Routing and Wavelength Assignment in optical networks by independent sets in conflict graphs

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1 Introduction

We consider two problems related to the *routing and wavelength assignment problem* (RWA) in *wavelength division multiplexing (WDM) optical networks.* For a given network topology, represented by an undirected graph $\mathcal{G}$, the RWA problem consists in establishing a set of traffic demands (or connection requests) in this network. Traffic demands may be of three types: static (permanent and known in advance), scheduled (requested for a given period of time) and dynamic (unexpected). In this communication, we deal with the case of *scheduled lightpaths demands* (SLDs), which is relevant because of the predictable and periodic nature of the traffic load in real transport networks (more intense during working hours, see [4]).

An SLD can be represented by a quadruplet $s = (x, y, \alpha, \beta)$, where x and y are some vertices of $\mathcal{G}$ (source and destination nodes of the connection request), and where α and β denote the set-up and tear-down dates of the demand. The routing of $s = (x, y, \alpha, \beta)$ consists in setting up a lightpath between x and y, i.e. a path between x and y in $\mathcal{G}$ and a wavelength w. In order to satisfy the SLD s, this lightpath must be reserved during all the span of $[\alpha, \beta]$.

The same wavelength must be used on all the links travelled by a lightpath (*wavelength continuity constraint*). Moreover, at any given time, a wavelength can be used at most once on a given link; in other words, if two demands overlap in time, they can be assigned the same wavelength if and only if their routing paths are disjoint in edges (*wavelength clash constraint*).

The two problems that we consider here are the following:

- minimize the number of wavelengths necessary to satisfy all the demands;
- given a number of wavelengths, maximize the number of connection demands that can be satisfied with this number of wavelengths.

These problems are NP-hard (see [3]), and have been extensively studied (see, among others, [1], [2], [4], [5] and the reference therein). In both problems, a solution is defined by specifying, for each SLD, the lightpath chosen for supporting the connection (i.e. a path and a wavelength), so that there is no conflict between any two lightpaths (let us recall that two lightpaths are in conflict if they use the same wavelength, they have at least one edge in common and the corresponding demands overlap in time). To solve these problems, we design a modelisation of the problem as the search of successive independent sets (IS) in some conflict graphs. Then we apply a descent heuristic improved by a post-optimization method.

2 Independent sets in conflict graphs

To solve these problems, we build a *conflict graph* $\mathcal{H}$ defined as follows. For each SLD $s = (x, y, \alpha, \beta)$, we compute a given number k of paths between x and y in $\mathcal{G}$ (for instance, $k = 5$): $C_s^1, C_s^2, ..., C_s^k$. We associate a vertex of $\mathcal{H}$ with each path C_s^i for each SLD s. Thus, if δ denotes the number of SLDs, the number of vertices of $\mathcal{H}$ is equal to $k\delta$. The edges of $\mathcal{H}$ are of two types:

- for each SLD s and for $1 \leq i < j \leq k$, all the edges $\{C_s^i, C_s^j\}$ are in $\mathcal{H}$; thus these edges induce, for each SLD s, a clique (i.e., a complete graph) on the vertices $C_s^1, C_s^2, ..., C_s^k$;
- for any SLD $s = (x, y, \alpha, \beta)$ and any other SLD $s' = (z, t, \gamma, \epsilon)$, we add the edges $\{C_s^i, C_{s'}^j\}$ for $1 \leq i \leq k$ and $1 \leq j \leq k$ if the time windows of s and s' overlap ($[\alpha, \beta] \cap [\gamma, \epsilon) \neq \emptyset$) and if the paths C_s^i and $C_{s'}^j$ are not edge-disjoint; such an edge $\{C_s^i, C_{s'}^j\}$ represents a conflict between s and s': it is not possible to assign a same wavelength to s and s' if we decide to route s thanks to C_s^i and s' thanks to $C_{s'}^j$.

Our algorithm consists in applying the following two steps successively:

- compute an independent set I in $\mathcal{H}$;
- remove from $\mathcal{H}$ all the cliques associated with the satisfied SLDs to obtain a new current conflict graph $\mathcal{H}$.

We perform this process in order to obtain a series of ISs I_1, I_2, ..., I_q in successive conflict graphs. This will provide a solution to our problem. Indeed, if the vertex C_s^i belongs to I_j, then we route s thanks to the path C_s^i with the j-th wavelength. Each I_j allows us to route $|I_j|$ SLDs with a same wavelength. We stop when all the SLDs are satisfied (first problem) or when q is equal to the prescribed number of wavelengths (second problem).

3 The heuristic to compute an independent set

To compute an IS in the current conflict graph $\mathcal{H}$, we apply an *iterative improvement method*, also called *descent* (we tried more sophisticated methods as simulated annealing, but these methods were too long to obtain interesting results). We start from an IS of cardinality 1, and we look for another IS of cardinality 2, 3, and so on, until reaching a value λ for which we do not succeed in finding an IS of cardinality λ. Then the method returns the last IS of cardinality $\lambda - 1$ as a solution.

To look for an IS I_λ of cardinality λ from an IS $I_{\lambda-1}$ of cardinality $\lambda - 1$, we add a random vertex to $I_{\lambda-1}$. Usually, we thus obtain a set I_λ inducing a subgraph containing some edges. Then we try to minimize the number of edges by performing elementary (or local) transformations, in order to find a set I_λ which will be an IS. It is for this minimization that we apply a descent.

The elementary transformation that we adopt consists in removing a vertex belonging to I_λ and simultaneously to add another vertex which does not belong to I_λ. Such a transformation is indeed accepted if the number of edges decreases.

When the descent stops, if the set I_λ still induces a subgraph which is not an IS, then we stop and we keep the previous IS $I_{\lambda-1}$ as the solution. Otherwise, we add a vertex and we apply the same process once again.

In fact, we improve this method in two manners. The first one consists, after the construction of each IS I, in trying to add extra SLDs with a greedy algorithm. For this, we consider each unsatisfied SLD s, and we look for a path in $\mathcal{G}$ that would allow us to route s with the current wavelength, i.e. a path which would not contain any edge of paths associated with another SLD s' routed with the same wavelength and of which the time window overlaps the one of s. Such a situation may occur since we limit ourselves to k paths in the construction of $\mathcal{H}$, while we look for a path in $\mathcal{G}$ to add extra SLDs.

The other improvement consists in applying a post-optimization method (already applied in [1]), after the computation of the series of ISs I_1, I_2, ..., I_q.

The aim is to reduce the overall values of the wavelengths in order to decrease the total number of wavelengths for the first problem or to make some place to extra SLDs, still unsatisfied, for the second problem. For this, given a wavelength w, we try to empty, at least partially, the set of SLDs routed with w, by assigning them lower wavelengths. So we change the wavelengths assigned to SLDs which are currently routed with the wavelenths $1, 2, ..., w-1$; in this process, all the SLDs with a current wavelength between $1, 2, ..., w-1$ will keep a wavelength in this interval. For the first problem, it may then happen that a wavelength becomes useless; then we remove it definitively and so the number of required wavelengths decreases. For the second problem, the changes involved in the assignments of the wavelenghts are such that, sometimes, we may route an SLD which was unsatisfied; so this process allows us to route extra SLDs.

4 Results

We experimentally study the impact of the formulation of the problem as the search of successive ISs in conflict graphs as well as the impact of the post-optimization method. The experiments are done on several networks, with numbers of SLDs up to 3000. The results, not detailed here, show that these two approaches are quite beneficial for both problems, when their results are compared to the one of the method developped in [5].

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