A Semi-analytical Method to Model Effective SINR Spatial Distribution in WiMAX Networks

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***Abstract*—The stationary probabilities of different modulation and coding schemes (MCS) are required for dimensioning an OFDMA based network. In this paper, we introduce a semi-analytical approach to find out these stationary probabilities for a WiMAX network in downlink (DL) with users served by the best base station (BS). Using Monte Carlo simulations, we find the spatial distributions of effective signal to interference-plus-noise ratio ($SINR_{eff}$) for different values of shadowing standard deviation (σ_{SH}). With the help of distribution fit, we show that generalized extreme value (GEV) distribution provides a good fit for different frequency reuse schemes. Furthermore, by applying curve fitting, we demonstrate that the parameters of GEV distributions, as a function of σ_{SH} values, can be expressed using polynomials. These polynomial can then be used off-line (in place of time consuming simulations) to find out GEV cumulative distribution function (CDF), and hence the stationary probabilities of MCS, for any desired value of σ_{SH}. We further show that these polynomials can be used for other cell configurations with acceptable deviation and significant time saving.**

Keywords: OFDMA, PUSC, IEEE 802.16e, WiMAX, $SINR_{eff}$, MIC, best base station.

I. Introduction

WiMAX, a broadband wireless access technology, is based on IEEE standard 802.16-2005 [1]. Physical layer of WiMAX is characterized by orthogonal frequency division multiple access (OFDMA). With OFDM, available spectrum is split into a number of parallel orthogonal narrowband subcarriers. These subcarriers are grouped together to form subchannels. The distribution of subcarriers to subchannels is done using three major permutation methods called: partial usage of subchannels (PUSC), full usage of subchannels (FUSC) and adaptive modulation and coding (AMC). The subcarriers in a subchannel for first two methods are distributed throughout the available spectrum while these are contiguous in case of AMC.

A slot, the basic and minimum resource unit of a WiMAX system, occupies place both in time (OFDM symbols) and frequency (subchannel) domains thus introducing both the time and frequency multiple access. One of the important features of IEEE 802.16 based network is assignment of MCS type to a user depending upon its $SINR_{eff}$ (cf. section III-B) value. Though the number of subcarriers possessed by a slot is fixed (i.e., forty eight), the number of bits it can transfer depends upon the MCS type used by the user. Therefore, cell throughput depends upon the probabilities of the possible MCS types. These MCS probabilities can be used in traffic analysis to obtain network dimensioning parameters (cf. section II). Since each MCS type is characterized by a $SINR_{eff}$ threshold value, we require CDF of $SINR_{eff}$ spatial distribution. Therefore an efficient way to obtain this CDF is always desired.

The study of SINR statistics in cellular environment is not recent. For examples, analytical/semi-analytical modeling of interference for mobile radio networks employing code division multiple access (CDMA) is given in [2]–[4]. However, the analysis carried out with single carrier in the physical layer can not be applied to multi carrier OFDMA based networks since the latter offers frequency diversity. System level simulations (SLS) have been used in [5]–[10] to find out percentage of MCS for an IEEE 802.16 based networks. The drawback of methods based purely on simulations is the excessive time consumption. In [11], an analytical method to calculate MCS probabilities and hence throughput in AMC mode of WiMAX has been proposed. However, the analysis does not take into account the shadowing effect. The authors of [12] present a semi-analytical method to calculate outage probabilities in OFDMA network (with no consideration of WiMAX specifications). In [13], an analytical calculation of symbol error rate for different MCS types is presented. To calculate symbol error, authors have not taken $SINR_{eff}$ into account. In short, a method is required by which modeling of $SINR_{eff}$ statistics in WiMAX networks can be carried out more efficiently.

In this paper, we propose a semi-analytical method to find out stationary probabilities of different MCS types for a mobile WiMAX network that can substitute a number of simulations. We start with Monte Carlo simulations and find out spatial distributions of $SINR_{eff}$ for some integral values σ_{SH}. It is shown that the probability density functions (PDF) of $SINR_{eff}$ can be approximated by GEV distributions [14]. We exhibit that GEV distributions' parameters can be expressed in terms of σ_{SH} using polynomial. Instead of simulations, these polynomials can then be used to find out GEV distribution, and hence MCS probabilities, for any desired value of σ_{SH} in the above range. Furthermore, we demonstrate the applicability of these polynomials for different values of cell range R and BS transmission power P_{Tx} and discuss the time efficiency offered.

