Multichannel phase unwrapping with Graph-cuts

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***Abstract*—Markovian approaches have proven to be effective for solving the multichannel phase unwrapping problem, especially when dealing with noisy data and big discontinuities. This paper presents a markovian approach to solve the phase unwrapping problem based on a new *a priori* model, the Total Variation, and graph-cut based optimization algorithms. The proposed method turns to be fast, simple and robust. Moreover, compared to other approaches, the proposed algorithm is able to unwrap and restore the solution in the same time, without any additional filtering. A set of experimental results on both simulated and real data illustrate the effectiveness of our approach.**

I. Introduction

In interferometric synthetic aperture radar systems, estimation of the phase is a crucial point since there exists a known relation between InSAR phase and height values of the ground [1]. It is known that the measured phase (*wrapped phase*) is given in the principal interval $[-\pi, \pi]$, so phase unwrapping (*PU*) problem has to be solved providing the absolute phase (*unwrapped phase*). This problem is known to be an ill-posed problem if the so-called *Itoh* condition (the absolute value of phase difference between neighboring pixels is less than π) is not satisfied [2]. Usually, in InSAR systems, *Itoh* condition is violated due to the presence of discontinuities and/or interferometric noise.

One of the methods proposed recently to solve the *PU* problem in case of *non-Itoh* condition is the multichannel Maximum a posteriori (MAP) estimation method described in [3]. In this approach, the *a priori* statistical term is modeled by an inhomogeneous Gaussian Markov Random Field (*GMRF*), i.e. *GMRF* with local hyperparameters [4]. The effectiveness of this method has been proved, even in presence of discontinuities, high sloped areas and low coherence areas. Anyway, it suffers of some limits in particular concerning the computational time and the optimization step (no guaranty of finding the global optimum).

In this work, we propose to improve this approach through a new *a priori* Total Variation (*TV*) based model and using energy optimization algorithms based on graph-cut theory. This new approach presented in this paper, gives similar solutions to the work of [3] in $1/10^{th}$ of the computation time. Furthermore, the global optimum for the considered energy function can be provided. The proposed method is validated both on simulated and real data, showing its effectiveness.

It is important to mention the work [5] that proposed graph-cut based optimization algorithms to solve the *PU* problem and [6] which is based on network programming optimization technique. We underline that these two approaches belong to minimum L^p norm unwrapping methods. Therefore, they do not optimally exploit statistical properties of the noise present on the data and they are not optimal from the information theoretical point of view. Moreover, differently from these approaches, we propose an algorithm that is able to unwrap and restore the solution at the same time.

In the next section, we introduce the multichannel phase unwrapping (*MCPU*) technique with an inhomogeneous *GMRF* model. In section III, we present our new approach based on *TV* model and graph-cut optimization algorithms. Finally, we present some results showing the reconstruction obtained on simulated and real data.

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II. Multichannel phase unwrapping

Multichannel phase unwrapping approach consists in combining two or more independent interferograms. These interferograms can be obtained in two different ways, multifrequency and multibaseline configurations [7]. The interferometric phase signals can be modeled as:

$$\phi_{p,c} = <\alpha_c h_p + w_{p,c}>_{2\pi};\ p \in \{1,...,N\}; c \in \{1,...,M\} \tag{1}$$

with $\alpha_c = \frac{4\pi B_c^{\perp}}{\lambda R_0 sin(\theta)}$ in case of multibaseline situation and $\alpha_c = \frac{4\pi B^{\perp}}{\lambda_c R_0 sin(\theta)}$ in case of multifrequency situation. The index p refers to the pixel position inside the image of size N, index c to the considered channel which is one of the M possible interferograms (frequencies or baselines), w is the phase decorrelation noise, $< . >_{2\pi}$ represents the $modulo{-}2\pi$ operation, λ is the wavelength, $B^{\perp}$ is the orthogonal baseline of the c^{th} SAR interferogram, θ is the SAR view angle and R_0 is the distance of the first antenna to the center of the scene. Defining these notations, the height reconstruction problem consists in estimating the height values h_p of the whole scene, using the $N \times M$ measured available wrapped phases $\phi_{p,c}$.
In the case of M statistically independent channels, the multichannel likelihood function is given by [3]:

$$F(\Phi|\mathbf{h}) = \prod_{p=1}^{N}\prod_{c=1}^{M} f(\phi_{p,c}|h_p;\alpha_c,\gamma_{p,c}) \tag{2}$$

where:

