiguous in case of AMC.

A slot, the basic and minimum resource unit of a WiMAX system, occupies place both in time (OFDM symbols) and frequency (subchannel) domains thus introducing both the time and frequency multiple access. One of the important features of IEEE 802.16 based network is assignment of MCS type to a user depending upon its $SINR_{eff}$ (cf. section III-B) value. Though the number of subcarriers possessed by a slot is fixed (i.e., forty eight), the number of bits it can transfer depends upon the MCS type used by the user. Therefore, cell throughput depends upon the probabilities of the possible MCS types. These MCS probabilities can be used in traffic analysis to obtain network dimensioning parameters (cf. section II). Since each MCS type is characterized by a $SINR_{eff}$ threshold value, we require CDF of $SINR_{eff}$ spatial distribution. Therefore an efficient way to obtain this CDF is always desired.

The study of SINR statistics in cellular environment is not recent. For examples, analytical/semi-analytical modeling of interference for mobile radio networks employing code division multiple access (CDMA) is given in [2]–[4]. However, the analysis carried out with single carrier in the physical layer can not be applied to multi carrier OFDMA based networks since the latter offers frequency diversity. System level simulations (SLS) have been used in [5]–[10] to find out percentage of MCS for an IEEE 802.16 based networks. The drawback of methods based purely on simulations is the excessive time consumption. In [11], an analytical method to calculate MCS probabilities and hence throughput in AMC mode of WiMAX has been proposed. However, the analysis does not take into account the shadowing effect. The authors of [12] present a semi-analytical method to calculate outage probabilities in OFDMA network (with no consideration of WiMAX specifications). In [13], an analytical calculation of symbol error rate for different MCS types is presented. To calculate symbol error, authors have not taken $SINR_{eff}$ into account. In short, a method is required by which modeling of $SINR_{eff}$ statistics in WiMAX networks can be carried out more efficiently.

In this paper, we propose a semi-analytical method to find out stationary probabilities of different MCS types for a mobile WiMAX network that can substitute a number of simulations. We start with Monte Carlo simulations and find out spatial distributions of $SINR_{eff}$ for some integral values of $\sigma_{SH}$. It is shown that the probability density functions (PDF) of $SINR_{eff}$ can be approximated by GEV distributions [14]. We exhibit that GEV distributions’ parameters can be expressed in terms of $\sigma_{SH}$ using polynomial. Instead of simulations, these polynomials can then be used to find out GEV distribution, and hence MCS probabilities, for any desired value of $\sigma_{SH}$ in the above range. Furthermore, we demonstrate the applicability of the polynomials for different values of cell range $R$ and BS transmission power $P_{Tx}$ and discuss the time efficiency offered.

Rest of the paper is organized as follows: section II gives a

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brief description of network dimensioning study and relative
details of IEEE 802.16e system. In section III, $SINR_{eff}$ com-
putation is discussed. The proposed semi-analytical method
is described in section IV. Numerical results are presented
in section V and finally section VI discusses the conclusion
of this paper.

II. NETWORK DIMENSIONING

The study of network dimensioning for mobile WiMAX net-
works can be divided into different components. As shown
in Fig. 1, we classify it into three components: Radio Coverage,
Throughput Calculation and Traffic Analysis. The work carried
out in this paper focuses on Radio Coverage and Throughput
Calculation blocks.

A. Radio Coverage

The input parameters to this block are: channel model,
network model and WiMAX configuration. These parameters
are mainly based on [15]. The output of this block is CDF of
$SINR_{eff}$ which can be obtained through Monte Carlo simu-
lations. The disadvantage of simulation approach is excessive
time consumption. In this paper, we intend to substitute the
simulation approach by a semi-analytical method.

We have considered distributed subcarrier permutation type
PUSC in our simulations. The analysis equally holds for sub-
carrier permutation type FUSC. Because of space limitation,
we only discuss reuse type 3x3x3 (shown in Fig. 2) in this
paper. However, method was also verified for five other reuse
types: 1x1x1, 1x3x1, 1x3x3, 3x1x1 and 3x3x1 and results can
be referred from [16]. The above mentioned six reuse types
have been proposed in [15] for WiMAX networks.

B. Throughput Calculation

Once the Radio Coverage block furnishes the CDF of
$SINR_{eff}$, we require thresholds values of different MCS
types to calculate MCS probabilities. Six different MCS types
have been considered in our simulation model: QPSK-1/2
(the most robust), QPSK-3/4, 16QAM-1/2, 64QAM-2/3 and
64QAM-3/4 (for the best radio conditions). $SINR_{eff}$ thresh-
old values for MCS types are given in Tab. I and have been
referred from [17]. If $SINR_{eff}$ of a mobile station (MS) is
less than the threshold of the most robust MCS (i.e., less than
2.9 dB), it can neither receive nor transmit anything and is
said to be in outage. We call outage as MCS type 0.