Rest of the paper is organized as follows: section II gives a

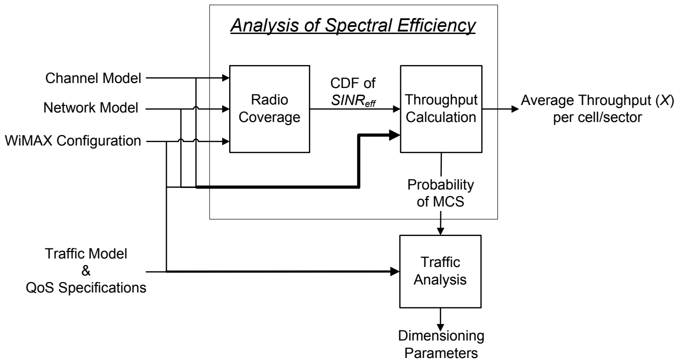

Fig. 1. System overview.

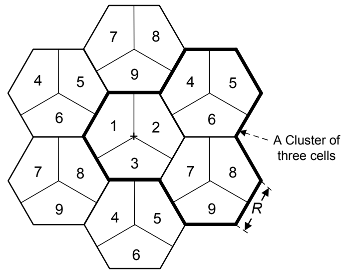

Fig. 2. Frequency Reuse Pattern 3x3x3 (R is the cell range).

brief description of network dimensioning study and relative details of IEEE 802.16e system. In section III, $SINR_{eff}$ computation is discussed. The proposed semi-analytical method is described in section IV. Numerical results are presented in section V and finally section VI discusses the conclusion of this paper.

II. Network Dimensioning

The study of network dimensioning for mobile WiMAX networks can be divided into different components. As shown in Fig. 1, we classify it into three components: *Radio Coverage*, *Throughput Calculation* and *Traffic Analysis*. The work carried out in this paper focuses on *Radio Coverage* and *Throughput Calculation* blocks.

A. Radio Coverage

The input parameters to this block are: channel model, network model and WiMAX configuration. These parameters are mainly based on [15]. The output of this block is CDF of $SINR_{eff}$ which can be obtained through Monte Carlo simulations. The disadvantage of simulation approach is excessive time consumption. In this paper, we intend to substitute the simulation approach by a semi-analytical method.

We have considered distributed subcarrier permutation type PUSC in our simulations. The analysis equally holds for subcarrier permutation type FUSC. Because of space limitation, we only discuss reuse type 3x3x3 (shown in Fig. 2) in this paper. However, method was also verified for five other reuse types: 1x1x1, 1x3x1, 1x3x3, 3x1x1 and 3x3x1 and results can be referred from [16]. The above mentioned six reuse types have been proposed in [15] for WiMAX networks.

TABLE I
Threshold of $SINR_{eff}$ values for six MCS types [17].

Index	MCS	bits per slot m_k	$\mathbf{SINR_{eff}}$ [dB]
0	Outage	0	< 2.9
1	QPSK 1/2	48	2.9
2	QPSK 3/4	72	6.3
3	16QAM 1/2	96	8.6
4	16QAM 3/4	144	12.7
5	64QAM 2/3	192	16.9
6	64QAM 3/4	216	18

B. Throughput Calculation

Once the *Radio Coverage* block furnishes the CDF of $SINR_{eff}$, we require thresholds values of different MCS types to calculate MCS probabilities. Six different MCS types have been considered in our simulation model: QPSK-1/2 (the most robust), QPSK-3/4, 16QAM-1/2, 64QAM-2/3 and 64QAM-3/4 (for the best radio conditions). $SINR_{eff}$ threshold values for MCS types are given in Tab. I and have been referred from [17]. If $SINR_{eff}$ of a mobile station (MS) is less than the threshold of the most robust MCS (i.e., less than 2.9 dB), it can neither receive nor transmit anything and is said to be in outage. We call outage as MCS type 0.

