

Cell Breathing, Sectorization and Densification in Cellular Networks

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Abstract—In this paper, we establish a closed form formula of the other-cell interference factor f for omni-directional and sectored cellular networks. That formula is based on a fluid model that approximates the discrete base stations (BS) entities by a continuum of transmitters which are spatially distributed in the network. Simulations show that the obtained closed-form formula is a very good approximation, even for the traditional hexagonal network. From f , we are able to derive the outage probability on the downlink as a function of the mobile density and the coverage range. From a maximum acceptable outage probability, we can deduce the link between cell coverage and mobile density and thus highlight with a new, easy and fast method the notion of cell breathing. At last, we show how an operator can use this approach in order to evaluate the impact of sectorization or BS densification on the cell coverage.

I. INTRODUCTION

In cellular systems, the geographical area covered by a base station is not constant. The size of a cell depends on the traffic managed by the base station: a cell shrinks when its load increases. Indeed, the higher is the number of users in the cell, the higher is the interference. As a consequence the lower the cell radius becomes. This is the phenomenon of cell breathing. This behaviour is particularly pronounced in CDMA networks and increases the complexity of network design. When coverage holes appear, the operator can for example sectorize existing omni-directional sites or densify the BS.

The estimation of cellular networks capacity mainly depends on the characterization of interference. An important parameter for this characterization is the other-cell interference factor f (OCIF). The knowledge of the OCIF allows the derivation of outage probabilities, capacity evaluation and then, the definition of Call Admission Control mechanisms. The impact of cell breathing on the coverage of CDMA cells was studied [5]. It was shown that under very specific deployment conditions where one cell is heavily loaded and is surrounded by lightly loaded cells, cell breathing may provide a small increase in capacity. The authors of [1] propose a power control algorithm to guarantee services for the area of a cell when the load increases. It allows to reduce the transmitted power. As a consequence, cell coverage is constant. In paper [6] cell-breathing effect and near-far unfair access problem are analyzed. Authors of [2] propose an algorithm, which provides capacity increase in heterogeneous CDMA/TDMA systems by controlling the cell-breathing, so that the interference level is reduced. The analysis proposed in [3] presents a cell breathing concept in cellular WLAN that performs load balancing.

Access points can dynamically adjust their transmission power in the aim to reduce the cell load. The paper [4] presents a load balancing technique by controlling the size of WLAN cells similar to cell breathing in cellular networks.

In this paper, we define OCIF as the ratio of total other-cell received power to the total inner-cell received power and we focus on the downlink. On this link, [7] [8] aimed at computing an average OCIF over the cell by numerical integration in hexagonal networks. In [10], other-cell interference is given as a function of the distance to the BS thanks to Monte-Carlo simulations. Chan and Hanly [12] precisely approximate the distribution of the other-cell interference. They however provide formulas that are difficult to handle in practice. Considering random networks, Baccelli et al. [16] provide spatial blocking probabilities in random networks by using gaussian approximation. In contrast to previous works in the field, the modelling key of our approach is to consider the discrete BS entities of a cellular network as a continuum.

This paper extends to sectored networks the framework proposed in [13] and [14] that provides a simple closed-form formula for f on the downlink as a function of the distance to the BS, the path-loss exponent, the distance between two BS, and the network size. We validate here the formulas by Monte Carlo simulations. We moreover generalize to sectored networks the analysis developed in [15], and show that it is possible to get a simple outage probability approximation by integrating f .

The outage probability is a function of the coverage range of a cell and the mobile density. Thus, setting a maximum admissible outage probability allows us to find a relation between the coverage range of a BS and the mobile density. With this relation, we are able to quickly analyze the effects of cell breathing, sectorization and densification. We also derive the optimal beam width of the sectored antennas.

In the next section, we introduce the interference model and notations. In section III, we provide the closed-form formula for the interference factor and the outage probability for both omni-directional and sectored networks. In section IV, we establish the relation between cell coverage and mobile density, we show how an operator could use our formulas to evaluate the impact of sectorization and densification.