$$f(\phi_{p,c}|h_p;\alpha_c,\gamma_{p,c}) = \frac{1}{2\pi}\frac{1-|\gamma_{p,c}|}{1-d^2}\left(1+\frac{dcos^{-1}(-d)}{(1-d^2)^{1/2}}\right) \tag{3}$$
$$d = |\gamma_{p,c}|\cos(\phi_{p,c} - \alpha_c h_p)$$

is the single-channel likelihood function and $\gamma_{p,c}$ is the coherence coefficient that depends on pixel p and on channel c. In (2) $\mathbf{\Phi} = [\mathbf{\Phi}_0^T \mathbf{\Phi}_1^T ... \mathbf{\Phi}_N^T]^T$ is the vector collecting all available wrapped phase values, $\mathbf{\Phi}_p = [\phi_{p,0}\phi_{p,1}...\phi_{p,M}]^T$ is the vector of the wrapped phases measured in pixels p for the M different channels and $\mathbf{h} = [h_0 h_1 ... h_N]^T$ is the vector containing the ground elevation values. Following the Bayes law, the Multichannel *MAP* estimation solution is given by:

$$\hat{\mathbf{h}} = argmax_{\mathbf{h}} F(\Phi|\mathbf{h}) g_\beta(\mathbf{h}) \tag{4}$$

where the function $g(.)$ is the prior probability of $\mathbf{h}$. A *MRF* is used to model $\mathbf{h}$, whose expression is given in this case by:

$$g_\beta(\mathbf{h}) = \frac{1}{Z(\beta)} e^{-E_{prior_\beta}(\mathbf{h})} \tag{5}$$

where $Z(\beta)$ is a normalization factor called the partition function, $E_{prior}(.)$ is the so-called energy function expressing relationship between pixels and $\beta = [\beta_0 \beta_1 ... \beta_N]$ is the hyperparameter vector [8] that will be discussed later. The energy function is defined in such a way to impose some constraint on the neighboring pixels.

Different prior energy functions can be used to model our problem. An effective model is the one proposed in [3], where, the energy E_{prior} is modeled by a local *GMRF* [9]:

$$E_{prior_\beta}(\mathbf{h}) = \sum_{p \sim q} \frac{(h_p - h_q)^2}{2\beta_{p,q}^2} \tag{6}$$

$p \sim q$ denotes that pixels p and q are neighbors within the neighborhood system $\mathcal{N}$ of the *MRF* model. The hyperparameter vector β represents in this case the local spatial variations of the unwrapped heights.

According to this approach, *MAP* estimation solution (4) needs a previous estimation of the local hyperparameters β which is performed using the Expectation Maximization algorithm (*EM*) [8]. Then, optimization step is carried out using a *semi-deterministic* solution. The *ICM* (Iterated Conditional Modes) [8] algorithm is used for this purpose and it is initialized with high-probability samples of the image generated in the hyperparameter estimation. This algorithm, although faster than simulated annealing, could not provide, in some cases, a global optimum.

III. Total Variation model and graph-cut optimization

In this section, we introduce a new, simple and effective energy function model, which combined with graph-cut based optimization algorithms allows to develop a fast and robust *MCPU* approach.

A. Total Variation based model

GMRF model used in [3] is a model well adapted to any kind of profile (smooth, non-smooth and big discontinuities) thanks to its local behavior and it has been proven that this model combined with the *ICM* optimization step gives good results. However, this approach suffers of some limits. First, it needs of estimating a parameter $\beta_{p,q}$ for each couple of pixels p, q which is computational heavy and excessively time demanding. Secondly, the *semi-deterministic* minimization approach doesn't guarantee, in case of very noisy and wrapped interferograms (energy function with many local minima), to reach the global minimum since the *ICM* is known to be sensitive to local minima. Therefore, we propose a new model for the prior energy, *TV* model, that leads to a faster phase unwrapping algorithm. This new approach is a trade off between the computation complexity, optimum quality and prior model adaptation.