Using the probabilities of MCS, the average cell throughput X [bps] in DL is given as:

$$X = \frac{N_S}{T_F} \sum_{k=1}^{K} m_k p_k, \tag{1}$$

where K represents the total number of considered MCS types. The other two variables, p_k and m_k, are respectively the probability and bits per slot for MCS type k, N_S is the number of slots in DL sub-frame in a cell (i.e., per three sectors) and T_F is the duration of TDD (time division duplex) frame.

Total bandwidth in our simulator has been set to 10 MHz. The number of OFDM symbols in a WiMAX TDD frame is considered to be 47 [15]. We assume two symbols fixed for common channel transmissions. The rest of 45 symbols are partitioned between DL and uplink sub-frames with DL part assuming two third of the symbols. Considering the symbol/bandwidth information, reuse type 3x3x3 and permutation type PUSC, there are $N_S = 450 \times 1/3$ slots in DL sub-frame, where $1/3$ appears because of reuse type.

C. Traffic Analysis

From *Throughput Calculation*, we get the available sector/cell throughput. However, utilization of this throughput depends upon the scheduling of different types of incoming traffic. A number of traffic types, characterized by application and QoS specifications, are defined in [15]. Unsolicited grant service (UGS) and best effort (BE) are two examples of these traffic types.

To carry out traffic analysis, MCS probabilities are required from *Throughput Calculation* block. In [18], we have exhibited how the MCS probabilities are utilized in traffic analysis.

Before the semi-analytical method is presented, we discuss the interference model used in the simulations.

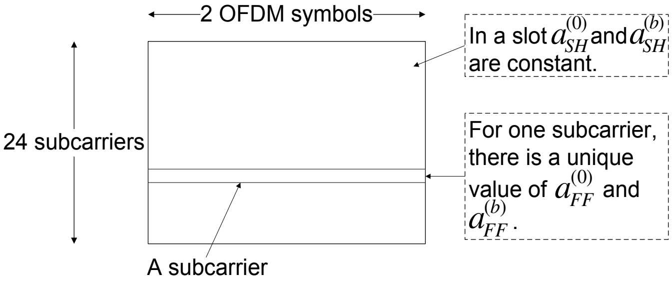

Fig. 3. Shadowing and fast fading over a PUSC slot.

III. Interference Model

A. Subcarrier SINR

SINR of a subcarrier n is given as:

$$SINR_n = \frac{P_{n,Tx} a_{n,Sh}^{(0)} a_{n,FF}^{(0)} \frac{K}{d^{(0)\alpha}}}{N_0 W_{Sc} + \sum_{b=1}^{B} P_{n,Tx} a_{n,Sh}^{(b)} a_{n,FF}^{(b)} \frac{K}{d^{(b)\alpha}}}, \quad (2)$$

where $P_{n,Tx}$ is the per subcarrier power, $a_{n,Sh}^{(0)}$ and $a_{n,FF}^{(0)}$ represent the shadowing (log-normal) and fast fading (Rayleigh) factors for the signal received from serving BS respectively, B is the number of interfering BS, K is the path loss constant, α is the path loss exponent and $d^{(0)}$ is the distance between MS and serving BS. The terms with superscript b are related to interfering BS. W_{Sc} is the subcarrier frequency spacing and N_0 is the thermal noise density. The values of pathloss constant and exponent are derived from COST231 Hata macro-urban path loss model [15].

B. Effective SINR

We compute $SINR_{eff}$ over the subcarriers of a slot. The physical abstraction model used for this purpose is mean instantaneous capacaity (MIC) [15]. For computation of $SINR_{eff}$, log-normal shadowing is drawn randomly for a slot and is same for all subcarriers of a slot. Since subcarriers of a subchannel (hence a slot) are not contiguous, fast fading is drawn independently for every subcarrier of a slot (Fig. 3). For fast fading, Rayleigh distribution has been considered in simulations.

IV. Semi-analytical Method

A systematic overview of the proposed semi-analytical method is depicted in Fig. 4. The method is divided into two steps: A) Simulations and Distribution/Curve Fitting and B) Off-line Application. In the following text, these steps are explained in detail.