II. INTERFERENCE MODEL AND NOTATIONS

We consider a cellular radio system and we focus on the downlink. BS have omni-directional antennas, so that a BS

covers a single cell. If a mobile u is attached to a station b (or serving BS), we write $b = \psi(u)$. The following power quantities are considered:

- $P_{b,u}$ is the transmitted power from station b towards mobile u (for user's traffic);
- $P_b = P_{cch} + \sum_u P_{b,u}$ is the total power transmitted by station b . In CDMA systems, P_{cch} represents the amount of power used to support broadcast and common control channels.
- $p_{b,u}$ is the power received at mobile u from station b ; we can write $p_{b,u} = P_b g_{b,u}$; $g_{b,u}$ designates the pathloss between station b and mobile u
- $S_{b,u} = P_{b,u} g_{b,u}$ is the useful power received at mobile u from station b (for traffic data); since we do not consider soft hand over, we can write $S_u = S_{\psi(u),u}$. [13] [14]

The total amount of power experienced by a mobile station u in a cellular system can be split up into several terms: useful signal (S_u), interference and noise (N_{th}). It is common to split the system power into two terms: $I_u = P_{int,u} + P_{ext,u}$, where $P_{int,u}$ is the *internal* (or own-cell) received power and $P_{ext,u}$ is the *external* (or other-cell) interference. Notice that we made the choice of including the useful signal S_u in $P_{int,u}$, and, as a consequence, it has to be distinguished from the commonly considered own-cell interference.

With the above notations, we define the interference factor in u , as the ratio of total power received from other BS to the total power received from the serving BS $\psi(u)$:

$$f_u = P_{ext,u}/P_{int,u}. \quad (1)$$

The quantities f_u , $P_{ext,u}$, and $P_{int,u}$ are location dependent and can thus be defined in any location x as long as the serving BS is known. In downlink, a coefficient α , may be introduced to account for the lack of perfect orthogonality in the own cell.

In this paper, we will use the signal to interference plus noise ratio (SINR) as the criteria of radio quality: γ_u is the SINR evaluated at MS u and γ_u^* is the SINR target for the service requested by MS u . This figure is a priori different from the SINR evaluated at mobile station u . However, we assume perfect power control, so $SINR = \gamma_u^*$ for all users.

A. CDMA system

With the introduced notations, the SINR experimented by u can thus be derived (see e.g. [9]):

$$\gamma_u^* = \frac{S_u}{\alpha(P_{int,u} - S_u) + P_{ext,u} + N_{th}} \quad (2)$$

From this relation, we can express the transmitted power for MS u , $P_{b,u} = S_u/g_{b,u}$, using relations $P_{int,u} = P_b g_{b,u}$ and $f_u = P_{ext,u}/P_{int,u}$ as:

$$P_{b,u} = \frac{\gamma_u^*}{1 + \alpha\gamma_u^*} (\alpha P_b + f_u P_b + N_{th}/g_{b,u}). \quad (3)$$

From this relation, the output power of BS b can be computed as follows:

$$P_b = P_{cch} + \sum_u P_{b,u}, \quad (4)$$

and so, according to Eq.3,

$$P_b = \frac{P_{cch} + \sum_u \frac{\gamma_u^*}{1 + \alpha\gamma_u^*} \frac{N_{th}}{g_{b,u}}}{1 - \sum_u \frac{\gamma_u^*}{1 + \alpha\gamma_u^*} (\alpha + f_u)}. \quad (5)$$

B. OFDMA system

In OFDMA, the data are multiplexed over a great number of subcarriers. There is no internal interference, so we can consider that $\alpha(P_{int,u} - S_u) = 0$. Since $P_{ext,u} = \sum_{j \neq b} P_j g_{j,u}$, the expression (2) can be written,

$$\gamma_u = \frac{P_{b,u} g_{b,u}}{\sum_{j \neq b} P_j g_{j,u} + N_{th}}, \quad (6)$$

so we have

$$\gamma_u = \frac{1}{f_u + \frac{N_{th}}{P_{b,u} g_{b,u}}}. \quad (7)$$

Since $\frac{N_{th}}{P_{b,u} g_{b,u}} \ll f_u$, typically for cell radii less than about 1 km, we can neglect this term and write

$$\gamma_u = \frac{1}{f_u}. \quad (8)$$

In cellular networks, coverage and capacity are closely related through the notion of cell breathing: the higher the number of mobiles in a cell, the smaller the cell radius. In order to analyze this effect, we analytically express the outage probability on the downlink as a function of the cell radius for both omni-directional and sectored sites. This analysis is based on the fluid model, presented in the next section.

III. FLUID MODEL

In this section, we first present the model, derive the closed-form formula for f_u for both omni-directional and sectored sites, and provide the outage probability on the downlink. The model is validated through Monte-Carlo simulations in a hexagonal network. We will denote $f_u = f_0$ for omnidirectional sites and $f_u = f_{sec}$ for sectored sites.

The key modelling step of the model we propose consists in replacing a given fixed finite number of BS by an equivalent continuum of transmitters which are spatially distributed in the network. This means that the transmitting power is now considered as a continuum field all over the network. In this context, the network is characterised by a MS density ρ and a base station density ρ_{BS} [13]. We assume that MS and BS are uniformly distributed in the network, so that ρ and ρ_{BS} are constant. As the network is homogeneous, all base stations have the same output power P_b .