The *TV* model introduced in 1992 [10] is one of the most used prior model in image processing due to its adequation to different contextual information. In *SAR* applications, *TV* model is mainly used for image restoration [11]. Our proposal is to apply it to the *PU* problem. The prior energy corresponding to the discretization of *TV* [12] can be written as follows:

$$E_{prior} = \beta \sum_{p \sim q} w_{p,q} |h_p - h_q| \tag{7}$$

where $w_{p,q}$ depends on neighbor connexity (1 for $4 - connexity$ and $\frac{1}{\sqrt{2}}$ for the 4 diagonal ones in the case of $8 - connexity$). Note that in this expression, β is a scalar and not a vector of hyperparameters as in the *GMRF* model. This makes the *TV* model a non-local model. The choice of a non-local prior energy is done in order to have a simplified model and a faster algorithm since it avoids the estimation of a vector of local hyperparameters. This choice is not as powerfull as the local one proposed in [3], since it is not local. However, between the existing non-local prior energy models, *TV* has been chosen because its main advantage, is that it does not penalize discontinuities in the image while simultaneously not penalizing smooth functions either [10]. As it is well adapted when dealing with strong discontinuities, it can be used in case of *InSAR* applications and particularly it well fits to urban areas.

We propose in the next section a fast *MCPU* algorithm based on graph-cut optimization method and TV prior.

B. Graph-cut based optimization

In the recent years, energy optimization with graph-cut has become very popular in computer vision [13]. Graph-cut optimization is successful because the exact minimum or an approximate minimum with certain guaranties of quality can be found in a polynomial time based on minimum-cut/maximum flow algorithms [14]. Compared to the classical optimization algorithm, *Simulated Annealing* [4], it provides comparable results with much less computational time and compared to the deterministic algorithm *ICM* [8], it avoids the risk of being trapped in local minima solution which can be far from the global one.

Two families of graph-cut based optimization algorithms have been developed in the recent years. The first one provides the global optimum of the Markovian energy with some constraints on the prior model [15], [12]. The second one provides a local optimum within a good quality and in more general cases of prior energies [13].

To solve the *MCPU* problem, we use two different graph-cut based algorithms for the optimization scheme. The first one is the one proposed in [15], that belongs to the exact optimization algorithm family. The second one is a non-exact optimization algorithm based on α-expansion move [13]. For the rest of the paper, we define a set of labels $\mathcal{L}$ which represents the discretized heights that have to be estimated.

1) Exact optimization algorithm: The exact optimization algorithm proposed by Ishikawa is based on two hypothesis, a linear order on the label set and convexity of the *a priori* energy function. The latter is well satisfied in our approach since the TV function is convex. A particular graph is constructed for which we compute the minimum cut. This graph $\mathcal{G}$ contains $N \times K$ nodes (N is the size of the image and K is the size of the label set) plus two special nodes s and t. For each pixel p, we associate K nodes that represent all the possible labels that the pixel p can take. Besides, $\mathcal{G}$ contains three families of edges: the data edges that represent the multichannel likelihood energy terms; the constraint edges, that ensure only one label is affected to a pixel p; the interaction edges between all neighbor pixels. The general capacity expressions associated to these edges are given in [15]. This graph construction ensures that the minimum cut on the graph provides the optimal configuration for our *MCPU* problem.

2) Non-exact optimization algorithm: α-expansion algorithm has been proposed by Boykov et al. [13]. This approximate optimization approach is iterative and based on the concept of α-expansion move. The latter, consists on changing a current configuration f by proposing to any set of image pixels to change their labels to α. It finds a new configuration $\hat{f}$ that minimizes the energy E over all labelings f' within this move. Thus, in each iteration of the algorithm, a particular graph $\mathcal{G}_\alpha$ is constructed where the minimum cut is computed. The structure of the graph is determined by the current labeling f and the label α. This graph contains N nodes plus two special nodes s and t. The set of edges contains two families of edges. Data edges that are related to the multichannel likelihood function and interaction edges that are related to the *a priori* function. Graph construction could be seen as a one level of Ishikawa graph [16]. All nodes are connected to the source and the sink weighted by the data function and all neighbour nodes are connected to each others and weighted by the prior function. To be graph representable, the *a priori* function needs to be a metric [13]. It is easy to prove that TV function satisfies this constraint. At convergence of the algorithm, it is proved in [13] that the expansion move algorithm produces a labeling f such that:

$$E(f^*) \leq E(f) \leq 2kE(f^*) \tag{8}$$

where f^* is the global minimum of E and k is a constant.

IV. Experimental results

In order to prove the effectiveness of the proposed method, we present some results obtained both on simulated and real data. The results presented were obtained with MATLAB coding (max-flow algorithm is implemented in C++ [1], using a PC Core 2 duo 2.66 Ghz with 2G memory). In all the presented cases, to perform automatic regularization parameter estimation β, we used the method based on the $L-curve$, in particular, the triangular method described in [17].