Using the probabilities of MCS, the average cell throughput
$X$ [bps] in DL is given as:

$$X = \frac{N_S}{T_F} \sum_{k=1}^{K} m_k p_k, \quad (1)$$

where $K$ represents the total number of considered MCS
types. The other two variables, $p_k$ and $m_k$, are respectively the
probability and bits per slot for MCS type $k$, $N_S$ is the number
of slots in DL sub-frame in a cell (i.e., per three sectors) and
$T_F$ is the duration of TDD (time division duplex) frame.

Total bandwidth in our simulator has been set to 10 MHz.
The number of OFDM symbols in a WiMAX TDD frame
is considered to be 47 [15]. We assume two symbols fixed
for common channel transmissions. The rest of 45 symbols
are partitioned between DL and uplink sub-frames with DL
part assuming two third of the symbols. Considering the sym-
bol/bandwidth information, reuse type 3x3x3 and permutation
type PUSC, there are $N_S = 450 \times 1/3$ slots in DL sub-frame,
where 1/3 appears because of reuse type.

C. Traffic Analysis

From Throughput Calculation, we get the available sector/cell throughput. However, utilization of this throughput
depends upon the scheduling of different types of incoming
traffic. A number of traffic types, characterized by application
and QoS specifications, are defined in [15]. Unsolicited grant
service (UGS) and best effort (BE) are two examples of these
traffic types.

To carry out traffic analysis, MCS probabilities are required
from Throughput Calculation block. In [18], we have exhibited
how the MCS probabilities are utilized in traffic analysis.

Before the semi-analytical method is presented, we discuss
the interference model used in the simulations.

<table>
<thead>
<tr>
<th>Index</th>
<th>MCS</th>
<th>bits per slot $m_k$</th>
<th>$SINR_{eff}$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Outage</td>
<td>0</td>
<td>&lt; 2.9</td>
</tr>
<tr>
<td>1</td>
<td>QPSK 1/2</td>
<td>48</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>QPSK 3/4</td>
<td>72</td>
<td>6.3</td>
</tr>
<tr>
<td>3</td>
<td>16QAM 1/2</td>
<td>96</td>
<td>8.6</td>
</tr>
<tr>
<td>4</td>
<td>16QAM 3/4</td>
<td>144</td>
<td>12.7</td>
</tr>
<tr>
<td>5</td>
<td>64QAM 2/3</td>
<td>192</td>
<td>16.9</td>
</tr>
<tr>
<td>6</td>
<td>64QAM 3/4</td>
<td>216</td>
<td>18</td>
</tr>
</tbody>
</table>

TABLE I

THRESHOLD OF $SINR_{eff}$ VALUES FOR SIX MCS TYPES [17].
A. Subcarrier SINR

SINR of a subcarrier $n$ is given as:

$$\text{SINR}_n = \frac{P_{n,Tx} a_{n,SH}^{(0)} a_{n,FF}^{(0)} K}{N_0 W_{Sc} + \sum_{b=1}^{B} P_{n,Tx} a_{n,SH}^{(b)} a_{n,FF}^{(b)} K},$$

where $P_{n,Tx}$ is the per subcarrier power, $a_{n,SH}^{(0)}$ and $a_{n,FF}^{(0)}$ represent the shadowing (log-normal) and fast fading (Rayleigh) factors for the signal received from serving BS respectively, $B$ is the number of interfering BS, $K$ is the path loss constant, $\alpha$ is the path loss exponent and $d^{(0)}$ is the distance between MS and serving BS. The terms with superscript $0$ are related to interfering BS. $W_{Sc}$ is the subcarrier frequency spacing and $N_0$ is the thermal noise density. The values of pathloss constant and exponent are derived from COST231 Hata macro-urban path loss model [15].

B. Effective SINR

We compute $\text{SINR}_{\text{eff}}$ over the subcarriers of a slot. The physical abstraction model used for this purpose is mean instantaneous capacity (MIC) [15]. For computation of $\text{SINR}_{\text{eff}}$, log-normal shadowing is drawn randomly for a slot and is same for all subcarriers of a slot. Since subcarriers of a subchannel (hence a slot) are not contiguous, fast fading is drawn independently for every subcarrier of a slot (Fig. 3). For fast fading, Rayleigh distribution has been considered in simulations.

IV. SEMI-ANALYTICAL METHOD

A systematic overview of the proposed semi-analytical method is depicted in Fig. 4. The method is divided into two steps: A) Simulations and Distribution/Curve Fitting and B) Off-line Application. In the following text, these steps are explained in detail.