We focus on a given cell and consider a round shaped network around this centre cell with radius R_{nw} . The half distance between two base stations is R_c (see Figure 1).

A. Omni-directional sites

For the assumed omni-directional BS network, we use a propagation model, where the path gain, $g_{b,u}$, only depends on the distance r between the BS b and the MS u . The power, $p_{b,u}$, received by a mobile at distance r_u can thus be written

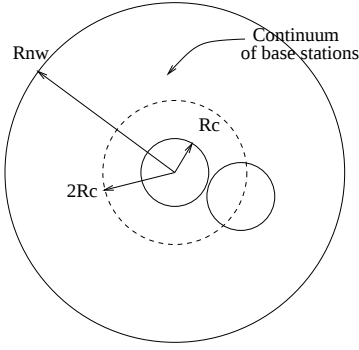


Fig. 1. Network and cell of interest in the fluid model; the distance between two BS is $2R_c$ and the network is made of a continuum of base stations.

$p_{b,u} = P_b K r_u^{-\eta}$, where K is a constant and $\eta > 2$ is the path-loss exponent.

Let's consider a mobile u at a distance r_u from its serving BS. Each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{BS}zdzd\theta$ base stations which contribute to $P_{ext,u}$. Their contribution to the external interference is $\rho_{BS}zdzd\theta P_b K z^{-\eta}$. We approximate the integration surface by a ring with centre u , inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$.

$$\begin{aligned} P_{ext,u} &= \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS} P_b K z^{-\eta} z dz d\theta \quad (9) \\ &= \frac{2\pi \rho_{BS} P_b K}{\eta - 2} [(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta}]. \end{aligned}$$

Moreover, MS u receives internal power from b , which is at distance r_u : $P_{int,u} = P_b K r_u^{-\eta}$. So, the interference factor $f_u = P_{ext,u}/P_{int,u}$ can be expressed by:

$$f_u = \frac{2\pi \rho_{BS} r_u^\eta}{\eta - 2} [(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta}]. \quad (10)$$

Note that f_u does not depend on the BS output power. This is due to the fact that we assumed an homogeneous network and so all base stations emit the same power. In our model, f only depends on the distance r to the BS and can be defined in each location, so that we can write f as a function of r , $f_0(r)$. Thus, if the network is large, i.e. R_{nw} is big in front of R_c , f_u can be further approximated by:

$$f_0(r) = \frac{2\pi \rho_{BS} r^\eta}{\eta - 2} (2R_c - r)^{2-\eta}. \quad (11)$$

This closed-form formula will allow us to fastly compute performance parameters of a cellular radio network. We notice that formula can be applied to *any wireless system* (TDMA, OFDMA) as long as ρ_{BS} represents the density of *interfering transmitting nodes*. However, before going ahead, we need to validate the different approximations we made in this model.

In this perspective, we compare the figures obtained with Eq. 10 with those obtained numerically by simulations. The simulator assumes an homogeneous hexagonal network. Figure 2 shows an example of such a network with the main

parameters involved in the study. The validation is done numerically by computing f in each point of the cell and averaging the values at a given distance from the BS. Figure 3 shows the simulated interference factor as a function of the distance to the base station. Simulation parameters are the following: $R = 1$ Km, η between 2.7 and 4, $\rho_{BS} = (3\sqrt{3}R^2/2)^{-1}$, the number of snapshots is 1000. Transmit power P_b and parameter K do not play any role in the simulation since thermal noise is neglected and all BS have the same output power (homogeneous network). Eq. 10 is also plotted for comparison. In all cases, the fluid model matches very well the simulations on an hexagonal network for various figures of the path-loss exponent. It allows calculating the influence of a mobile, whatever its position in a cell. The fluid model and the traditional hexagonal model are two simplifications of the reality. None is a priori better than the other but the latter is widely used, especially for dimensioning purposes. That is the reason why a comparison is useful.

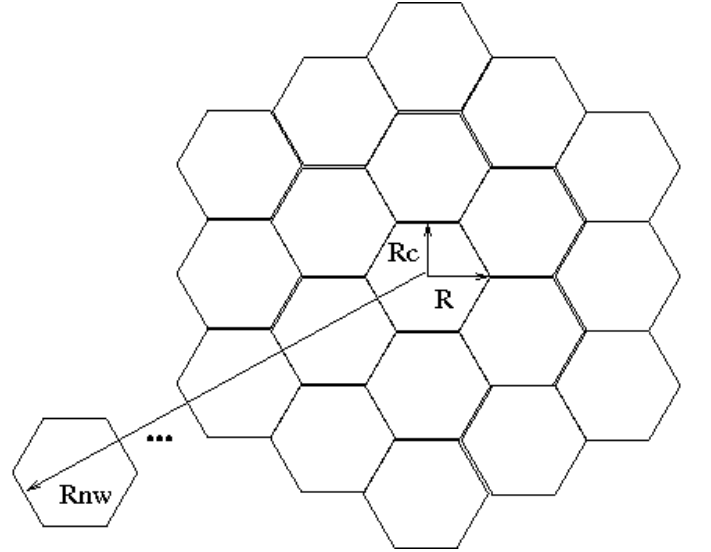


Fig. 2. Hexagonal network and main parameters of the study.