A. Simulations and Distribution/Curve Fitting

During this step, spatial distributions of $SINR_{eff}$ is obtained using Monte Carlo simulations for a given value of R, P_{Tx} and a specified range of σ_{SH} integral values. The parameters (mainly based on [15]) can be found in Tab. II. The details of simulator can be found in [16].

Each distribution of $SINR_{eff}$ is specific to a value of σ_{SH}. With the help of distribution fit (based on maximum likelihood estimation), the parameters of GEV distribution

TABLE II
Parameters and details of simulations.

Parameter	Value
Reuse type	3x3x3
No. of interfering BS	18 using wraparound technique
Spatial distribution of MS	Uniform random
Number of MS dropped per sector	1
Number of snapshots	10000
Carrier frequency f_c	2.5 GHz
Subcarrier spacing $\triangle f$	10.9375 kHz
TDD frame duration	5 ms
Thermal noise density N_0	-174 dBm/Hz
Shadowing standard deviation σ_{SH}	8.9 dB
Height of BS h_{BS}	32 m
Height of MS h_{MS}	1.5 m
Antenna beam pattern 3GPP2 $G(\psi)$, where ψ is the angle MS subtends with sector boresight	$G_{max} + \max\left[-12\left(\frac{\psi}{\psi_{3dB}}\right)^2, -G_{FB}\right]$
Antenna Gain (boresight) G_{max}	16 dBi
Front-to-back power ratio G_{FB}	25 dB
3-dB beamwidth ψ_{3dB}	$70°$

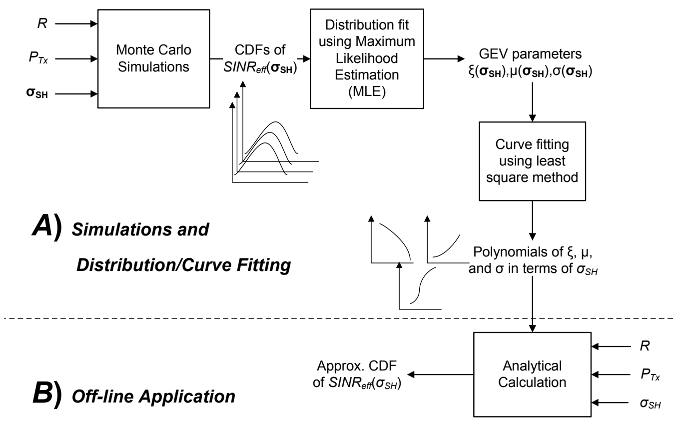

Fig. 4. Overview of proposed semi-analytical method.

(shape parameter ξ, scale parameter σ and location parameter μ), approximating the simulation PDFs, are acquired for each value of σ_{SH}.

In order to evaluate the distribution fit, the dissimilarity or error Ξ between GEV and simulation PDFs, φ_{GEV} and φ_{sim}, is quantified as follows [19]:

$$\Xi \triangleq \int_{-\infty}^{\infty} |\varphi_{GEV}(t) - \varphi_{sim}(t)|\, dt. \quad (3)$$

Since the area under a PDF is 1, the maximum value of error can be 2. Hence the value of error can be between 0 and 2 i.e., $0 \leq \Xi \leq 2$.

Once it is verified that simulation PDFs of $SINR_{eff}$ can be approximated by GEV PDFs, three GEV parameters are then separately plotted against the integral values of σ_{SH}. With the help of curve fitting (using least square method), distinct polynomials, expressing each parameter in terms of σ_{SH}, are found.

B. Off-line Application

To calculate $SINR_{eff}$ distribution for any desired value (integral/non-integral) of σ_{SH} in the range specified in section IV-A, we no longer require to carry out time consuming Monte

Carlo simulations. It is sufficient to find out GEV parameters through polynomials for that value of σ_{SH}. Then using GEV CDF and thresholds values of $SINR_{eff}$ for different MCS types of Tab. I, probabilities of these MCS can be obtained. These MCS probabilities are used to calculate sector/cell throughput by applying Eq. 1. In section V, we also show that results obtained through this method are applicable for various values of R and P_{Tx}.