A. Simulations and Distribution/Curve Fitting

During this step, spatial distributions of $\text{SINR}_{\text{eff}}$ is obtained using Monte Carlo simulations for a given value of $R$, $P_{Tx}$ and a specified range of $\sigma_{SH}$ integral values. The parameters (mainly based on [15]) can be found in Tab. II. The details of simulator can be found in [16].

Each distribution of $\text{SINR}_{\text{eff}}$ is specific to a value of $\sigma_{SH}$. With the help of distribution fit (based on maximum likelihood estimation), the parameters of GEV distribution

<table>
<thead>
<tr>
<th>TABLE II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PARAMETERS AND DETAILS OF SIMULATIONS.</strong></td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Reuse type</td>
</tr>
<tr>
<td>No. of interfering BS</td>
</tr>
<tr>
<td>Spatial distribution of MS</td>
</tr>
<tr>
<td>Number of MS dropped per sector</td>
</tr>
<tr>
<td>Number of snapshots</td>
</tr>
<tr>
<td>Carrier frequency $f_c$</td>
</tr>
<tr>
<td>Subcarrier spacing $\Delta f$</td>
</tr>
<tr>
<td>TDD frame duration</td>
</tr>
<tr>
<td>Thermal noise density $N_0$</td>
</tr>
<tr>
<td>Shadowing standard deviation $\sigma_{SH}$</td>
</tr>
<tr>
<td>Height of BS $h_{BS}$</td>
</tr>
<tr>
<td>Height of MS $h_{MS}$</td>
</tr>
<tr>
<td>Front-to-back power ratio $G_{FB}$</td>
</tr>
<tr>
<td>3-dB beamwidth $\psi_{3dB}$</td>
</tr>
</tbody>
</table>

Fig. 4. Overview of proposed semi-analytical method.

(1) Using Maximum Likelihood Estimation (MLE)

(2) Using least square method

Since the area under a PDF is 1, the maximum value of error can be 2. Hence the value of error can be between 0 and 2 i.e., $0 \leq \Xi \leq 2$.

Once it is verified that simulation PDFs of $\text{SINR}_{\text{eff}}$ can be approximated by GEV PDFs, three GEV parameters are then separately plotted against the integral values of $\sigma_{SH}$. With the help of curve fitting (using least square method), distinct polynomials, expressing each parameter in terms of $\sigma_{SH}$, are found.

B. Off-line Application

To calculate $\text{SINR}_{\text{eff}}$ distribution for any desired value (integral/non-integral) of $\sigma_{SH}$ in the range specified in section IV-A, we no longer require to carry out time consuming Monte Carlo simulations. In the following text, these steps are explained in detail.

A. Simulations and Distribution/Curve Fitting

During this step, spatial distributions of $\text{SINR}_{\text{eff}}$ is obtained using Monte Carlo simulations for a given value of $R$, $P_{Tx}$ and a specified range of $\sigma_{SH}$ integral values. The parameters (mainly based on [15]) can be found in Tab. II. The details of simulator can be found in [16].

Each distribution of $\text{SINR}_{\text{eff}}$ is specific to a value of $\sigma_{SH}$. With the help of distribution fit (based on maximum likelihood estimation), the parameters of GEV distribution...
Carlo simulations. It is sufficient to find out GEV parameters through polynomials for that value of $\sigma_{SH}$. Then using GEV CDF and thresholds values of $SINR_{eff}$ for different MCS types of Tab. I, probabilities of these MCS can be obtained. These MCS probabilities are used to calculate sector/cell throughput by applying Eq. 1. In section V, we also show that results obtained through this method are applicable for various values of $R$ and $P_{Tx}$.

V. NUMERICAL RESULTS

In this section, we present the numerical results. For Monte Carlo simulations, range of $\sigma_{SH}$ is considered to be 4.5, ..., 12. Other input parameters are $R = 1500$ m and $P_{Tx} = 43$ dBm. An $SINR_{eff}$ distribution is obtained for each value of $\sigma_{SH}$. Using distribution fitting, GEV parameters are determined for each of these distributions. As an example, in Fig. 5, approximation of $SINR_{eff}$ PDF (obtained through simulation) by a GEV PDF for $\sigma_{SH} = 9$ dB is shown. As can be noticed, the two distributions only have a dissimilarity error of 0.052 which is 2.6% of the maximum possible error.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig5.png}
\caption{$SINR_{eff}$ distribution through simulation and GEV polynomial for $\sigma_{SH} = 9$ dB, $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.}
\end{figure}

GEV parameters, obtained through distribution fitting, are separately plotted against $\sigma_{SH}$ values in Figs. 6, 7 and 8. With the help of curve fitting, polynomials of the curves approximating these plots are found and are also given in the figures. As can be noted in the figures that the degree of all polynomials never exceeds four. These polynomials can instantaneously give values of GEV parameters for any value of $\sigma_{SH}$.