Note that the considered network size can be finite and chosen to characterize each specific local network's environment (urban or country, macro or micro cells). We notice moreover that our model can be used even for great distances between the base stations: We validated the model considering a distance of 2 Km between the BS. We conclude that our approach is accurate even for a very low base station's density.

B. Sectorized sites

We define a site as a geographical location where a base station is located. A cell is the area covered by a BS. An omni-directional site has one BS and one cell. A q -sectorized site has q BS with directional antenna and q cells. In a sectorized site, a cell can be denoted a sector. We focus on $q=3$.

Figure 4 represents a three-sectorized network. Each arrow represents an antenna direction. The circles represent the zone covered by a site, and the zones S_1 , S_2 and S_3 (central site) represent the zones covered by an antenna. We consider a homogeneous sectorized network with a BS density: $\rho_{BS,3} =$

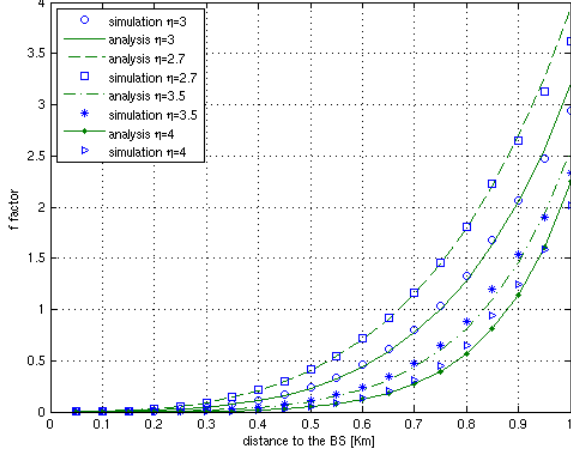


Fig. 3. Interference factor vs. distance to the BS; comparison of the fluid model with simulations on an hexagonal network with $\eta = 2.7, 3, 3.5, 4$.

$\frac{S_{omni}}{S_{sec}} \rho_{BS}$, where ρ_{BS} is the omni-directional network BS density, S_{omni} and S_{sec} are the omni-directional and sector cells surfaces.

Using now a model where the pathloss $g_{b,u}$ is a function of the location (r, θ) (between the serving base station b and the mobile u), the interference factor is a function of (r, θ) . The normalized antenna gain $G(\theta)$ represents the effect of directional antennas in sectored BS. We have $0 \leq G(\theta) \leq 1$, ($G(0) = 1$). We denote $G_s(\theta)$ the antenna gain for the sector s . For the three-sectored sites, $G(\theta)$ has an angular $\frac{2\pi}{3}$ symmetry. We can write $G_s(\theta) = G(\theta + \frac{(s-1)2\pi}{3})$.

A mobile station u belonging to the sector S_1 (see Figure 4) at the position (r, θ) receives internal power from b : $P_{int,u} = P_b K r_u^{-\eta} G_1(\theta)$. We choose the axis origin such as θ is the angle between the azimuth of the sector and the direction BS-MS (see Figure 4 central zone, sector S_1). As the network is homogeneous, all base stations have the same output power P_b . We focus on a given cell and consider a round shaped network around this centre cell with radius R_{nw} .

In a sectored network, antennas other than the serving one, but belonging to the same site contribute to interference. So, to calculate the interference factor in a given sector (sector 1, Figure 4) we have to consider the total radio power coming from the other sites of the network (denoted 1 to 18 in this case) and the power coming from the other antennas belonging to the same site (sectors S_2 and S_3).

Considering the other sites, each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{BS,3}zdzd\theta$ base stations which contribute to $P_{ext,u}$. Their contribution to the external interference is $\rho_{BS,3}zdzd\theta P_b K z^{-\eta} G(\theta)$. Like in the omni-directional case, we approximate the integration surface by a ring with centre u , inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$.