[1]by V. Kolmogorov, http://www.cs.cornell.edu/People/vnk/software.html

A. Simulated data

In this section, we will perform three different experiments. In the first one, we will show the effectiveness of the proposed approach concerning both the model and the optimization algorithms based on graph-cuts. In the second one, we analyze which kind of optimization algorithm (exact or non-exact) has to be used for different phases that have to be unwrapped. Finally a comparison with another multichannel PU method is presented.

In the first experiment, we consider a height profile (64×64 pixels) with a maximum height of $140m$ exhibiting both smooth and discontinuous areas Fig.1(a). We used two frequencies ($5GHz$ and $9GHz$) to generate interferograms and we added interferometric noise with a coherence of $\gamma = 0.5$. In Fig.1(b), we show the $5GHz$ noisy interferogram. Moreover, for each working frequency we generated 4 azimuth looks allowing to generate a total of $M = 8$ independent interferograms. It is important to note that the profile is ambiguous for both the working frequencies. In fact, there are height jumps of 150 m, corresponding to phase jumps of about 1.33π at $5GHz$ and 2.4π at $9GHz$, which violate the *Itoh* condition [2]. For this reason, a classical single frequency phase unwrapping method would fail. The multichannel approach can overcome this problem. In figures 1(c) 1(d) we show the results

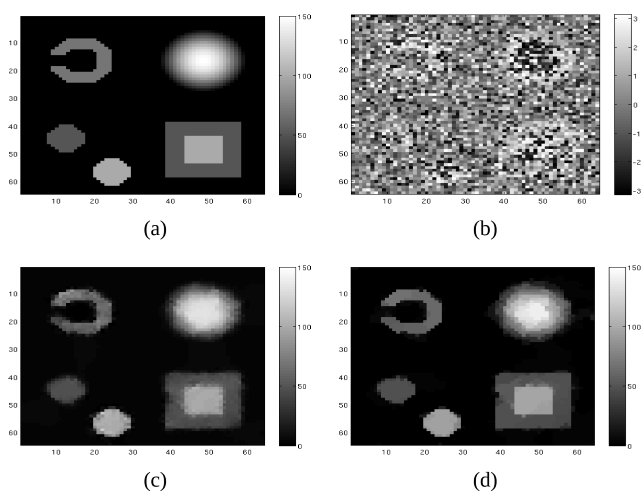

Fig. 1. Comparison between results of *GMRF* prior model and *TV* prior model. (a) Original profile, (b) noisy interferogram, reconstruction with (c) the multichannel approach of [3], (d) the proposed multichannel approach.

obtained respectively with the multichannel approach of [3] and with the new proposed algorithm. It can be seen that both reconstruction profiles appear very similar to the reference (Fig. 1(a)). Note that the proposed approach works very well when the profile is flat or when there is a discontinuity, while the reconstruction quality decreases, compared to the one provided by the approach of [3], when we face a smooth but not flat profile, such as the gaussian profile in the top right corner. The normalized reconstruction square error defined by $e_{\mathbf{h}} = \frac{\|\hat{\mathbf{h}}-\mathbf{h}\|^2}{\|\mathbf{h}\|^2}$, where $\hat{\mathbf{h}}$ is the estimated profile and $\mathbf{h}$ is the reference profile is equal to 2.05×10^{-2} in the first case and 1.93×10^{-2} in the second. It is very interesting to see, looking at the square object at the right bottom of Fig. 1(d), how the *TV* model is well adapted for this kind of profile which simulates a typical urban building as we will see next in real data experiments.

Although reconstruction results are mainly the same using the previous and new approach, their performances in term of execution time are not comparable. In fact, while the new algorithm takes less than one minute, the classical one is 10 times slower due to the local hyperparameter estimation. Another point has to be noted. Since the optimization algorithm based on graph-cut used in this experiment is Ishikawa algorithm, we are sure to obtain the exact solution. Whereas, using *ICM* in the optimization step as in [3] we still risk of being trapped in local minima solution. Note that in this case, due to the energy function used in this experiment, also α-expansion algorithm provided the global solution.