V. Numerical Results

In this section, we present the numerical results. For Monte Carlo simulations, range of σ_{SH} is considered to be $4, 5, ..., 12$. Other input parameters are $R = 1500$ m and $P_{Tx} = 43$ dBm. An $SINR_{eff}$ distribution is obtained for each value of σ_{SH}. Using distribution fitting, GEV parameters are determined for each of these distributions. As an example, in Fig. 5, approximation of $SINR_{eff}$ PDF (obtained through simulation) by a GEV PDF for $\sigma_{SH} = 9$ dB is shown. As can be noticed, the two distributions only have a dissimilarity error of 0.052 which is 2.6% of the maximum possible error.

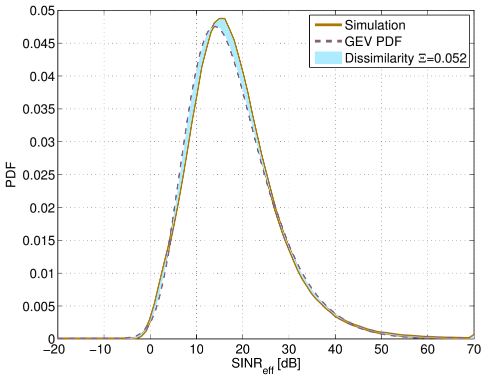

Fig. 5. $SINR_{eff}$ distribution through simulation and GEV polynomial for $\sigma_{SH} = 9$ dB, $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

GEV parameters, obtained though distribution fitting, are separately plotted against σ_{SH} values in Figs. 6, 7 and 8. With the help of curve fitting, polynomials of the curves approximating these plots are found and are also given in the figures. As can be noted in the figures that the degree of all polynomials never exceeds four. These polynomials can instantaneously give values of GEV parameters for any value of σ_{SH}.

To validate off-line application (cf. section IV-B), we choose an arbitrary value $\sigma_{SH} = 7.5$ dB. We calculate the GEV parameters through polynomials and get PDF, MCS probabilities and cell throughput. For the same value of σ_{SH} and assuming the values of $R = 1500$ m, $P_{Tx} = 43$ dBm, we find the PDFs, MCS probabilities and cell throughput through simulations. Furthermore, we also check the applicability of results obtained through GEV parameters, with $\sigma_{SH} = 7.5$ dB, for various cell configurations. For this purpose, we fix $\sigma_{SH} = 7.5$ dB and carry out simulations for different values of R and P_{Tx}. The maximum value of R is considered to be 2000 m beyond which outage probability increases rapidly [20]. PDFs, MCS probabilities and average cell throughput are obtained

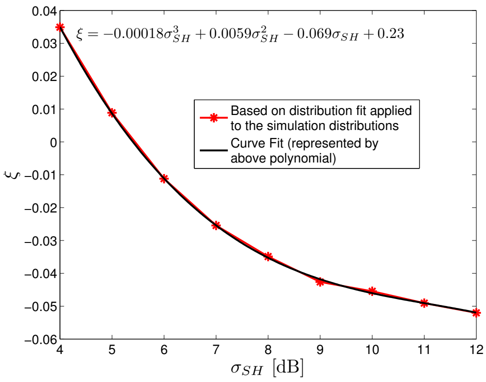

Fig. 6. Shape parameter ξ of GEV distribution versus σ_{SH} for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

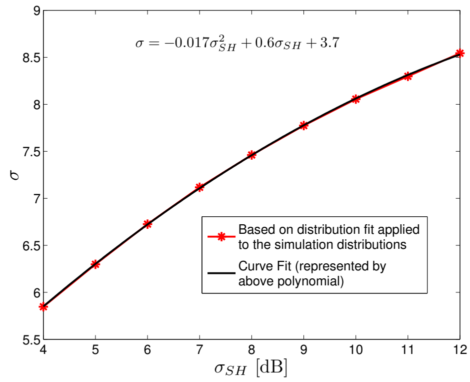

Fig. 7. Scale parameter σ of GEV distribution versus σ_{SH} for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

through simulations with different configurations are compared with those obtained through GEV parameters.

The results of validation and applicability for various cell configurations are given in Fig. 9 and Tab. III. For MCS probabilities, maximum difference was found to be 0.06 (for MCS 64QAM-3/4) with simulation configuration of $R = 1000$ m, $P_{Tx} = 43$ dBm, which is 13% of the value of MCS 64QAM-3/4 probability. As far as cell throughput and PDF error are concerned, the percentage error w.r.t maximum possible error never exceeds 5% and cell throughput does not differ more than 5.47% for all cell configurations.