In this case, the interference factor depends on the location of the mobile. We denote it $f_{sec}(r, \theta)$. So, considering the whole network, the interference factor defined as $f_{sec} = \frac{P_{ext,u}}{P_{int,u}}$ can be expressed by:

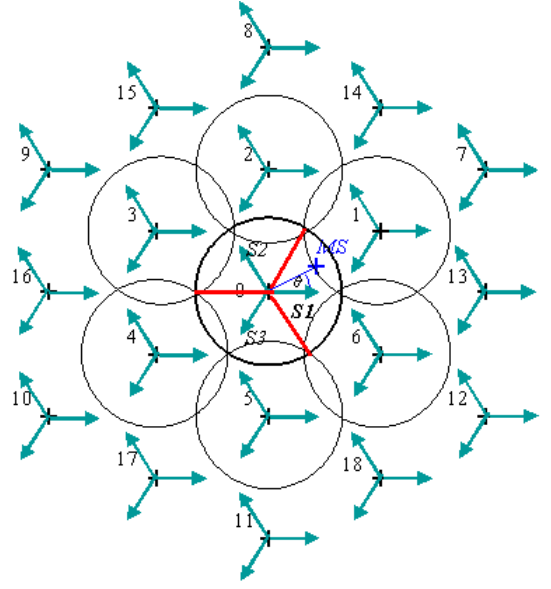


Fig. 4. Sectored network.

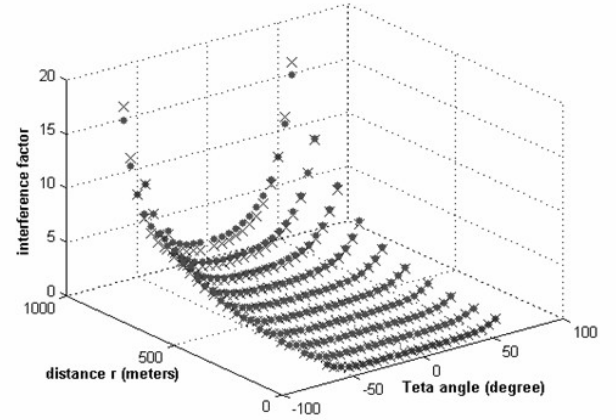


Fig. 5. Interference factor vs. r and θ : comparison between fluid model and simulations.

$$f_{sec}(r, \theta) = \frac{1}{z^{-\eta} G_1(\theta)} \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{BS,3} z^{-\eta} \times G(\theta) z dz d\theta + \frac{1}{z^{-\eta} G_1(\theta)} \sum_{j=2}^3 r_u^{-\eta} G_j(\theta). \quad (12)$$

We recognize in this expression the expression of the interference factor for omni-directional sites $f_0(r)$. Denoting moreover

$$a(\theta) = \frac{3}{2\pi} \frac{\int_0^{2\pi} G(\theta) d\theta}{G_1(\theta)} \text{ and } b(\theta) = \frac{\sum_{j=2}^3 G_j(\theta)}{G_1(\theta)} \quad (13)$$

So the expression (12) can be written as:

$$f_{sec}(r, \theta) = a(\theta) f_0(r) + b(\theta). \quad (14)$$

The interference factor in a given sector of a three-sectored network can be expressed as a *linear function of the omni-directional one*. The parameters $a(\theta)$ and $b(\theta)$ only depend on the angle, the number of sectors and the normalized antenna gain $G(\theta)$.

To validate our approach, we follow an analogue method as the one used for an omni-directional network. Considering the parameters values $\eta = 3$, $R = 1$ Km, and $R_{nw} = 5R$, and typical patterns $G(\theta)$ for antennas of beam width 60° and 65° , we compare in Figure 5 the interference factor values obtained with the fluid model and with simulations in the sectored hexagonal network. We observe that the simulated values of the interference factor (crosses) are very close to the fluid analytical ones (points), for each angle between -60° to 60° (beam width) and whatever the distances r (from 0 to 1000 m).

C. Outage probabilities

In this section, we compute the outage probability using the Gaussian approximation for both omni-directional and sectored networks. Closed-form formulas for the mean and standard deviation of f over a cell are provided.

1) *Outage probability definition*: For a given number of MS per cell, n , outage probability, $P_{out}^{(n)}$, is the proportion of configurations, for which the needed BS output power exceeds the maximum output power: $P_b > P_{max}$. If noise is neglected and if we assume a single service network ($\gamma_u^* = \gamma^*$ for all u), we deduce from Eq.5:

$$P_{out}^{(n)} = Pr \left[\sum_{u=0}^{n-1} (\alpha + f_u) > \frac{1-\varphi}{\beta} \right], \quad (15)$$

where $\varphi = P_{ch}/P_{max}$ and $\beta = \gamma^*/(1 + \alpha\gamma^*)$.