In the second experiment, we compare the two graph-cut based optimization algorithms, Ishikawa and α-expansion. The analysis will be based on two characteristics of the wrapped interferogram, size of interferogram and the phase rate. First, we fix the phase rate and we consider different interferogram sizes. We use the reference profile of the first experiment Fig. 1(a) with the same system parameters (coherence, frequencies and channels) and a fixed value of β. Results given by the two optimization algorithms are the same (global solution is provided). The computational times are shown in table I. We note two important aspects. First, we can see that α-expansion is faster than Ishikawa algorithm in all the considered cases. Secondly, there exist a threshold after which Ishikawa algorithm becomes extremely time and memory consuming. In fact, α-expansion based optimization algorithm is computed in polynomial time, since the graph constructed is linear with respect to the number of pixels, $O(N)$, and algorithm used for the maximum flow computation is polynomial. However, for Ishikawa algorithm, although maximum flow can be computed in polynomial time, algorithm has only a pseudo-polynomial time complexity, since the number of nodes grows linearly with respect to the number of labels ($O(N \times L)$ in case of TV prior model).

TABLE I
OPTIMIZATION TIME IN SECONDS FOR α-EXPANSION AND ISHIKAWA ALGORITHMS USING DIFFERENT INTERFEROGRAM SIZES.

Time / Size	64×64	128×128	192×192	256×256
α-expansion (sec)	2	10	30	44
Ishikawa (sec)	24	102	232	350

Then, we fix the size and we consider different interferogram phase rates. In this case, we will base our analysis on the profile of Fig.2(a) (160×160 pixels) which is a Gaussian profile. The maximum height of the profile is $90rad$. The system parameters are the same as for the first experiment. In Fig. 2(b), we show the least ambiguous noisy interferogram. Due to the large phase rate, large enough to produce aliasing, the energy function presents many local minima that are very near to the global one. This property violates the distance condition (8) needed for the α-expansion algorithm to reach the global solution. In this case, Ishikawa is more adapted and provides the best solution for the *PU* problem. Figures

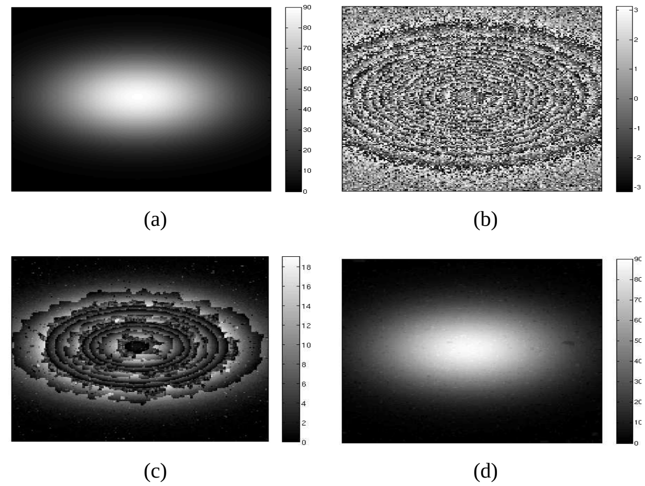

Fig. 2. Comparison between exat and non-exact solutions. (a) Original Gaussian profile, (b) noisy interferogram, reconstruction with (c) α-expansion algorithm, (d) Ishikawa algorithm.

2(c) 2(d) represent respectively the reconstruction obtained using α-expansion and Ishikawa algorithms. Using Ishikawa approach, we obtain a normalized square reconstruction error of 9.4×10^{-4}.

Finally, we compared the proposed approach to another *MCPU* method, the Phase Difference (PD) based algorithm proposed by Fornaro et al [18]. We considered a realistic profile of 458×157 pixels representing the mountains around the Isolation Peak in Colorado (see Fig. 3(a)). Starting from the latter, we simulated $M = 6$ interferograms (different baselines), with a coherence $\gamma = 0.5$. The vector containing the used baseline lenghts, normalized to a reference one, is $[\frac{8}{13}, \frac{11}{13}, 1, \frac{17}{13}, \frac{20}{13}, \frac{23}{13}]$. Note that such data sets can be unwrapped using a single channel interferograms only in case of very high coherence values. Fig. 3(b) shows the least ambiguous noisy interferogram (the one obtained using a baseline ratio equal to $\frac{8}{13}$). The unwrapped results, obtained using the PD algorithm and Ishikawa + TV algorithm are shown respectively in Figures Fig. 3(c) 3(d). The Ishikawa + TV method is able to unwrap and restore the profile in the same time with a better accuracy compared to the PD algorithm. The good restoring capabilities of our approach can be appreciated, considering Figures 3(e) 3(f), where the reconstruction error maps obtained using the two methods are shown. In the case of PD reconstruction, a 5×5 median filter has been applied before calculating the reconstruction error map. This has been done in order to compare our result with the best achievable result by PD algorithm. Even after median filtering, the PD solution is still not as accurate as the one provided by our method. Note that, no additional filtering is required by our algorithm to provide the shown results.