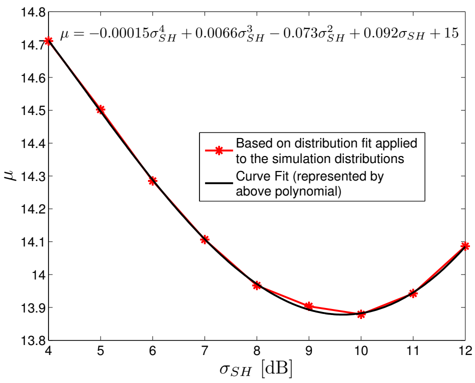

Fig. 8. Location parameter μ of GEV distribution versus σ_{SH} for $R = 1500$ m, $P_{Tx} = 43$ dBm and reuse 3x3x3.

TABLE III

COMPARISON OF RESULTS OBTAINED THROUGH SIMULATION AND GEV PARAMETERS FOR $\sigma_{SH} = 7.5$ dB.

Simulation Configuration		**Dissimilarity Ξ**	**Percentage w.r.t max error**	**Throughput X [Mbps]**		**Percentage difference**
$\mathbf{P_{Tx}}$ [dBm]	$\mathbf{R}$ [m]			**Sim**	**GEV**	
43	1000	0.095	4.73	5	4.73	5.47
43	1250	0.073	3.65	4.96	4.73	4.57
43	1500	0.056	2.83	4.88	4.73	3.17
43	1750	0.058	2.92	4.78	4.73	1.18
43	2000	0.1	5	4.66	4.73	1.35
40	1500	0.065	3.27	4.75	4.73	0.49
46	1500	0.075	3.77	4.96	4.73	4.72

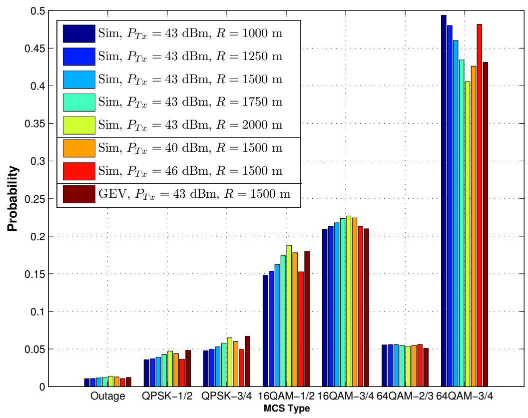

Fig. 9. MCS Probabilities for $\sigma_{SH} = 7.5$ dB and reuse 3x3x3.

The simulations were run on a computer with following specifications: 3 GHz Intel Core 2 Duo processor, 2 GB RAM and 4 MB shared L2 cache. Time taken by one Monte Carlo simulation was about 5 hours. Time required for semi-analytical method is around $N_{SH} \times 5$ hours, where N_{SH} is the length of vector $\sigma_{\mathbf{SH}}$. If MCS distributions are required for N different scenarios (each defined by specific values of σ_{SH}, R and P_{Tx}), our proposed method always requires fixed duration which is equal to $N_{SH} \times 5$ hours while the same task carried out by Monte Carlo simulations will require $N \times 5$ hours.

VI. CONCLUSION

In this paper, we have proposed a semi-analytical method to model SINR statistics in mobile WiMAX cellular networks. We have shown that $SINR_{eff}$ distribution, obtained through system level Monte Carlo simulations, can be successfully approximated by a GEV distribution. It is further illustrated that the parameters of GEV distribution can be expressed using simple polynomials in terms of σ_{SH}. These polynomials can be used to calculate the GEV parameters for any desired value of σ_{SH}. These parameters can be used to estimate $SINR_{eff}$ distribution and hence the MCS stationary probabilities. The results can be used for a number of network configurations with sufficient accuracy. As a result, we no longer require exhaustive simulations to derive distribution of $SINR_{eff}$.

ACKNOWLEDGMENT

This work was partially sponsored by Alcatel-Lucent Bell Labs France.

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