2) *Gaussian Approximation*: In order to compute these probabilities, we rely on the Central Limit theorem and use a Gaussian approximation. As a consequence, we need to compute the spatial mean, μ_f , and standard deviation, σ_f , of f_u , when mobile u is uniformly distributed over a disk of radius R_e . R_e is the radius of the zone covered by the BS. In the generic case, it is not related to R_c , the half inter-BS distance or R the cell radius (see Figure 2). Indeed, if, because of cell breathing, there are coverage holes, $R_e < R$. For a continuous (or full) coverage however, $R_e = R = 2/\sqrt{3}R_c$.

So, considering the covered disk, the outage probability can be approximated by:

$$P_{out}^{(n)} = Q \left(\frac{\frac{1-\varphi}{\beta} - n\mu_f - n\alpha}{\sqrt{n}\sigma_f} \right), \quad (16)$$

where Q is the error function. As a consequence, we need to compute the spatial mean and standard deviation of f_u .

3) *Omnidirectional Cells*: Using the expression of f_u given by the fluid model ($f_u = f_0(r)$ given by Eq. 11), we integrate $f_0(r)$ on a disk of radius R_e . We denote $\mu_f = \mu_{f_0}$ and $\sigma_f = \sigma_{f_0}$.

As MS are uniformly distributed over the disk of radius R_e , the probability density function (pdf) of r is: $p_r(t) = \frac{2t}{R_e^2}$. We now can show that:

$$\begin{aligned} \mu_{f_0} &= \frac{2\pi\rho_{BS}}{\eta-2} \int_0^{R_e} t^\eta (2R_c - t)^{2-\eta} \frac{2t}{R_e^2} dt \\ &= \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta^2-4} \left(\frac{R_e}{R_c} \right)^\eta \\ &\quad \times {}_2F_1(\eta-2, \eta+2, \eta+3, R_e/2R_c), \end{aligned} \quad (17)$$

where ${}_2F_1(a, b, c, z)$ is the hypergeometric function, whose integral form is given by:

$${}_2F_1(a, b, c, z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a} dt,$$

and Γ is the gamma function. Note that for $\eta = 3$, we have the simple closed formula:

$$\mu_{f_0} = -2\pi\rho_{BS}R_c^2 \left(\frac{\ln(1-\nu/2)}{\nu^2} + \frac{16}{\nu} + 4 + \frac{4\nu}{3} + \frac{\nu^2}{2} \right),$$

where $\nu = R_e/R_c$. In the same way, the variance of $f_0(r)$ is given by:

$$\sigma_{f_0}^2 = E[f_0^2] - \mu_{f_0}^2 \quad (18)$$

$$\begin{aligned} E[f_0^2] &= \frac{2^{4-2\eta}(2\pi\rho_{BS}R_c^2)^2}{(\eta+1)(\eta-2)^2} \left(\frac{R_e}{R_c} \right)^{2\eta} \\ &\quad \times {}_2F_1(2\eta-4, 2\eta+2, 2\eta+3, \frac{R_e}{2R_c}). \end{aligned} \quad (19)$$

4) *Sectored Cells*: The coverage area is now $\pi R_e^2/3$. As MS are uniformly distributed over the equivalent sector, we can write from (14). We denote $\mu_f = \mu_{f_{sec}}$ and $\sigma_f = \sigma_{f_{sec}}$:

$$\begin{aligned} \mu_{f_{sec}} &= \frac{3}{\pi R_e^2} \int_0^{R_e} \int_{-\pi/3}^{\pi/3} (a(\theta)f_0(r) + b(\theta))rdrd\theta \\ &= \frac{3}{\pi R_e^2} \int_{-\pi/3}^{\pi/3} a(\theta) \int_0^{R_e} f_0(r)rdr + \frac{3}{2\pi} \int_{-\pi/3}^{\pi/3} b(\theta)d\theta \\ &= \frac{3}{2\pi} \mu_{f_0} \int_{-\pi/3}^{\pi/3} a(\theta)d\theta + \frac{3}{2\pi} \int_{-\pi/3}^{\pi/3} b(\theta)d\theta, \end{aligned} \quad (20)$$

For any function $u(\theta) \in \{a^2(\theta), b^2(\theta), a(\theta)b(\theta)\}$:

$$E[u(\theta)] = \frac{3}{2\pi} \int_{-\pi/3}^{\pi/3} u(\theta)d\theta. \quad (21)$$