To validate off-line application (cf. section IV-B), we choose an arbitrary value $\sigma_{SH} = 7.5$ dB. We calculate the GEV parameters through polynomials and get PDF, MCS probabilities and cell throughput. For the same value of $\sigma_{SH}$ and assuming the values of $R = 1500$ m, $P_{Tx} = 43$ dBm, we find the PDFs, MCS probabilities and cell throughput through simulations. Furthermore, we also check the applicability of results obtained through GEV parameters, with $\sigma_{SH} = 7.5$ dB, for various cell configurations. For this purpose, we fix $\sigma_{SH} = 7.5$ dB and carry out simulations for different values of $R$ and $P_{Tx}$. The maximum value of $R$ is considered to be 2000 m beyond which outage probability increases rapidly [20]. PDFs, MCS probabilities and average cell throughput are obtained through simulations with different configurations are compared with those obtained through GEV parameters.

The results of validation and applicability for various cell configurations are given in Fig. 9 and Tab. III. For MCS probabilities, maximum difference was found to be 0.06 (for MCS 64QAM-3/4) with simulation configuration of $R = 1000$ m, $P_{Tx} = 43$ dBm, which is 13% of the value of MCS 64QAM-3/4 probability. As far as cell throughput and PDF error are concerned, the percentage error w.r.t maximum possible error never exceeds 5% and cell throughput does not differ more than 5.47% for all cell configurations.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6.png}
\caption{Shape parameter $\xi$ of GEV distribution versus $\sigma_{SH}$ for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig7.png}
\caption{Scale parameter $\sigma$ of GEV distribution versus $\sigma_{SH}$ for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig8.png}
\caption{Location parameter $\mu$ of GEV distribution versus $\sigma_{SH}$ for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.}
\end{figure}
TABLE III

COMPARISON OF RESULTS OBTAINED THROUGH SIMULATION AND GEV PARAMETERS FOR σ_{SH} = 7.5 dB.

<table>
<thead>
<tr>
<th>Simulation Configuration</th>
<th>Dissimilarity Ξ</th>
<th>Percentage w.r.t max error</th>
<th>Throughput X [Mbps]</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTx [dBm]</td>
<td>Rx [m]</td>
<td></td>
<td></td>
<td>Sim</td>
</tr>
<tr>
<td>43</td>
<td>1000</td>
<td>0.045</td>
<td>4.73</td>
<td>5</td>
</tr>
<tr>
<td>43</td>
<td>1250</td>
<td>0.074</td>
<td>3.65</td>
<td>4.96</td>
</tr>
<tr>
<td>43</td>
<td>1500</td>
<td>0.056</td>
<td>2.83</td>
<td>4.88</td>
</tr>
<tr>
<td>43</td>
<td>1750</td>
<td>0.028</td>
<td>2.92</td>
<td>4.78</td>
</tr>
<tr>
<td>43</td>
<td>2000</td>
<td>0.1</td>
<td>5</td>
<td>4.66</td>
</tr>
<tr>
<td>40</td>
<td>1500</td>
<td>0.065</td>
<td>3.27</td>
<td>4.75</td>
</tr>
<tr>
<td>46</td>
<td>1500</td>
<td>0.017</td>
<td>3.17</td>
<td>4.96</td>
</tr>
</tbody>
</table>

The simulations were run on a computer with the following specifications: 3 GHz Intel Core 2 Duo processor, 2 GB RAM, and 4 MB shared L2 cache. Time taken by one Monte Carlo simulation was about 5 hours. Time required for semi-analytical method is around NSH × 5 hours, where NSH is the length of vector σ_{SH}. If MCS distributions are required for N different scenarios (each defined by specific values of σ_{SH}, R and PTx), our proposed method always requires fixed duration which is equal to NSH × 5 hours while the same task carried out by Monte Carlo simulations will require N × 5 hours.

VI. CONCLUSION

In this paper, we have proposed a semi-analytical method to model SINR statistics in mobile WiMAX cellular networks. We have shown that SINR_{eff} distribution, obtained through system level Monte Carlo simulations, can be successfully approximated by a GEV distribution. It is further illustrated that the parameters of GEV distribution can be expressed using simple polynomials in terms of σ_{SH}. These polynomials can be used to calculate the GEV parameters for any desired value of σ_{SH}. These parameters can be used to estimate SINR_{eff} distribution and hence the MCS stationary probabilities. The results can be used for a number of network configurations with sufficient accuracy. As a result, we no longer require exhaustive simulations to derive distribution of SINR_{eff}.

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REFERENCES


Fig. 9. MCS Probabilities for σ_{SH} = 7.5 dB and reuse 3x3x3.