B. Real data

We tested our new proposed algorithm on a real data set of an urban scenario. We used a set of 8 L-Band E-SAR interferograms (2 interferograms for each of the four polarizations) acquired on the city of Dresden. The smallest

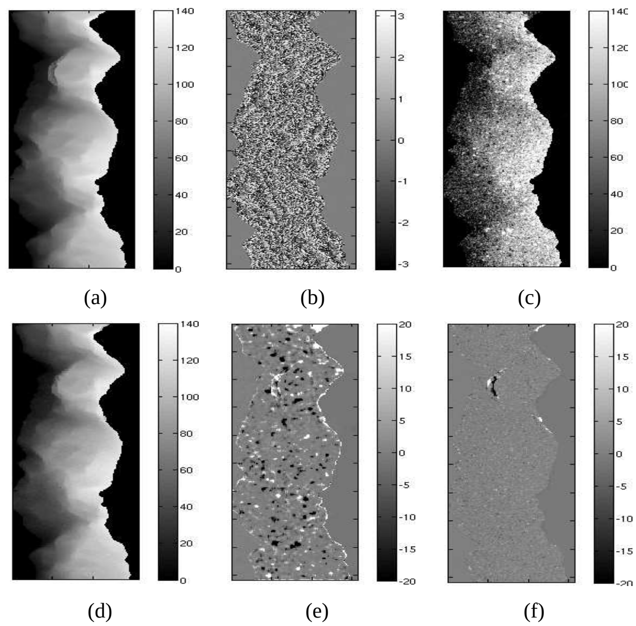

Fig. 3. Comparison between PD algorithm and the proposed approach. (a) Original profile, (b) least ambiguous noisy interferogram, (c) reconstruction using the PD algorithm, (d) reconstruction using the proposed algorithm, (e) reconstruction errors map for the PD algorithm solution, after filtering with a 5×5 median filter, (f) reconstruction errors map using the proposed approach.

orthogonal baseline is of about $8.42m$ and the biggest is of about $28.34m$. We applied our new approach based on the *TV* model and Ishikawa algorithm. Even if the height of the building in the first interferogram is less than the ambiguous height ($h_{amb} \in \{38.5m, 11.5m\}$) Fig.4(a), due to the presence of noise, the Itoh condition is violated in some areas. So, if we apply a classic single channel approach, the solution will show some errors in these areas. Using our approach, it is possible to retrieve the correct height of the building. Note that the height of the roof building is almost constant Fig. 4(c). Moreover, it is possible to note the goodness of the *TV* model since contours of buildings are recovered very efficiently Fig.4(d).

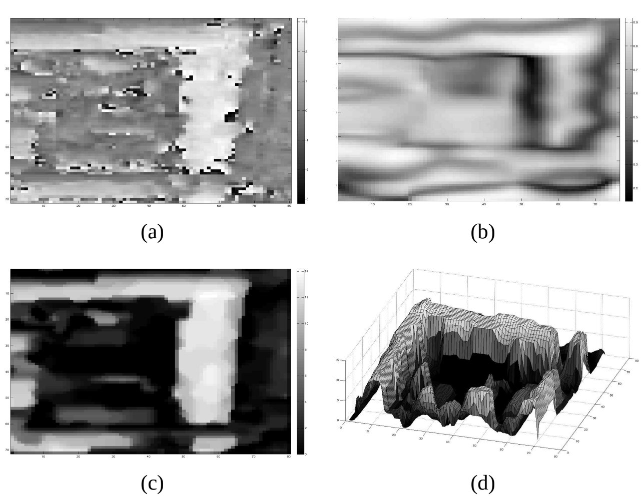

Fig. 4. 3D reconstruction of real data. (a) Noisy interferogram (least ambiguous one), (b) coherence map of the previous interferogram, (c) reconstruction with the proposed approach, (d) 3D view of the reconstruction.

V. Conclusion

In this paper, we developed a new multichannel phase unwrapping methodology based on TV prior model and graph-cut optimization algorithms. The proposed algorithm overcomes the limits that characterize other *MCPU* approaches. Moreover, with Ishikawa optimization algorithm, we are able to reach the exact energy optimum. We have tested this approach on simulated data and we obtained good results both in term of reconstruction error and computational time. As it has been proved to be effective dealing with high discontinuities, we also tested the proposed method on real *InSAR* urban data. Future work will focus on the implementation of the proposed method using graph cut based algorithms that are both robust and not excessively memory consuming.

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