We denote $E[u(\theta)] = E[u]$ so we have

$$\mu_{f_{sec}} = E[a] \mu_{f_0} + E[b]. \quad (22)$$

In the same way, the variance of $f_{sec}(r, \theta)$ is given by:

$$\sigma_{f_{sec}}^2 = E[f_{sec}^2] - \mu_{f_{sec}}^2 \quad (23)$$

$$\begin{aligned} E[f_{sec}^2] &= E[a^2] E[f_0^2] + E[b^2] + 2E[ab] E[f_0] \\ &= E[a^2] E[f_0^2] + E[b^2] + 2E[ab] \mu_{f_0}. \end{aligned} \quad (24)$$

IV. CELL BREATHING

During the dimensioning process, the cell radius is determined by taking into account a given value of outage probability. Indeed, the provider defines the maximum outage probability it accepts for his network, and then he calculates the number of base stations needed to cover a given zone. Considering a maximum value of the outage probability, we determine the decrease of cell radius when the load of the cell increases (i.e. the number of mobiles increases).

A. Cell breathing characterization

Let consider a maximum value of outage probability $A = P_{out}^{(n)}$. From Eq. 16, we can write:

$$Q^{-1}(A) = \frac{\frac{1-\varphi}{\beta} - n\mu_f - n\alpha}{\sqrt{n}\sigma_f}. \quad (25)$$

Denoting $a = \frac{1-\varphi}{\beta}$, that equation can be expressed as:

$$(\alpha + \mu_f)^2 n^2 - (2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(A)^2) n + a^2 = 0. \quad (26)$$

As ρ is the mobile density, we can write $n = \rho\pi R_e^2$ (omnidirectional cell) $n_{sec} = \frac{1}{q}\rho\pi R_e^2$ (q-sectored cell), $q=3$ (resp 1) for tri-sectored cell (resp. omnidirectional cell), where n (resp. n_{sec}) is the maximum number of mobiles in a cell for maximum outage probability A for omni-directional cells (resp. sectored cells). We now obtain the following equation:

$$\begin{aligned} (\alpha + \mu_f)^2 \frac{(\pi R_e^2)^2}{q^2} \rho^2 \\ - (2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(A)^2) \frac{\pi R_e^2}{q} \rho^2 + a^2 = 0 \end{aligned} \quad (27)$$

This equation has two solutions. The maximum mobile density can be expressed as:

$$\begin{aligned} \rho = & \left[(2a(\alpha + \mu_f) + \sigma_f^2 Q^{-1}(A)^2) \right. \\ & \left. + \sqrt{\sigma_f^2 Q^{-1}(A)^2 (\sigma_f^2 Q^{-1}(A)^2 + 4a(\alpha + \mu_f))} \right] \\ & \times \frac{1}{2(\alpha + \mu_f)^2} \frac{q}{\pi R_e^2} \end{aligned} \quad (28)$$

Since μ_f and σ_f depend on the cell radius R_e , this equation allows us to determine the zone covered by a BS when the number of mobiles increases.

B. Effect of cell breathing and sectorization

Numerical values in Figure 6 shows the kind of results we are able to obtain instantaneously thanks to the simple formulas derived in this paper for voice service ($\gamma_u^* = -16$ dB), $\varphi = 0.2$, $\alpha = 0.7$, $\eta = 3$ and $R_e = 1$ Km in a CDMA network. Antenna gain pattern is $G(\theta) = -\min\left[12\left[\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_m\right]\right]$, with $A_m = 20$ [19], where θ_{3dB} is the angle for which the received power is divided by 2 and characterizes the beam width.

Figure 6 shows the mobile density as a function of the coverage range of base stations for both omni-directional and sectored sites, and, in the latter case, for different antenna beam width (θ_{3dB}). In this figure, the BS density is supposed to be constant, i.e., the inter-BS distance $2R_c$ is constant. The curves show that the coverage range shrinks when the traffic (characterized here by the density of mobiles ρ) increases.

We observe that for omni directional cells (dotted line without marks), the maximum density of mobiles is 10 mobiles/Km² for a coverage range of $R_e = 1$ Km, and when the density is 20 mobiles/Km² the coverage range reaches only 0.8 Km. As a consequence since the inter-BS distance is constant ($= 2R_c$), the zone is not completely covered.

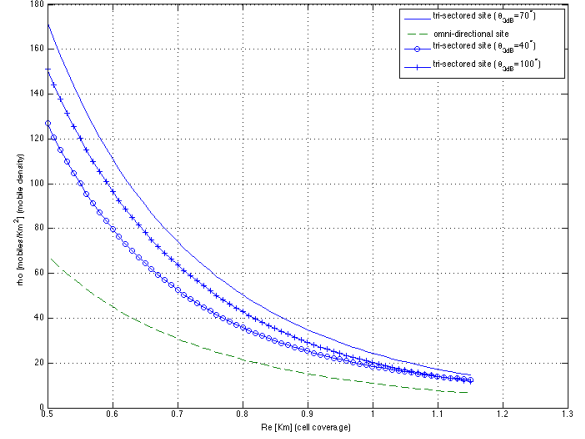


Fig. 6. Effect of cell breathing and sectorization.

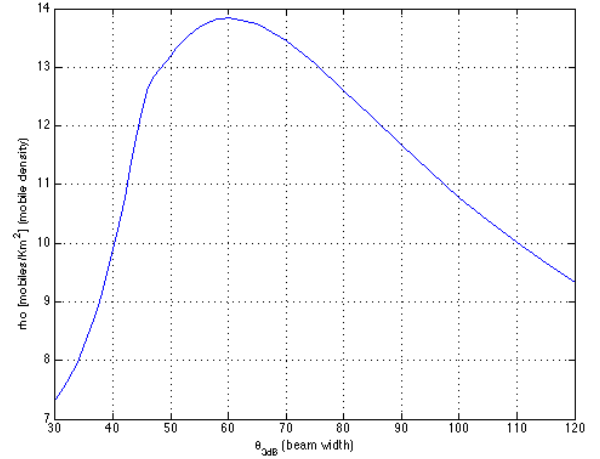


Fig. 7. Optimal beam width.

Due to cell breathing, coverage holes appear. Though the cell breathing also reduces the zone covered by sectored networks (three top curves), its impact is less important. Indeed, we observe for example that to reach 40 mobiles per Km², the coverage range decreases until 0.62 Km with omni-directional cells, and only until 0.86 Km with sector cell $\theta_{3dB} = 70^\circ$.

These curves also provide the maximum mobile density for a given coverage range. As an example, in a sectored network with $\theta_{3dB} = 70^\circ$, a coverage range of 1 Km is feasible if the mobile density does not go beyond 20 mobile per Km².

C. Optimal beamwidth

Figure 7 shows the maximum mobile density ρ as a function of the antenna beam width in a sectored network, when $R_e = 1$ Km. It is clear that a narrow beam width does not provide sufficient useful signal to mobiles of the cell. On the contrary, a broad beam width creates a lot of interference on other sectors and cells. Our analytical study shows that the optimal beam width is 60° .

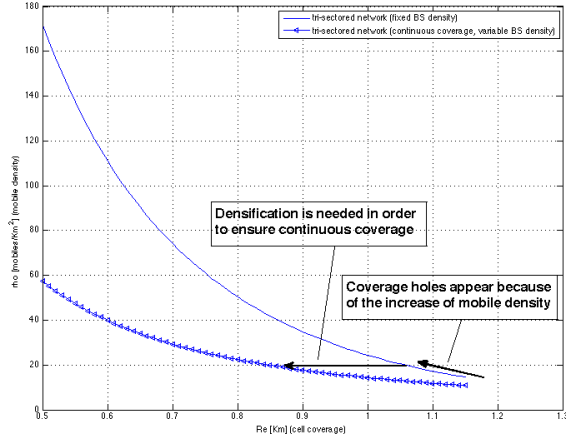


Fig. 8. Base station densification in a sectored network.

D. Base station densification

Figure 8 shows the effect of densification in a sectored network. The solid line without marks is again the mobile density as a function of the coverage range R_e when BS density is constant. The solid line with marks shows the mobile density as a function of the coverage range while assuming a continuous coverage, i.e., at each point the ratio R_e/R_c is constant. When the coverage range decreases, the inter-BS distance $2R_c$ decreases and the BS density thus increases.

Let us assume that a network has been dimensionned in order to have a full coverage for a mobile density of 17 mobiles per Km^2 (the coverage range is thus here equal to 1.15 Km, see the right extremity of the curve). When the mobile density increases, the coverage range decreases along the solid line without marks. As the inter-BS distance is constant, coverage holes appear. If the operator wants to have again a full coverage (for example here when coverage range is 1.05 Km), he has to densify the network and thus jump on the solid lines with marks. This latter curve gives him the new coverage range (here 0.87 Km) and thus the new inter-BS distance to reach thanks to densification.

V. CONCLUSION

In this paper, we established a closed form formula of the other-cell interference factor f for omni-directional and sectored cellular networks, as a function of the mobile location. It allowed a spatial integration of f leading to closed-form formula for the global outage probability on the downlink. That formula is based on a fluid model of wireless networks which considers transmitting nodes as a continuum. From a maximum outage probability, we derive a relation between cell coverage and mobile density. Numerical results show how an operator can combat coverage holes induced by cell breathing by sectorizing omni-directional sites and/or by densifying the base-stations. From our analysis, the optimal beam width for sectored cell can also be obtained.

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